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PARLIAMENTARY ELECTIONS: IMPACT OF VOTING ABROAD*

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Elections are the cross point of Law and Mathematics

The paper deals with the mathematics of the Bulgarian parliamentary elections. The impact of voting abroad is studied under different hypotheses and ways to overcome the disproportions in the regional apportionment of seats are considered. The history of voting abroad is briefly outlined. This study is partially motivated by the "super cheap" seats for one of the political parties (474 votes in the 2013 elections and 864 votes in the 2014 elections) which led to social and political unrest. An extended and adapted version of this paper shall be presented to the leaders of the political parties as well as to the Speaker of the Parliament.

Bulgarian parliamentary elections. There were 8 parliamentary elections in Bulgaria for Ordinary National Assembly of 240 members of the parliament (MP) during the period 1991–2014. Several variants of proportional electoral systems have been used in these elections (in 2009 the system was mechanically mixed with 31 MP elected in single constituencies). The voting is carried out in 31 regions in the country and abroad. A party takes part in the distribution of seats if it has at least 4% of the valid votes. There are independent candidates as well but no one has been elected so far.

The electoral system acts in two steps. First the total number of seats of each party is determined at nationwide level. This is done by the D'Hondt method (1991, 1994, 1997, 2001, 2005) and the Hare-Niemeyer (HN) method (2009, 2013, 2014). Next the seats of each party are personified by 31 regional party lists, where in each region the number of seats is preassigned proportionally to the population. This has been realized by different bi-proportional algorithms (for such algorithms see also [1, 2]). The present algorithm has been used in 2013 and 2014 and is now a part of the Electoral Code adopted on March 5, 2014. Unfortunately, it contains inaccuracies and may not work in certain cases. A correct version of the algorithm is given in [5] and also in the site `apa.bg`. This correct version is realized as a computer code in MATLAB environment and is used for the calculations presented further on in this paper.

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The mathematical aspects of the Bulgarian parliamentary elections are considered in a number of papers, see e.g. [3, 4, 5] to mention some.

History of voting abroad (VA). In Bulgaria VA takes place in parliamentary, presidential and European elections. The first VA was for electing the 7th Great National Assembly in 1990 but there is no reliable data for this vote. On the contrary, for 8 parliamentary elections (1991, 1994, 1997, 2001, 2005, 2009, 2013 and 2014) there is very detailed data provided by the Central Election Commission (CEC). Such detailed information – polling station by polling station and party list by party list, is available for all elections (including local ones) in Bulgaria since 1991.

VA in parliamentary elections has impact on the seat distribution only since 2001 when it was allowed to vote outside Bulgarian embassies and consulates. The VA for presidential elections (1992, 1996, 2001, 2006, 2011) and European elections (2007, 2009, 2014) had no effect on the electoral results so far.

Suppose that n parties $\Pi_1, \Pi_2, \ldots, \Pi_n$ had past the 4% barrier. Denote by $\mathbf{c} = [c_1, c_2, \ldots, c_n]$ and $\mathbf{a} = [a_1, a_2, \ldots, a_n]$ the vectors of votes cast for these parties in the country and abroad, respectively, and let

$$\mathbf{v} = \mathbf{c} + \mathbf{a} = [v_1, v_2, \ldots, v_n] = [c_1 + a_1, c_2 + a_2, \ldots, c_n + a_n]$$

be the total vector of votes.

Next denote by

$$\mathbf{h} = \mathbf{H}(\mathbf{v}) \text{ and } \mathbf{d} = \mathbf{D}(\mathbf{v}) \quad (\|\mathbf{h}\| = \|\mathbf{d}\| = 240)$$

the vectors of seats produced by the Hare-Niemeyer (HN) and D'Hondt methods, respectively, where $\|\cdot\|$ is the 1-norm. If M seats are to be distributed we denote the result produced by the HN method as $\mathbf{H}(M, \mathbf{v})$. Thus $\mathbf{H}(\mathbf{v}) = \mathbf{H}(240, \mathbf{v})$.

The number of parties passing the 4% barrier is given at Table 1 below.

Table 1. Number of parties passing the 4% barrier

Elections	1991	1994	1997	2001	2005	2009	2013	2014
# of parties	3	5	5	4	7	6	4	8

This gives an average of 5.25 parties in the Bulgarian parliament since 1991.

The impact of VA on the nationwide allocation of party seats is analyzed below.

1991. Only three parties (a record!) past the barrier: 1 – Union of Democratic Forces (UDF), 2 – Bulgarian Socialist Party (BSP) and 3 – Movement for Rights and Freedoms (MRF). The total vote vector was

$$\mathbf{v}^{1991} = [1903567, 1836050, 418168].$$

The VA was negligible and had no effect on the national distribution of seats which, according to the then used D'Hondt method, was

$$\mathbf{D}(\mathbf{v}^{1991}) = \mathbf{D}(\mathbf{c}^{1991}) = [110, 106, 24].$$

The HN method would give the same result, i.e.

$$\mathbf{H}(\mathbf{v}^{1991}) = \mathbf{H}(\mathbf{c}^{1991}) = [110, 106, 24].$$

1994. In these elections VA again had no effect on the distribution of seats which

was based on the total vector

$$\mathbf{v}^{1994} = [2262943, 1260374, 338478, 283094, 245849],$$

where the corresponding parties now were: 1 – BSP, 2 – UDF, 3 – Bulgarian Agricultural National Union-Peoples Union (BANU-PU), 4 – MRF and 5 – Bulgarian Business Block (BBB). The distribution of seats was

$$\mathbf{D}(\mathbf{v}^{1994}) = \mathbf{D}(\mathbf{c}^{1994}) = [125, 69, 18, 15, 13].$$

If the HN method had been used, the distribution would be slightly different:

$$\mathbf{H}(\mathbf{v}^{1994}) = \mathbf{H}(\mathbf{c}^{1994}) = [124, 69, 19, 15, 13].$$

Thus the D'Hondt method 'takes' one seat from the third party and 'gives' it to the first one compared with the distribution by the HN method.

1997. Again 5 parties past the barrier: 1 – UDF (in a slightly augmented format), 2 – BSP, 3 – MRF (in coalition), 4 – Euro Left and 5 – BBB. The vote vectors were

$$\mathbf{v}^{1997} = [2223714, 939308, 323429, 234058, 209796]$$

and

$$\mathbf{a}^{1997} = [15409, 2066, 6362, 839, 522].$$

The VA has no effect on the D'Hondt distributions of seats:

$$\mathbf{D}(\mathbf{v}^{1997}) = \mathbf{D}(\mathbf{c}^{1997}) = [137, 58, 19, 14, 12].$$

If however the HN method had been used, there would be slight differences. First, the distribution according to the total vote would be

$$\mathbf{H}(\mathbf{v}^{1997}) = [136, 57, 20, 14, 13].$$

And second, the VA now affects the result:

$$\mathbf{H}(\mathbf{c}^{1997}) = [136, 58, 19, 14, 13].$$

Thus, for the first time one party (MRF) would win one seat and another party (BSP) would lose one seat due to the VA if the HN method was used.

The conclusion for the 1991, 1994 and 1997 elections is that *VA had no impact on the distribution of seats.* This is due to the fact that in these elections VA took place only in the Bulgarian embassies and consulates and was only a little part of the total vote. Since 2001 the situation has dramatically changed due to the then addopted "Law for electing MP". VA had enlarged its territory and now there are hundreds of polling stations abroad. The principal beneficence from VA is MRF.

2001. There were four parties above the barrier: 1 – National Movement Simeon the Second (NMSS), 2 – UDF, 3 – BSP and 4 – MRF. The vote vectors were

$$\mathbf{v}^{2001} = [1952513, 830338, 783372, 340395],\ \mathbf{a}^{2001} = [85433, 37373, 31901, 78769].$$

The D'Hondt and HN methods produce the same seat distribution

$$\mathbf{D}(\mathbf{v}^{2001}) = \mathbf{H}(\mathbf{v}^{2001}) = [120, 51, 48, 21].$$

Without VA the distribution would be

$$\mathbf{D}(\mathbf{c}^{2001}) = \mathbf{H}(\mathbf{c}^{2001}) = [122, 52, 49, 17].$$

Thus MRF won 4 seats (!), NMSS lost 2 seats and UDF and BSP lost 1 seat as a result of VA.

This was the *first significant impact of VA* in Bulgarian political history. It is due

to the fact that the governing coalition UDF allowed voting not only at embassies and consulates (i.e. on Bulgarian territory) but also at other places, sometimes without due control on behalf of the Bulgarian authorities. It must be stressed that although NMSS had largest result of 85 433 votes abroad it nevertheless loses two seats for both D'Hondt and HN methods. This is an example of the so called *population paradox*.

2005. There were 7 parties passing the barrier: 1 – BSP, 2 – NMSS, 3 – MRF, 4 – "Ataka" and three parts of the former right, namely 5 – United Democratic Forces (UDF), 6 – Democrats for Strong Bulgaria (DSB) and 7 – Union of Free Democrats (UFD). The vote vectors were

$$\mathbf{v}^{2005} = [1129196, 725314, 467400, 296848, 280323, 234788, 189268]$$

and

$$\mathbf{a}^{2005} = [6125, 13432, 40626, 1567, 3547, 5026, 1479].$$

The then used D'Hondt method produced

$$\mathbf{D}(\mathbf{v}^{2005}) = [82, 53, 34, 21, 20, 17, 13].$$

Thus, the famous government formula "3–5–8" took place.

Without VA the result would be

$$\mathbf{D}(\mathbf{c}^{2005}) = [83, 53, 31, 22, 20, 17, 14].$$

Hence, VA gives 3 seats to MRF and takes one seat from BSP, "Ataka" and UFD.

At the same time the HN method would produce slightly different results:

$$\mathbf{H}(\mathbf{v}^{2005}) = [82, 52, 34, 21, 20, 17, 14].$$

Without VA the HN method gives

$$\mathbf{H}(\mathbf{c}^{2005}) = \mathbf{D}(\mathbf{c}^{2005}) = [83, 53, 31, 22, 20, 17, 14]$$

and MRF again wins 3 seats from VA.

This was the last use of the D'Hondt method which had been replaced by the HN method for parliamentary, European and local elections.

2009. There were 6 parties passing the barrier: 1 – Citizens for European Development of Bulgaria (GERB), 2 – BSP, 3 – MRF, 4 – "Ataka", 5 – "Blue Coalition" (formed by former UDF and DSB) and 6 – "Order, Law, Justice". There were also 31 pseudo-plurality regions which strongly contorted the seat distribution. That is why below we present the hypothetical results following only from the proportional part of the vote. Under this hypothesis the vote vectors are

$$\mathbf{v}^{2009} = [1678641, 748147, 610521, 395733, 285662, 174582]$$

and

$$\mathbf{a}^{2009} = [33426, 3882, 93926, 6258, 9058, 1585].$$

The then used HN method would give

$$\mathbf{H}(\mathbf{v}^{2009}) = [103, 46, 38, 24, 18, 11]$$

while without VA the result would be

$$\mathbf{H}(\mathbf{c}^{2009}) = [105, 48, 33, 25, 18, 11].$$

Hence the VA gives 5 seats to MRF and takes 2 seats from GERB and BSP and 1 seat from "Ataka".

The D'Hondt method would produce slightly different results:

$$\mathbf{D}(\mathbf{v}^{2009}) = [105, 46, 38, 24, 17, 10]$$

with VA and

$$\mathbf{D}(\mathbf{c}^{2009}) = [106, 48, 33, 25, 17, 11]$$

without VA.

Therefore for both D'Hondt and HN methods the VA "gives" 5 seats to MRF. This is due to the record number of votes (93 926) for MRF from abroad and mainly from Republic of Turkey.

2013. There were 4 parties passing the barrier: 1 – GERB, 2 – BSP, 3 – MRF and 4 – "Ataka". The vote vectors were

$$\mathbf{v}^{2013} = [1081605, 942541, 400466, 258481],\ \mathbf{a}^{2013} = [23090, 4907, 54353, 3018].$$

The HN method and the D'Hondt methods both gave

$$\mathbf{H}(\mathbf{v}^{2013}) = \mathbf{D}(\mathbf{v}^{2013}) = [97, 84, 36, 23].$$

Without VA the result would be

$$\mathbf{H}(\mathbf{c}^{2013}) = \mathbf{D}(\mathbf{c}^{2013}) = [98, 87, 32, 23].$$

Hence VA gives 4 seats to MRF and takes 1 seat from GERB and 3 seats from BSP. In particular without VA the formation BSP + MRF would have 119 seats instead of 120.

2014. There was a record number of 8 parties passing the barrier: 1 – GERB, 2 – BSP, 3 – MRF, 4 – Reformation Block (containing parts of the former UDF), 5 – Patriotic Front, 6 – "Bulgaria Without Censorship", 7 – "Ataka" and 8 – "Alternative for Bulgarian Revival" (ABR). The vote vectors were

$$\mathbf{v}^{2014} = [1072491, 505527, 487134, 291806, 239101, 186938, 148262, 136223]$$

and

$$\mathbf{a}^{2014} = [38868, 3635, 59938, 15523, 5150, 1672, 2429, 2637].$$

The HN method produced

$$\mathbf{H}(\mathbf{v}^{2014}) = [84, 39, 38, 23, 19, 15, 11, 11].$$

Without VA the result would be be

$$\mathbf{H}(\mathbf{c}^{2014}) = [84, 41, 35, 23, 19, 15, 12, 11].$$

Hence VA gave 3 seats to MRF and took away 2 seats from BSP and 1 seat from "Ataka".

The D'Hondt method would produce slightly different results:

$$\mathbf{D}(\mathbf{v}^{2014}) = [85, 40, 38, 23, 19, 14, 11, 10]$$

with VA and

$$\mathbf{D}(\mathbf{c}^{2014}) = [85, 41, 35, 22, 19, 15, 12, 11]$$

without VA.

Thus for both the D'Hondt and HN methods the VA "gives" 3 seats to MRF.

Summary for VA. To estimate the quantitative effect of VA on the election results since 1994 we introduce the ratio $\eta = \|\mathbf{a}\|/\|\mathbf{v}\|$ and the rounded (to the nearest integer) value $\mu = \text{round}(240\mu)$ of eventual number of MP due to VA. If a political solution to introduce a region for VA is taken, this number may correspond to the eventual number

of seats in this region.

Table 2. Measures of the effect of VA

Year	1994	1997	2001	2005	2009	2013	2014
$\eta(\mu)$	0.0047	0.0064	0.0598	0.0216	0.0380	0.0318	0.0423
Seats	1	2	14	5	9	8	10

Since MRF is the main beneficiary from VA, we show the electoral results and the impact of VA on MRF for all parliamentary elections in Table 3.

Table 3. Seats for MRF

Elections	1991	1994	1997	2001	2005	2009	2013	2014
total seats	24	15	19	21	34	38	36	38
seats from VA	0	0	0	4	3	5	4	3

This gives an average of 3.80 seats for MRF since 2001 due to VA.

The electoral system since 2005. The country is divided into $m = 31$ electoral regions $R_1, R_2, \ldots, R_m$. In each region R_i a number of seats r_i is preassigned proportionally to its population. Denote $\mathbf{r} = [r_1; r_2; \ldots; r_m]$ (recently there is an additional requirement $r_i \geq 4$).

Let parties $\Pi_1, \Pi_2, \ldots, \Pi_n$ have votes $v_1, v_2, \ldots, v_n$ so that $v_j/V_0 \geq 0.04$ (the 4% barrier), where V_0 is the sum of all valid votes. We have $v_j = c_j + a_j$, or $\mathbf{v} = \mathbf{c} + \mathbf{a}$, where c_j and a_j are the votes for party Π_j in the country and abroad, respectively. The party seats at national level are determined from $\mathbf{s} = [s_1, s_2, \ldots, s_n] = \mathbf{H}(\mathbf{v})$ (in 2009, 2013 and 2014) and $\mathbf{s} = \mathbf{D}(\mathbf{v})$ (in 1991–2005).

The data for distribution of seats among party lists Π_{ij} is the $m \times n$ matrix $\mathbf{V} = [v_{ij}]$ and the vector $\mathbf{a} = [a_1, a_2, \ldots, a_n]$, where v_{ij} is the number of votes for the party list Π_{ij} of party Π_j in region R_i. The data is augmented as the $(m+1) \times n$ matrix $\mathbf{W} = [\mathbf{V}; \mathbf{a}]$ of votes in country regions and abroad.

The result is the $m \times n$ non-negative integer matrix $\mathbf{S} = [s_{ij}]$, where s_{ij} is the number of seats assigned to Π_{ij}. The matrix $\mathbf{S}$ satisfies $\sum(\mathbf{S}) = \mathbf{s}$ and $\mathrm{sum}(\mathbf{S}^\top) = \mathbf{r}$, where $\mathrm{sum}(\mathbf{S})$ is the column sum of $\mathbf{S}$.

The bi-proportional algorithm used in 2013 and 2014 elections is now a part of the Electoral Code (addopted March 5, 2014). It has some flaws, the correct version being described in [5], see also `apa.bg`.

Severe differences in price of seats. Let V be the sum of votes cast in the country for parties passing the 4% barrier. Then the *average price* of one seat is $V/240$ [votes/seat] (or [v/s]) and this quantity varied during the years from approximately 18 000 [v/s] to 12 000 [v/s]. Due to several reasons (one of which is the absence of RVA) severe differences in the price of seats have been observed since 2001.

For example, in 2005 elections BSP took 1 "super expensive" seat with the impressive 62 206 votes (!) in R_{23} (price 62 206 [v/s]), while in the same region DSB took 5 seats with only 36 536 votes (price 7 307 [v/s]), see [3] for more details. It took several days for the leaders of BSP to assimilate this strangely looking result. The leadership of DSB

had not been happier either – in both cases MP became people not very favorite to the leaders, while many favorites did not become MP.

Later on MRF took 1 seat with only 474 votes in 2013 (in R_5) and 1 seat with only 864 votes in 2014 (in R_{10}). Many other "super cheap" seats had been distributed in 2013 and 2014 elections and not only for MRF. It is particularly unpleasant when a party takes a cheap seat with say less than 1 000 votes and at the same time a party with 7-8 000 votes gets no seat in the same region.

Such disproportions are hardly accepted by both people and politicians and they may lead to tensions and even rebelion in the society – there were such cases after the 2014 elections. Thus *a minor mathematical problem can have great political and social impact.* We have to admit, however, that the public is much more impressed by cheap seats of MRF rather than by similar seats of other parties.

Region for voting abroad (RVA). In the methods for seat allocation used since 2005 the VA is accounted only at national level and this leads to discordances in the regional seat allocation. A possible way to overcome this problem is to introduce RVA as R_{32}. This is motivated also by constitutional reasons: nowadays voters abroad cannot vote for particular party lists (and cannot use preferences) as well as for independent candidates which violates their constitutional rights.

A problem here is the determination of the number r_{32} of seats in RVA. A way to overcome the problem is to define r_{32} a posteriori on the basis of votes cast. However, the same rule has to be applied to the electoral regions in the country. Thus the number of seats allocated to electoral regions is determined proportionally to the number of votes cast (including invalid ones). Similar rule is applied for federal elections in Swiss. This a posteriori determination of regional seats has even a certain advantage: the voters know that the number of seats in their region depends on the vote turnout and are thus motivated to vote. This approach gives 9, 8 and 10 seats to RVA for the 2009, 2013 and 2014 elections, respectively (see also Table 2 above).

Another approach is to preassign the value of r_{32}, e.g. 4, 6 or 9, based on political arguments. In this case the remaining $240 - r_{32}$ seats are distributed among the 31 regions in the country by the HN method.

The introduction of RVA should in principle relax the problem but it may as well worsen some of the quantitative measures of the seat distribution. As an extreme example consider the case when we preassign 1 seat to RVA, i.e. $r_{32} = 1$. Then the party with most votes abroad takes this seat at eventually very high price. In 2009 this price should reach the unthinkable 93 926 [v/s]!

National party lists. When the country is treated as one electoral region (as at the first step of the bi-proportional algorithms for seat allocation used in Bulgaria) it is reasonable to have nationwide party lists instead of regional ones. This possibility however was omitted as an option in 1990. Now the political system is fitted to the use of regional lists and we are only allowed to make some improvements in the bi-proportional algorithms.

We have to admit that *low prices of seats are inevitable* with or without RVA and the Bulgarian politicians must be aware of this fact. This is illustrated by the following example which is not very far from a possible real situation. Suppose that we have $V =$ 3 000 000. Then the 4%–barrier is 120 000 votes and the average price of one seat in the

country is 12 500 [v/s]. A small party with slightly more than 120 000 votes eventually passes the barrier (like ARB in 2014) and if its electors are evenly distributed it should have from 1 000 to about 10 000 votes in each region. Thus obtaining cheap seats with price from 1 000 [v/s] to 2 000 [v/s] is quite probable. Indeed, in 2014 elections ARB had 1 seat with 1 260 and 1 272 votes in R_{22} and in R_5, respectively.

A possible way to overcome very low prices of seats is to introduce national party lists together with the regional ones (more details are published at `apa.bg`). The algorithm acts as follows. Let $\mathbf{V}$ be the $31 \times n$ matrix of votes in the country, $\mathbf{a}$ be the vector of votes abroad and $\mathbf{S}$ be the seat matrix produced by the present bi-proportional algorithm. Then matrix $\mathbf{S}$ is updated to $\mathbf{S}^*$ as shown below.

First the votes abroad are distributed in the country regions updating the columns of the matrix $\mathbf{V}$. Let $\mathbf{V}^* = [v^*_{ij}]$ be the matrix with columns

$$\mathbf{V}^*_{\bullet j} = \mathbf{V}_{\bullet j} + \mathbf{H}(a_j, \mathbf{V}_{\bullet j}).$$

The average price of 1 seat is $p = \|\mathbf{v}\|/240$ [v/s]. The seats $s_{ij} \geq 2$ are preserved, i.e. $s^*_{ij} = s_{ij}$. The seats $s_{ij} = 1$ with $v^*_{ij} = p$ are also preserved. If $s_{ij} = 1$ but $v^*_{ij} < \beta p$, where $\beta \leq 1$ is a given constant, we set $s^*_{ij} = 0$. Let n_j be the number of regional seats for party Π_j that were set to 0 in this way. Then party Π_j has shortage of $s_j - n_j$ seats which are taken from a national party list.

The analysis shows that when there are 7–8 parties passing the barrier then more than half of the MP may be elected from national party lists.

Measures for estimating seat distributions. Several measures to estimate the quality of seat distributions were proposed in [3, 5].

Unpleasant effects in a seat distribution are the *regional discordances* when some of the inequalities

$$(v_{ij} - v_{ik})(s_{ij} - s_{ik}) \geq 0$$

are violated. The number of such discordances is denoted by D. A desirable property of a distribution is that the number D is small.

Two other measures for quality of a seat distribution are the number $v_{\min}$ of minimum votes among the lists with 1 seat and the number $v_{\max}$ of maximum votes among the lists with no seats.

A set of measures is connected with the price v_{ij}/s_{ij} of 1 seat among lists with $s_{ij} \geq 1$. Let $p_{\max}$ and $p_{\min}$ be the maximum and minimum prices among all lists and set

$$\rho = \frac{p_{\max}}{p_{\min}}.$$

In the tables below we present the corresponding measures for 2013 elections (4 parties passing the barrier) and 2014 elections (8 parties passing the barrier).

Here *reg* and *par* stay for the numbers of the region and of the party for which the corresponding extreme value occurs. When something happens in RVA R_{32} we also show the corresponding values computed for regions in the country only (there are two such cases for Table 3 and one - for Table 4). The notation *floating* means that the number of seats in all regions (including RVA) is obtained a posteriori proportionally to the votes cast instead of being preassigned a priori.

Conclusions. The mathematics involved in Bulgarian parliamentary elections is quite simple but the political impact of its application is significant. More precisely, we

Table 4. Elections 2013

Seats in RVA	D	$v_{\min}$ (reg-par)	$v_{\max}$ (reg-par)	ρ (reg-par/reg-par)
0	2	474 (5-3)	7166 (14-4)	41.37 (10-1, 5-3)
4	0	1128 (24-3)	7166 (14-4)	20.47 (32-1, 24-3)
in the country				17.38 (10-1, 24-3)
6	0	1156 (23-3)	7166 (14-4)	16.96 (10-1, 23-3)
8 (floating)	0	1553 (25-3)	7166 (14-4)	11.36 (29-3, 25-3)
9	0	4907 (32-2)	7166 (14-4)	4.00 (10-1, 32-2)
in the country		5046 (7-4)		3.89 (10-1, 7-4)

Table 5. Elections 2014

Seats in RVA	D	$v_{\min}$ (reg-par)	$v_{\max}$ (reg-par)	ρ (reg-par/reg-par)
0	46	864 (10-3)	8015 (20-2)	23.32 (10-1, 13-3)
4	43	1012 (14-3)	8015 (20-2)	38.41 (32-1, 14-3)
in the country				19.91 (10-1, 14-3)
6	39	1260 (22-8)	8015 (20-2)	15.99 (10-1, 22-8)
9	34	1260 (22-8)	8015 (20-2)	15.99 (10-1, 22-8)
10 (floating)	35	1260 (22-8)	8015 (20-2)	15.99 (10-1, 22-8)

can make the following observations.

1. The beneficiary from VA is MRF. Since 2001 MRF has had a total of 167 MP including 19 MP (or 11.4%) due to VA. For the next elections we may expect about 4 seats for MRF from VA.
2. The D'Hondt method and the HN method produce very close results (usually ± 1 seat and very rarely ± 2 seats) for all parliamentary elections in Bulgaria. According to a previous result of the authors, for the Bulgarian electoral system (240 MP and 4% barrier) the difference between seats for a party for both methods can reach 5.
3. The number D of discordances increases with the number of parties passing the 4%-percent barrier and is very small when 4 or less such parties enter the parliament.
4. The introduction of RVA in principle relaxes the problem of distributing the regional seats but may as well worsen some of the quantitative measures for estimating the seat allocation.
5. The increase of the number r_{32} of seats in RVA decreases the number D (with one exception at lines 6 and 7 of Table 4), increases the minimum price $p_{\min} = v_{\min}$ of one seat and keeps constant the number $v_{\max}$ of votes for a list with no seats.
6. The ratio ρ decreases with the increase of the number r_{32} with one exception (lines 3 and 4 in Table 4). This phenomenon arises when the number of seats in RVA is relatively small and has been explained above, see also the next issue.
7. The introduction of RVA relaxes the seat allocation problem when either r_{32} is obtained using the votes cast (floating seats) or when it is preassigned but is close to this quantity. The expected value for r_{32} if determined proportionally to the votes cast is 8.

8. The value $r_{32} = 6$ for an a priori number of seats in RVA seems suitable from both political and mathematical points of view. Indeed, it corresponds to the results of previous five elections (where the average value of r_{32} is 8) and produces acceptable values for the quantitative measures of the seat distribution.

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ПАРЛАМЕНТАРНИ ИЗБОРИ: ВЛИЯНИЕ НА ВОТА В ЧУЖБИНА

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Работата е посветена на българските парламентарни избори. Изследавани са влиянието на вота в чужбина и начините да се намалят диспропорциите при разпределяне на мандатите в многомандатните изборни райони. Проследена е също историята на проблема. Това изследване е отчасти мотивирано от свръх евтините мандати, спечелени от една политическа сила (474 гласа на изборите през 2013 и 864 гласа на изборите през 2014), които доведоха до политическо и обществено недоволство. Разширена и адаптирана версия на тази работа ще бъде представена на ръководствата на политическите сили и на председателя на Народното събрание.