МАТЕМАТИКА И МАТЕМАТИЧЕСКО ОБРАЗОВАНИЕ, 2015
MATHEMATICS AND EDUCATION IN MATHEMATICS, 2015
Proceedings of the Forty Fourth Spring Conference
of the Union of Bulgarian Mathematicians
SOK "Kamchia", April 2–6, 2015

PEDAGOGICAL SCENARIOS IN THE STYLE OF EXPERIMENTAL MATHEMATICS*

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Even though experiments are used in mathematics research from the inception of mathematics, the term "experimental mathematics" was recently coined. Experimental mathematics is a new area of modern mathematics. The main feature of this field is obtaining of mathematical results by means of computer experiments. This field of mathematics is accessible not only for scientists, but also for lay mathematicians and for students. The purpose of the present paper is to show pedagogical scenarios for organization of students' activities in the style of experimental mathematics. These scenarios may be used in additional mathematical education: a club "Experimental mathematics", a math camp at a school for gifted children; and also in basic educational programs, if specific classes - "computer workshops in math" will be included in the curricula.

1. Introduction. Experiments are used in mathematics research from the inception of mathematics. Experiments are the basis of many mathematics methods: the quadrature method, the method of indivisibles (B. Cavalieri), the method of mechanical theorems (Archimedes) and others.

The term "experimental mathematics" was first used in 1969 at the opening of the Ural Branch of the Russian Academy of Sciences. But the meaning of this term is still under discussion among scientists from many countries.

Mark McEvoy [1] called this term a new brand of Mathematics. The term "experimental mathematics" refers to research related to obtaining mathematical propositions using computer technology. This term emphasizes the difference between the methods used in this new field of mathematical research and the methods of theoretical mathematics. The main method is a computer experiment. Features of the computer experiment in mathematics are disclosed in the Manifesto of Experimental Mathematics, developed by the leader of Experimental Mathematics group. The Manifesto is posted on the Experimental mathematics website [2]. This group was created in 2008 in the laboratory of nonlinear wave processes of the RAS Institute of Oceanology and the Department of Differential Equations and Mathematical Physics, Peoples' Friendship University of Russia.

This field of mathematics is accessible not only for scientists, but also for lay mathematicians and for students. Evidence of this are theorems, which are created by lay

*Key words: experimental mathematics, mathematical education, pedagogical scenarios, dynamic geometry software.

mathematicians, and which are placed in the Encyclopedia of Triangle Centers of C. Kimberling [3], as well as experience of pedagogical work of Experimental mathematics Club in Russia (since 1983), which is headed by G. B. Shabat [4].

The high hopes for the use of methodology of experimental mathematics in mathematics education were reflected in the Concept of development of mathematical education in the Russian Federation [5]. Dynamic geometry software (DGS) is recommended to be used for these purposes. Dynamic geometry software (DGS) represents a separate class of software for research and educational purposes. The first program of this class (Cabri-géométre) was created in 1985. At present, there are more than fifty programs of such kind. Only four programs of this class are used in Russia. They are The Geometer's Sketchpad (1989), GeoNext (1999), GeoGebra (2002), 1C: Mathematical Constructor (2006). Dynamic geometry software has a lot of opportunities to support cognitive activity of students.

DGS is used:

– to construct a virtual model of an object under consideration;

– to explore its new properties;

– to verify hypotheses;

– to monitor the progress of reasoning and analytical calculations;

– to develop an idea how to solve a problem.

In this paper, we present two pedagogical scenarios in the style of experimental mathematics with DGS, and describe how to use them in mathematics education.

2. Chain of tasks in DGS for intellectual workout. DGS offers many tools for construction of geometric figures. We can manage the DGS tools. We can remove tools from the Toolbar and insert new ones. We can create new tools. This allows us to explore the communication tools, the possibility of varying their sets in the course of solving problems to build dynamic drawings. Such tasks are used to assist students in mastering DGS tools, as well as to provide intellectual workout.

For example, we present a sequence of construction problems in DGS. The result of the solution of each problem is a new tool, which can be inserted into the Toolbar and can be used for the next problems in this sequence. At the start, the DGS GeoGebra Toolbar includes five tools: “Point on Object”, “Intersect”, “Segment”, “Ray”, “Circle with Centre through Point”.

Level 1 task. You have to construct: 1) a segment AB; 2) a circle with center A and radius AB; 3) a regular triangle.

When students solve this task, we insert the new tool – “Equilateral triangle” on the Toolbar and remove tool “Circle with Centre through Point” from the Toolbar.

Level 2 task. You have to construct a segment AB: 1) mark its midpoint; 2) construct its perpendicular bisector.

The method of solving this task is shown in Figure 1.

When this task is solved, we insert the new tools “Midpoint” and “Perpendicular bisector”.

Level 3 task. 1) You have to restore a segment via its midpoint and one of its ends. 2) You have to construct a line AB, and mark a point C on it. Construct a perpendicular line to AB through point C.

The method of solving this task is shown in Figure 2.

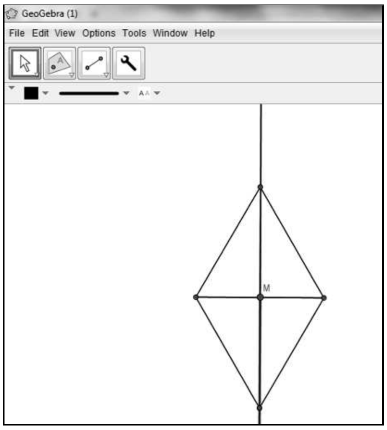

Fig. 1

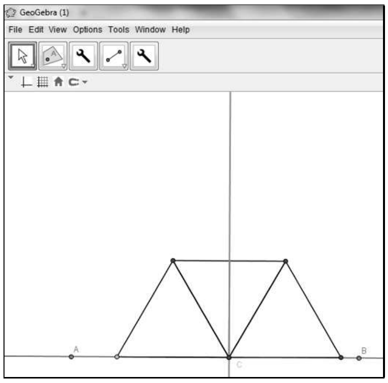

Fig. 2

When this task is solved, we insert the new tool "Perpendicular line through point on a line"

Level 4 task. You have to construct a line AB and point C, not on this line. Then you have to construct perpendicular line to AB through point C.

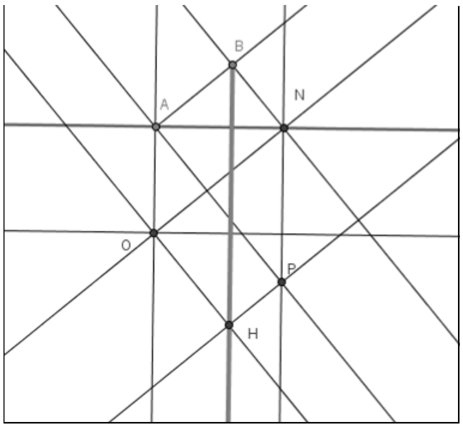

Fig. 3

A similar sequence of tasks can be found on the Internet [6], [7]. They may be different depending on the level of geometrical competence of students and limit of time for solving tasks.

3. Pedagogical scenario for solving research problems with DGS. We offer students to solve the problem: "Two travelers Smith and John found themselves on a deserted island, which has a triangular form. Smith waits for a ship on the capes (in points of triangle) every day, but John catches fish in the sea near the beaches (segments of triangle). They understand that they will have to live on the island for a long time. They want to build huts, so that the sum of the distances to the points/segments was the shortest. Is there a place for their common hut on the island? If they have to live separately, where each of them must build a house? Are the results dependent on the form of the island (the form of a triangle)?".

First step. We offer students to start research from a equilateral triangle. We give them the cardboard's sheets of triangular form to mark the places where the huts must be constructed. Most students think that John's hut can be only at the center of the triangle. We offer students to test this hypothesis on the computer (Figure 4).

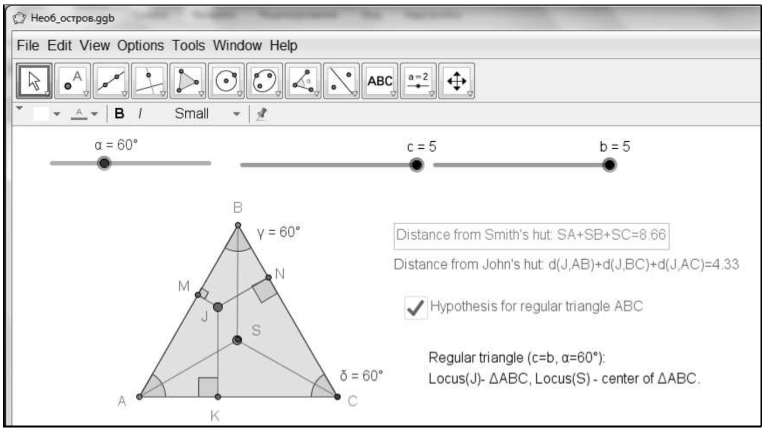

Fig. 4

After verification of hypotheses, we discuss to find ideas of proof. One of the ways of proof is presented on Figure 5.

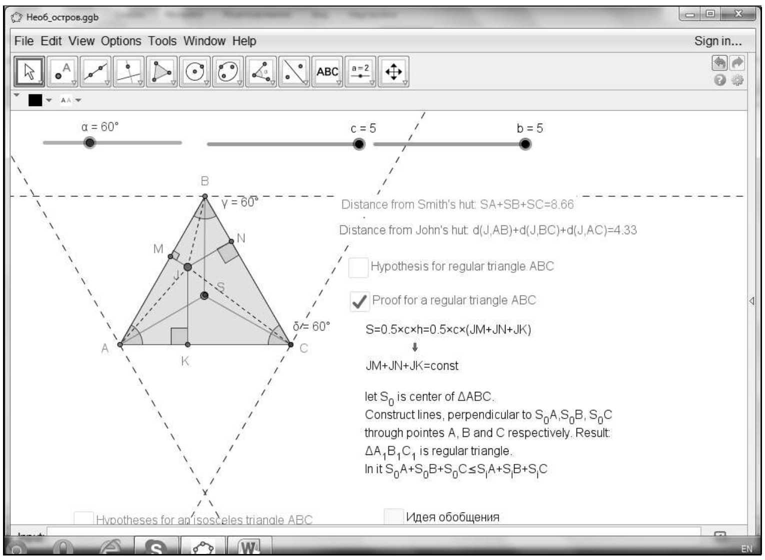

Fig. 5

Second step. We offer the students to explore the case of an isosceles triangle ($c = b$, $\alpha \neq 60°$. Students conduct a computer experiment and come to the conclusions presented on Figure 6.

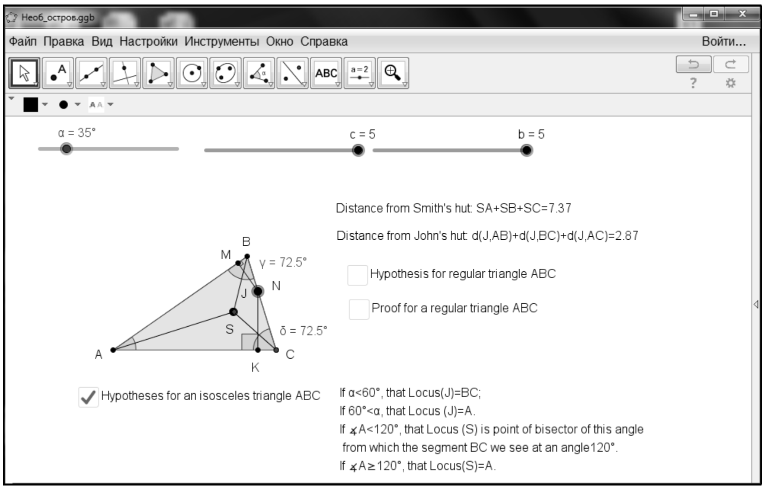

Fig. 6

Further on, students solve two problems: 1) The problem of searching ideas for the proof of hypotheses. 2) The problem of finding a way to construct the point from which the segment is seen at an angle of 120°.

The result of the second task is very important for generalization. It is the base for the construction of the Torricelli point.

Third step. We offer students to explore case of arbitrary triangles. For solving this problem, we discuss the distinction between the notions of "distance from a point to a line" and "distance from a point to a segment" with students. We teach students to build a dynamic drawing that visualizes this distinction. Students are offered to find information that will help to assess whether these results are new or known in mathematics.

There are similar research problems, like "The Pirates of The Caribbean" [8], "Findings of archaeologists" [9], "The New Adventures of Pinocchio" [10].

4. Use of pedagogical scenarios. We think, that similar pedagogical scenarios may be used in the system of additional mathematical education. For their testing, we used two forms of extra-curricular courses: *Experimental Mathematics club* in M.V. Lomonosov Northern (Arctic) Federal University for students from secondary school (7 students from 7–8th grades; 2 hours a week, total 34 hours); a math camp at the Physics and Mathematics Lyceum No. 17, Severodvinsk, Russia (two groups of 15 students all from 8th grades and two groups of 10 and 11 students from 7th grades; 2 hours with each group). All classes were conducted according to the same plan:

Step of classes	Learning content	Time
1. Intellectual workout	Solution of task sequence on construction in DGS, similar to the series, which are presented in paragraph 2 of this paper.	10 min
2. Problem statement	Presentation of the story of the problem, reading conditions of the problem, performing the situation.	5 min
3. Pre-computer solution of the problem	Conducting of natural exploration experiments or experiments with material models in the classroom or outdoors.	10 min
4. Computer solution of the problem	Constructing a model in DGS and conducting an exploration and verification of computer experiments.	20 min
5. Post-computer solution of the problem	Theoretical understanding of hypotheses, their deductive proof.	40 min
6. Statement of problems for independent work	Plenary discussion about the possibilities of generalizing the results and about directions for further development of ideas.	5 min
	Total:	90 min

The experiment confirmed our hypothesis that the context of games and the use of computers for problem solving are very interesting for students. During the classes they achieved skills of conducting natural and computer experiments, of variation of the problem, and of formulation of hypotheses. They internalize the logic of research in the style of Experimental Mathematics. However, they have great difficulty in the theoretical validation of the results, so they do not like this step. Students' interest in post-computer problem solving depends on whether they have a high level of mathematical competence. Factors that can increase the interest towards post-computer solving could be a discrepancy between the results of the computer and natural experiments, or an unexpected result.

We think that similar scenarios can be used in basic educational programs. For this, specific classes – "computer workshops in math" should be included in the curricula.

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ПЕДАГОГИЧЕСКИ СЦЕНАРИИ В СТИЛА НА ЕКСПЕРИМЕНТАЛНАТА МАТЕМАТИКА

Мария Шабанова, Мария Павлова

Въпреки че в математиката експерименти се използват още от самото ѝ зараждане, терминът „експериментална математика“ е въведен сравнително наскоро. Експерименталната математика е нова област на модерната математика, чиято основна характеристика е получаването на математически резултати чрез компютърни експерименти. Тази област на математиката е достъпна не само за учените, но и за обикновените математици и ученици. Целта на настоящата статия е да покаже дидактически сценарии за организиране на учебните дейности на учениците в стила на експерименталната математика. Тези сценарии могат да се използват в допълнителното математическо образование: в клуб „Експериментална математика“, на математически лагер в училище за надарени деца, но също и в основните образователни програми, ако в учебната програма се въведе специален предмет „компютърни семинари по математика“.

Ключови думи: експериментална математика; математическо образование; педагогически сценарии; динамичен геометричен софтуер.