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A BOMBIERI-VINOGRADOV TYPE EXPONENTIAL SUM RESULT*

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We prove a Bombieri-Vinogradov type result for linear exponential sums over primes. Then we apply it to improve a previous result of Dimitrov and the author.

1. Introduction and statements of the result. In this paper we consider the exponential sum:

$$W(x)=\sum_{d\sim D}\xi(d)\sum_{\substack{n\sim x\\ n\equiv a\,(d)}}\Lambda(n)e(\alpha n) \tag{1.1}$$

Here $\alpha\in\mathbb{R}$, $\Lambda(n)$ is the von Mangoldt function, $k\sim K$ means $K\le k<2K$,

$$x\ge 1,\ D=D(x)\ge 1,\ z=z(D)\ge 1,\ P(z)=\prod_{p\le z}p$$

and $\xi(d)$ is a real function such that

$$\begin{aligned}&\xi(d)\ll 1, \quad \text{if} \quad d|P(z) \quad \textit{and} \quad d\le D;\\ &\xi(d)=0, \quad \text{otherwise.}\end{aligned} \tag{1.2}$$

The necessity of estimation of such sums arises in some applications of the sieve method.

Let α have a rational approximation a/q satisfying

$$\left|\alpha-\frac{a}{q}\right|<\frac{1}{q^2}, \quad \text{where} \quad (a,q)=1, \quad q\ge 1. \tag{1.3}$$

If we only use that $\xi(d)\ll 1$ for $d\le \dfrac{x^{1/3}}{(\log x)^A}$ then Tolev [7, Lemma 1], and Dimitrov, Todorova [1, Lemma 6.4], proved that:

$$W(x)\ll\left(\frac{x}{q^{1/4}}+\frac{x}{(\log x)^{A/2}}+x^{3/4}q^{1/4}\right)\log^{37}x\,.$$

In the case when the function $\xi(d)$ is well factorable, K. Matomäki [6] received that:

$$W(x)\ll(\log x)^C x^{3/4+\eta}\left(\frac{x}{q}+q+D^2+x^{7/9+4\eta}+\min\left\{D^{4+20\eta},\frac{x}{D}\right\}\right)^{1/4-\eta}.$$

Here $\eta>0$, $C=C(\eta)>0$.

Using the conditions (1.2) we prove

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Theorem 1.1. *Suppose* $\alpha \in \mathbb{R}$ *with a rational approximation* $\dfrac{a}{q}$ *satisfying* (1.3), $\xi(d)$ *are real numbers satisfying* (1.2), $z = \dfrac{x^{2/15}}{(\log x)^A}$, $(\log x)^A < q < \dfrac{x}{(\log x)^A}$, $A > 148$ *and* $2D \le z^3$. *Then*

$$W(x) \ll \left(\frac{X}{q^{1/4}} + \frac{X}{(\log X)^{A/4}} + X^{3/4} q^{1/4} \right) (\log x)^{37} . \tag{1.4}$$

Using the above theorem we improve a previous result of Dimitrov and Todorova [1] and prove that

Theorem 1.2. *Let* B *be an arbitrary large and fixed and*

$$\lambda_i \in \mathbb{R},\ \lambda_i \neq 0, i = 1, 2, 3\,;$$
$$\lambda_1, \lambda_2, \lambda_3 \quad \text{not all of the same sign;}$$
$$\lambda_1/\lambda_2 \in \mathbb{R} \setminus \mathbb{Q}\,;$$
$$\eta \in \mathbb{R}\,.$$

Then, there are infinitely many ordered triples of primes p_1, p_2, p_3 *such that*

$$|\lambda_1 p_1 + \lambda_2 p_2 + \lambda_3 p_3 + \eta| < [\log(\max p_j)]^{-B} \tag{1.5}$$

and

$$p_1 + 2 = P_7', \quad p_2 + 2 = P_7'', \quad p_3 + 2 = P_7'''.$$

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ОЦЕНКА ОТ ТИПА НА БОМБИЕРИ-ВИНОГРАДОВ ЗА ЕКСПОНЕНЦИАЛНИ СУМИ ВЪРХУ ПРОСТИ ЧИСЛА

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