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TEACHING CATEGORY THEORY TO UNDERGRADUATES

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With this short note I would like to promote a discussion on the teaching of category theory to undergraduate students and share my experience. I will argue that it is due time to introduce category theory in the undergraduate mathematics curriculum as an elective course.

1. Introduction. Mathematics is one of the most conservative disciplines and for a good reason. The proof of Hippasus¹ that $\sqrt{2}$ is irrational 2500 years ago is still valid and will be valid ‘until the end of the Universe’. Thus, mathematics operates on grand scales and many mathematicians will be reluctant to include the category theory, which was born 74 years ago in the undergraduate mathematical curriculum. I will try to argue that it is time to overcome this reluctance.

In order to make my point I will recall very briefly the story of Linear Algebra. From the two millennia old Chinese text *Nine Chapters of Mathematical Art*, through the work of Leibnitz, Gauss, and many more, what constitutes now Linear Algebra was a scattered collection of results. It was put in a form that is close to what we know today in the book of G. Peano in 1888 following the work of H. Grassmann. It was not till the 50’s and 60’s of 20th century that Linear Algebra entered the undergraduate math curricula in the USA. It is well known that W. Heisenberg in 1925 when he invented Quantum Mechanics was surprised to find that he needed a non commutative operation and it was M. Born and P. Jordan who told him that he was doing matrix multiplication. At present Linear Algebra together with Calculus forms the groundwork of any mathematics curriculum. The reason is that both are not only mathematically important but have vast applications.

Category theory was born in 1945 and the first textbooks appeared in the late 60’s and early 70’s (some of the early textbooks are [6, 39, 45, 8, 32, 2, 22]) while the journal *Cahiers*² devoted to category theory was founded in 1958 by C. Ehresmann, a student of E. Cartan. Fifty years have past. The pace of time is accelerating so the period 1971–2019 in fact is much longer than the period 1888–1960. Why category theory is considered as too abstract, not relevant for applications, even harmful for young people, absurd to enter the curriculum on an equal par with linear algebra? There are subjective, purely human hate-love attitudes³ to category theory similar to the attitudes towards Grothendieck,

¹The legend attributes the proof to Hippasus of Metapontum, around 520 BC. He was a member of the Pythagorean school. The legend says that he lost his life for this discovery.

²*Cahiers de Topologie et Geometrie Differentielle Categoriqes* <http://cahierstgdc.com/>.

³See the discussion ‘unpopularity of category theory’ <https://nforum.ncatlab.org/discussion/1927>. The fact that the excellent journal ‘Theory and Applications of Categories’ is in practice ignored by the AMS is commented in the *n*-Lab entry about TAC.

the greatest mathematician of all times for some or an evil genius for others, who had big impact on the development of category theory in the 60's to the 80's. Another reason why it is taking category theory much longer than linear algebra to enter the curriculum is that category theory is doing a paradigm shift in mathematics and such shifts never go smoothly. Set theory also took a while to catch on but all of us today are brought up with a set theoretic mind set. Yes, set theory provides a very economical means based only on one relation 'is-an-element-of' but the shift of emphasis to functions/morphisms/arrows and category theory equips us from the start with a structural approach and most of mathematics is the study of structures. Another reason is that even though the impact of category theory on pure mathematics is momentous this is not the case for areas of mathematics such as differential equations, optimization theory, industrial and engineering mathematics (with the exception of computer science) while in all of these areas linear algebra is crucial. A lively discussion "categories-first-or-categories-last-in-basic-algebra" related to this note can be found at <https://mathoverflow.net/questions/41057/>. One of the arguments against teaching category theory early is that one needs examples from all of pure mathematics (as in the book of MacLane) thus first learn all of (pure) mathematics and then study category theory. But this is absurd – the very fact that category theory permeates and forces us to rethink all of (pure) mathematics, which is a great feature, is used against the teaching of category theory. Because linear algebra is applied in functional analysis should we study first functional analysis and only then linear algebra? The question is absurd but isn't the previous argument absurd too? The definition of a category is simpler than the definition of a vector space hence developing categories axiomatically is not more difficult than developing vector spaces axiomatically. (An intuition for vectors comes from physics but how many of today's math students have a good command of physics?)

I will give a lightning overview of the history of category theory and its application, followed by resources and ways of teaching it, and end with a conclusion.

2. Very short and incomplete history of category theory. Category Theory originated in 1945 with the publication [15] by Saunders Mac Lane and Samuel Eilenberg. The notion of 'adjoint functors', accepted as the most important mathematical notion coming from category theory, one on an equal footing with 'continuous function', was introduced in 1958 by Daniel Kan [25]. In the same year Kan introduced the concept of (co)limit. Category theory was born because of the needs of algebraic topology and homological algebra, thus it is not surprising that the first major development of category theory was the theory of abelian categories by A. Grothendieck and his school in the 1950's. Grothendieck reworked all of algebraic geometry in the 50's and 60's in the language of category theory. Another major development of the 60's and 70's is topos theory. Topoi were introduced by Grothendieck, coming from geometry and topology, and by W. Lawvere, coming from set theory, logic, and foundations of mathematics. In 1963 MacLane introduced monoidal categories – the categories with the biggest potential in applied mathematics. In the 70's P. May introduced operads as a generalization of the work on A_∞ algebras by J. Stasheff in the 60's. In the 80's and 90's and further one sees a flurry of activity in higher category theory and categorification (from Grothendieck's "Pursuing stacks" and the homotopy hypothesis, through the Baez-Dolan hypothesis, to Khovanov's categorification of knot polynomials and Lurie's higher topoi, etc). For much more see the Wikipedia article "Timeline of category theory and related mathematics".

Besides the development of category theory itself category theory has become the language of most of contemporary mathematics. Algebraic topology is the study of functors from topological to algebraic categories. Some fundamental dualities such as Gelfand duality or Stone duality are best expressed in categorical language as equivalence of appropriate categories. Some important developments that cannot even be formulated without category theory are for example, Morita equivalence, quantum groups (Drinfeld et al.), deformation quantization and mirror symmetry (Kontsevich), topological quantum field theories [50] (Segal, Atiyah 1988, Witten 1980's). Studying representations of groups in isolation is not much use while from the category of representations one can reconstruct the group (Tannaka-Krein reconstruction). Sheaf theory has practically merged with category theory. In the late 2010's S. Awodey and coworkers and V. Voevodski independently have introduced Homotopy Type Theory as a new foundation for mathematics.

3. Applied category theory. Many of the applications of category theory to physics could be viewed also as development of pure mathematics, thus besides the already mentioned quantum groups, deformation quantization, and topological field theory, one should not miss the Doplicher-Roberts reconstruction theorem in algebraic quantum theory, braid statistics in 2-dimensional conformal field theory and braided tensor categories, the Fuchs-Runkel-Schweigert theorem on rational 2d conformal field theory.

The development in categorical quantum mechanics initiated by S. Abramsky and B. Coecke has evolved to a very active area of research (e.g. the book of Coecke and Kissinger [13] and the very recent one of Heunen and Vicary [21]). Contextuality is one of the most important quantum properties, probably the main reason for the exponential speedup of quantum computers. The work of Abramsky and coworkers on a sheaf theoretic interpretation of contextuality in quantum mechanics is very promising. Motivated by the Kochen-Specker theorem (a manifestation of contextuality) the topos approach to quantum mechanics was developed by Isham-Butterfield-Doering and Heunen-Landsman-Spitters (e.g., [29]). A different category theory based approach to the foundation of quantum mechanics and quantum logic is effectus theory developed by B. Jacobs and coworkers [11].

The pioneer in the application of category theory to biology is R. Rosen [41, 44]. His ideas have been continued by A. Cornish-Bowden, T. Haruna, Y. Gunji, J.-C. Letelier, A.H. Louie, T. Nomura, etc. A recent categorical language for genetics is developed in [49]. The book [20] gives a categorical view on neurobiology. A categorical approach to cognition and neuroscience is being developed by S. Phillips and W. H. Willson. The collection of works of A. Ehresmann and P. Vanbremeersch, e.g., [14] deserves a special mention. They view live, neurobiology, and cognition as hierarchical categories evolving via functorial dynamics. The colimits⁴ play a key role in their approach.

The works on applications of category theory to (theoretical) computer science are so many that even a long overview cannot do them justice. One can say that category theory is becoming the main mathematical tool in computer science. The observation that Cartesian closed categories provide a semantics for the simply typed lambda calculus is a key result of J. Lambek [27]. Thus no wonder that a functional programming language such as Haskell from its conception relies crucially on notions and results of category

⁴The colimits are the categorical way to build by 'gluing' more complex structures from simpler ones.

theory. In particular, following the very influential paper of E. Moggi [37] the nontrivial categorical notion of a monad takes a prominent role in Haskell. I will only mention a few textbooks [4, 17, 36, 40, 51]. Haskell code and an ordinary text take turns in the very popular blog-style lecture series on category theory by B. Milewski (the book [36] is a byproduct). Programming languages are only one example of synergy between computer science and category theory but there are many others. For instance, monoidal categories give a language for concurrency [38]. Category theory provides a unified formalism for data bases (e.g., see works of D. Spivak). In bidirectional transformations, e.g., synchronization of databases, lenses and more generally optics play a prominent role [43]. Categorical coalgebras are central for the development of the calculus of infinite structures, e.g. streams, [23].

Philosophy and category theory go very well along: in some of his papers Lawvere starts with a philosophical motivation, Awodey is at the Philosophy Department of CMU, Corfield (one of the founders of n -category-cafe) is a philosopher. A few recent books on the subject are [26, 34, 28]. There is a growing body of work on linguistics and monoidal categories associated with the names of B. Coecke, M. Sadrzadeh, etc. One should not miss the four volumes of G. Mazzola on the Topos for Music [35].

An important development in the last decade which, will bring category theory into engineering, learning, and economic applications, is the study of open ‘things’ (networks, dynamical systems, games, learning, etc.). Driving forces in this are Baez and his coworkers, Spivak, Fong, Hedges, etc. Some examples of this can be seen in the recent book [18]⁵.

The categorical approach to probability theory started, as so many other things, by Lawvere with his 1962 secretive manuscript⁶ where one finds the origins of the Girmonad [19]. Independently a categorical approach to statistics was developed by N. N. Cencov [10]. Recently such developments have been carried by P. Panangaden, E. E. Doberkat, B. Fong, B. Jacobs, K. Sturtz, A. Sokolova, T. Avery, K. Adachi, Y. Ryu, F. Dahlqvist, V. Danos, I. Garnier, T. Fritz, etc.

One of the last bastions of mathematics resisting the advances of category theory are differential and integral calculus, optimization, differential equations. These are subjects of major importance for applications and applied mathematics constitutes the bulk of publications, financing, and people in mathematics. Hence, no wonder that the sentiments against category theory are popular in math departments. Things are changing here also. An example of such developments on the side of pure mathematics is the ‘universal property of the unit interval’ due to P. Freyd and Leinster ([42]). In applied and computational mathematics automatic differentiation plays a key role.⁷ The recent papers of C. Elliott [16] gives a categorical account of automatic differentiation. A different, but again related to differentiation, development by R. F. Blute, R. Cockett,

⁵Some of most recent developments can be followed in conferences on Applied Category Theory (see the home page of Baez) or the Symposium on Compositional Structures SYCO 1-5 <http://events.cs.bham.ac.uk/syco/strings3-syco5/>. In particular, note the talk of D. Ghica “Teaching category theory in primary school”. This is a nice experiment but I am not advocating it.

⁶Now available at <https://ncatlab.org/nlab/files/lawvereprobability1962.pdf>

⁷In machine learning and deep networks the key algorithm is backpropagation which is based on a reverse mode automatic differentiation.

G. S. H. Cruttwell, J. S. P. Lemay, R. A. G. Seely, etc., gives a categorical semantics of differential lambda calculus (e.g., [12]).

4. Resources for teaching category theory. In chronological order first comes the list of textbooks from the 1970's [6, 39, 45, 8, 32, 2, 22] of which one should single out MacLanes text – the ‘bible’ of category theory. The three volumes of Borceux [7] cover more than MacLane but are what the title says – a ‘handbook’ not a textbook. Again from the 1990's one has the textbooks on category theory and computer science such as [4, 51, 40] which could also be used as a main text or as a supplementary material. In the last decade several excellent texts appeared [3, 46, 31, 42] and to this list I will add the future book by P. Smith available in almost final form from <https://www.logicmatters.net/categories/>. The book [48] provides a much needed recent addition of an introductory textbook aimed at students with little mathematical background while [18] gives further examples of recent applications of category theory in the sciences.

The book [30] deserves a special mention. W. Lawvere (one of the driving forces in category theory) and S. Schanuel taught a course on category theory to high school students and the book is the delightful result of this course. A book on the subject addressing the general public is [9] based on the understanding that (monoidal) categories are ubiquitous because processes are ubiquitous and an example of a composing process in time and space is cooking. A big advocate of monoidal categories as describing processes is B. Coecke (the book [13] has the ambition to introduce quantum theory with a graphical language (string diagrams for dagger categories) that should be accessible to a very large audience).

A very nice example of the way category theory starts to enter into standard undergraduate courses is the 2009 book of P. Aluffi with the provocative title *Algebra: Chapter 0* [1]. This is a textbook intended to cover the material of a first course in abstract algebra where in parallel are introduced also the relevant categorical concepts. As such it can be used also to teach/study category theory.

I will mention quickly the electronic resources for category theory that one can use in a course. The *Wikipedia* though controversial and uneven has plenty of very well written articles on category theory. The main venue for discussing category related topics (and not only) is the *n-category cafe*. It started as a community blog of J. Baez, U. Schreiber, and D. Corfield and now has ten hosts. Baez's Azimuth project contains plenty of stuff on category theory related to ‘real applications’ like networks or control. A very important side product of the ‘cafe’ is the wiki ‘nLab’ devoted to all aspects of category theory and its applications in physics and computer science. The articles are not introductory but nevertheless this wiki has established itself as a basic reference for category theory. One should also have in mind the Q&A sites <https://math.stackexchange.com/> and <https://mathoverflow.net/>, the first being open to students and professionals while the second addresses mainly research questions. Young people love to watch videos so having the course ‘The Catsters’ (E. Chang and S. Willerton) at YouTube is a wonderful resource. A big list of resources can be found in P. Smith's [logicmatters.net/categories/](https://www.logicmatters.net/categories/) and on the ‘Category Theory Mailing List’ <https://www.mta.ca/~cat-dist/>. A valuable addition is the course of B. Milewski <https://bartoszmilewski.com>.

5. Teaching category theory. I have taught three times category theory at AUBG. The first time was in 2007 using the textbook [51]. During the first accreditation of

the mathematics program⁸ members of the accreditation team voiced unofficially strong disapproval of teaching categories to undergraduates⁹. I got the message and did not offer it again but in 2016 and in 2019 students asked that I teach it and I happily responded.

I am targeting an audience of math majors (some doing a double major also in computer science) in their third or fourth year. As a prerequisite I ask for Linear Algebra and strongly advise them to have taken some abstract course, e.g., Introduction to Abstract Algebra. But some of the students are only computer science majors or second year students (this is not uncommon at a liberal arts university) and they manage better than part of the fourth year math majors. In 2016 and 2019 I based the course on the book of Leinster [31]. This book is very appropriate for such a course providing the basic minimum of categorical concepts ‘adjoints’, ‘representables’, and ‘limits’. For additional reading I recommend Simmons [46], Awodey [3], and Spivak [48] while Riehl [42] and MacLane could be texts for deeper involvement. The text of Simmons is great for a very first introduction, with plenty of relatively easy exercises, but it lacks the Yoneda Lemma and representables. Since this lemma is a basic pillar of category theory I would not consider basing a course on [46] alone. The course does not cover Kan extensions, ends and coends, and regrettably there is not much time left in one semester to develop more seriously the monoidal categories. One of the properties of category theory is that every one of the basic notions (co)limits, representables, adjoints, or Kan extensions subsumes the rest. Leinster follows the sequence adjoints-representables-limits but I prefer to start with (co)limits thus I don’t follow the book verbatim. Students have seen examples of (co)limits in concrete situations elsewhere and it is good to begin with something familiar. Moreover all limits/colimits can be expressed as terminal/initial objects in appropriate categories and a terminal/initial object is probably the simplest notion¹⁰. Exercises are extremely important and the books I use together provide a big pool but of course one has to come up with new exercises.¹¹ The student evaluations on the course were positive.

6. Conclusion. Category theory is a paradigm shift in mathematics of grand proportions. It has reorganized first the closest areas such as homological algebra, algebraic geometry, representation theory, etc., and has spread its influence to most of abstract mathematics, large portions of theoretical physics, and much of theoretical computer science. In recent years we see categorical thinking and methods entering economics, linguistics, engineering, etc.

The time has come category theory to be considered not as something exotic but as something normal for the undergraduate curriculum. It should be taught as an elective course to 3rd or 4th year undergraduates with a prerequisite of linear algebra and some abstract mathematical course (e.g., abstract algebra). The resources are available (introductory textbooks, numerous lecture notes, many electronic resources). It took Moses

⁸AUBG has both an American and a Bulgarian accreditation and the second requires separate accreditation of each major. The math major at AUBG was established in 2001.

⁹I thank the referee for pointing out that courses on category theory were given at Sofia University by I. Prodanov., G. Dimov, and S. Petkova. To the best of my knowledge this has happened more than 3 decades ago.

¹⁰An object is terminal if from *every* other object to the terminal there *exists a unique* arrow.

¹¹It is in every book that the limit of a diagram with only two objects (not mentioning the identity arrows) is the product but what is the limit on a diagram with a single object? You never see this question because the answer is not interesting but some students are taken aback by it.

and the Israelites 40 years wandering in the desert before reaching the Promised Land. It is time after 75 years to accept the changes category theory brought to science and mathematics. My short answer to the question “why teach category theory to undergraduates” is: it is mathematically important; it has its own important results and at the same time it gradually permeates most of mathematics, creating a common structure and reversing the trend of compartmentalization; it is interesting for application; pedagogical textbooks exist; young people are enthusiastic about it.

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ПРЕПОДАВАНЕ НА ТЕОРИЯ НА КАТЕГОРИИТЕ НА БАКАЛАВРИ

Александър Ганчев

С тези кратки бележки искам да започна обсъждане на въпроса за включване на курс по теория на категориите в бакалавърските програми и да споделя своя опит. Ще приведа аргументи, че е настъпило времето курсове по теория на категориите да се включат като избираеми предмети в бакалавърски програми.