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POSSIBILITIES FOR USING AI IN MATHEMATICS EDUCATION*

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Artificial intelligence could be used as a powerful and innovative tool in mathematics education. It is poised to transform the way of learning and teaching this subject. The main objective of our study is to provide a more complete and thorough understanding of the role and impact of using AI in mathematics education by determining the trends, the AI methods, the technological applications and the opportunities for utilizing AI by teachers and students. The potential benefits and threats caused by the use of AI are also discussed.

Keywords: artificial intelligence, mathematics education, intelligent tutoring systems.

ВЪЗМОЖНОСТИ ЗА ИЗПОЛЗВАНЕ НА АІ В ОБУЧЕНИЕТО ПО МАТЕМАТИКА

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Изкуственият интелект може да се използва като мощен и иновативен инструмент в обучението по математика. Той е способен да трансформира начина на учене и преподаване на този предмет. Основната цел на нашето изследване е да осигури по-пълно и задълбочено разбиране на ролята и въздействието на използването на AI в обучението по математика чрез определяне на тенденциите, методите на AI, технологичните приложения и възможностите за използване на AI от учители и ученици. Потенциалните ползи и рискове, свързани с използването на изкуствения интелект, също са разгледани.

Ключови думи: изкуствен интелект, обучение по математика, интелигентни системи за обучение

1. Introduction. Technology has been tremendously developing in recent times, which affects all areas of life, including education. Artificial intelligence (AI) could help learners understand the lessons and topics they find difficult and it may also serve as a powerful tool for teachers not only to support the education process, but to cultivate critical thinking [10] and problem-solving skills in their students. One of the most important school subjects is Mathematics as it has application in various fields such as natural sciences, economics, social sciences, medicine, computing, banking and many others. However, according to the results of many exams like PISA and the maturity exams in a lot of countries, including Bulgaria, it appears to be a challenge for a large number of students. Technological advances, especially AI systems, give an opportunity for enhancing the understanding of mathematics by diagnosing difficulties in learning and providing personalised support [8, 9]. Such systems could also increase the interest in mathematics through gamification and more impressive visualization, especially in functional analysis and geometry.

2. Existing research concerning the application of AI in mathematics education. The role of artificial intelligence in education has been an object of many studies, but few of them give a holistic picture of the usage of AI in mathematics education. In [3] Gwo-Jen Hwang and Yun-Fang Tu have made a review of 129 relevant articles published in the quality journals indexed by the Social Sciences Citation Index (SSCI) from the Web of Science (WoS) database published from 1996 to the end of 2020. They find that 45.24% of the works discuss "intelligent tutoring systems", 28.57% – "profiling and prediction", 21.43% – "adaptive systems and personalization" and only 4.76% talk about "assessment and evaluation".

Another study [7] analyses publications in journals indexed in ScienceDirect, Scopus, SpringerLink, ProQuest, and EBSCOhost for the period 2017-2021. After a thorough analysis, it is found that from over 900 publications identified by title, abstract and keywords, only 20 fully explain the use of AI in mathematics education and clearly describe the data obtained. A reason for the small number of publications, according to [7], is that authors focus more on the use of AI in other fields of education and mathematics is less concerned. However, the considered in that research publications indicate that AI is helping students and teachers to improve the quality and effectiveness of learning and teaching by implementing solutions like robots, AI systems, teachable agents, autonomous agents, machine learning models, and digital technology devices.

3. Opportunities for using AI in mathematics education. Teachers and students could use numerous means to support mathematics learning – from information sources on the Internet (e-textbooks, reference books, blogs, forums, presentations, 118 videos, etc.) to systems using artificial intelligence. In general, AI systems suitable for mathematics education can be divided into two large groups – AI-based calculators and Intelligent tutoring systems (ITS). The first type are applications and they are used to a great extent by students to get help in solving specific mathematical problems, while the second type covers complex systems that can adapt the presented educational material depending on the individual needs and learning style of the student [4].

a. Opportunities for students

AI-based calculators. In present days, there are numerous AI-based calculators that can be used by teachers and students. They differ in their architecture (web applications, desktop applications or mobile applications), features, and price, but their principle of operation is similar:

- The user captures the task with the camera of their device or uses a formula editor or chat to enter it;

- The system then uses optical character recognition (OCR) and artificial intelligence to recognize the handwritten or editor-typed task;

– An AI-based module extracts the information and turns it into a mathematical formulation;

- Through another AI-based module, the task is solved and the solution is presented to the user. It should be noted that some systems do not provide a step-by-step solution for all types of problems, and in some cases, they only give answers (Microsoft Math Solver). There are systems which, when they cannot solve the problem, display a link to external ones allowing the user to see the solution (Google Socratic).

In our research, the functional capabilities and characteristics – working with photos, presence of an equation editor, display of the solution step by step, license for use, and Bulgarian language support – of nine AI-based calculating systems (Google Socratic [19], Microsoft Math Solver [25], Photomath [27], Symbolab [29], Nerd AI [26], Question.AI [28], Answer.AI [16], ChatGPT 3.5 [18] and MathGPT Chat [23]) are tested and analysed. Their abilities to solve certain classes of problems have been examined: quadratic equations, trigonometric expressions, systems of linear equations, systems of quadratic equations, ordinary integrals, integration by parts and logarithmic equations (Table 1).

The results show that among the AI-based calculating systems only ChatGPT 3.5 does not support working with task pictures. Seven of the studied programs have an equation editor for entering problem formulas, and two programs (ChatGPT 3.5 and MathGPT Chat) require the use of chat to enter problem conditions. The next functional possibility that is investigated is whether AI-based calculators can show the solution step by step, which is useful for student learning – again, seven of the systems output step-by-step solutions, with three of the programs having some limitations: Google Socratic displays step-by-step solutions for some tasks, and for others it provides a link to other programs supporting that feature; Microsoft Math Solver provides step-by-step solutions for some problems and only the correct answer for others; MathGPT Chat displays detailed postsolutions, but if they are very long, it displays only a part of them and offers to upgrade to a paid version to view the full solutions. The analysis of the types of tasks that AIbased calculators can solve shows that they can all solve quadratic equations, systems of linear equations and ordinary integrals. For trigonometric expressions, all programs did well, with eight showing a stepwise solution and only Microsoft Math Solver producing a single answer. The result of the logarithmic equations is similar – eight of the programs solve the problem (seven show a step-by-step solution, and Microsoft Math Solver – only an answer). The special thing about this task is that ChatGPT has given two different answers to the task set in Bulgarian and in English (in English the answer is correct, in Bulgarian – wrong). This case shows that students should be critical of the information offered by the AI-based calculators and it is mandatory to verify it. The task, where integration by parts should be applied, the solution has turned out to be a little more complicated – six showed a step-by-step solution, Microsoft Math Solver – only an answer, Google Socratic shows a link to another program, and Answer. AI gives a wrong answer. The system of quadratic equations turns out to be the most complex of all the tasks. Five programs solve it and show a step-by-step solution, Google Socratic bring up a link to another program, Microsoft Math Solver and Photomath fail to solve it, and Question.AI gives a wrong answer. Regarding the usage license – most programs offer free access, two of the programs (Symbolab and Nerd AI) offer a time-limited trial period after which a certain price must be paid, MathGPT Chat has limits on the number of uses, as well as on the length of the solutions displayed (for full solutions one must upgrade to a paid version), and Answer. AI requires user activity to be used for free. It has also been analysed whether AI-based calculating systems support Bulgarian language. Four of the programs are found to be the case – Google Socratic, Question.AI, ChatGPT 3.5 and MathGPT Chat. Undoubtedly, the communication and the proposed solutions to the tasks in Bulgarian language would greatly facilitate Bulgarian users (especially in the lower classes) when using these AI-based calculators.

The paper [11] states that it is not yet clear how these AI-based calculators will fit into current mathematics education programs. According to some analysts, if such systems are allowed without restrictions, this will lower the knowledge of students. Others believe that the role of such tools should be discussed by the educational community, while others note that the availability of these tools changes the goals of mathematics education.

Intelligent tutoring systems. Intelligent tutoring systems are computer-based tools that use artificial intelligence to create an adaptive learning environment. This environment must simultaneously correspond to the level and needs of the learner, as well as to what is included in the educational program. ITS analyse vast amounts of data to assess individual strengths and weaknesses, identify knowledge gaps, and provide customized content and exercises to address specific needs. Unlike AI-based calculators, intelligent tutoring systems are much more complex and interactive tools.

In [12], it is stated that ITS most often include four different modules: an "expert knowledge" module, which includes rules and terms, task databases, and error databases; a "student" module that stores information about student's knowledge and behaviour, a "tutor" module for choosing the best pedagogical action and an interface for interacting with the student.

When used in the educational process, ITS provide personalized instruction and feedback to students, adapting to their individual learning styles and needs. By analysing student performance data, AI algorithms can identify areas where a student is struggling and provide targeted support to help them overcome those challenges. This level of customization allows students to progress at their own pace, ensuring they have the knowledge and skills they need before moving on to the next learning unit.

Table	1
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	Google Socratic	Microsoft Math Solver	Photomath	Symbolab	Nerd AI	Question.AI	Answer.AI	ChatGPT 3.5	MathGPT Chat
Working with photos	Yes	Yes	Yes	Yes	Yes	Yes	Yes	No	Yes
Equation editor	Yes	Yes	Yes	Yes	No	Yes	No	Chat	Chat
A step-by- step solution	Yes/Link	For some tasks	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Quadratic equations	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Trigono- metric expressions	Yes	Only answer	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Systems of linear equations	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Systems of quadratic equations	Link to other programs with solution	No	No	Yes	Yes	Wrong	Yes	Yes	Yes
Ordinary in- tegrals	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Integration by parts	Link to other programs with solution	Only answer	Yes	Yes	Yes	Yes	Wrong	Yes	Yes
Logarithmic equations	Yes	Only answer	Yes	Yes	Yes	Yes	Yes	Yes*	Yes
License to use	Free	Free	Free	Paid 76.99 lv. per year	Paid 14.99 lv. per week	Free	Free ^{***}	Free	Free ^{**}
Support of Bulgarian language	Yes	No	No	No	No	Yes	No	Yes	Yes

The designations are as follows:

Yes - the task is solved and in addition to the answer, a step-by-step solution is visualized;

No – the task cannot be solved;

Wrong – the task is solved incorrectly;

* - ChatGPT has given two different answers to the same task, set in Bulgarian and in English (in English the answer is correct, in Bulgarian – wrong);
** - MathGPT Chat has limits on the number of solved problems, as well as the length of the

** – MathGPT Chat has limits on the number of solved problems, as well as the length of the displayed solutions (for longer solutions, it displays a part of the solution and offers to switch to the paid Pro version);

*** - Answer.AI requires activity from the user (daily use, viewing ads, sharing on social networks) and gives rewards so that the program can be used for free.

Examples of ITS systems for mathematics education are MATHia [22], ALEKS [15], ASSISTments [17], Knewton Alta [21], i-Ready [20], Matific [24], etc.

One of the essential issues in the use of ITS is the provision of multilingual support in order to achieve more effective learning. Some modern ITS (e.g. Knewton Alta) support only English, which limits their use to English-speaking countries.

For multilingual support ITS developers apply different solutions – using external applications to translate text information into different languages or built-in multilingual support in the systems. For example, MATHia does not have built-in multilingual support, while ALEKS supports English and Spanish. For visualisation of text information in other languages, these systems use Google Translate, but sometimes the translations are not accurate.

Other ITS work with a limited number of built-in languages. i-Ready supports English, Spanish, Portuguese, and Haitian. The Gradescope system, which is used to intelligently grade student work (written assignments, exams and practices), has a limited set of built-in languages, too – English, Japanese, Korean, Portuguese, Spanish and Turkish, and it is used in the respective countries. Matific has built-in support for more than 40 languages (including Bulgarian) and it is used in over 50 countries, most of which are members of the European Union.

In [13] it is stated that there is a study conducted in 2021 which is based on data from 100,000 students in the US state of Florida. It finds that the use of MATHia in lower secondary school leads to better results in algebra, especially for the weaker students. In the same source [13], they claim that ALEKS is effective when used to supplement traditional instruction. According to a research by the University of Western Sydney [1], the use of Matific in the Australian primary school classrooms increases students' results by 34%.

Authors of [8] state that some ITS are being developed as gamification strategies to improve students' attention. This approach also increases learners' engagement and active participation in the educational process and helps them develop their interpreting and decision-making skills.

In Ecuador, over 14,000 students have used an online platform with built-in AI capabilities and access to over 400 courses from different curricula. The results of the study [14] show that when starting the program, students have about 25% of the necessary knowledge in mathematics, while after 16 weeks of training their level of preparation has increased significantly to 68.7%, which corresponds to a one-year course of study.

b. Opportunities for teachers

Educators need to be aware of the possibilities and limitations that the use of AI offers in teaching mathematics in order to deliver the educational material in a state-of-the-art way. By doing so, they will prepare their students for successful career after graduation as AI enters more and more areas of life.

In addition to the benefits of AI systems for teaching mathematics to students, AIbased tools can also be used effectively by educators to support the organization of the educational process. These tools can create lesson plans [11], suggest assignments and solutions, collect data on learners' success, offer an individual approach to learning for each student. Teachers can analyse the information provided by AI-based tools about learner performance and make appropriate decisions about their teaching methods. In this way, 122 the time spent by teachers on preparation is reduced and the traditional mathematics education could be greatly improved [6].

For example, ChatGPT can be asked to generate problems [5, 11] and solutions to them, as well as give wrong solutions (so that students can find where the error in the solution is).

Another case in which AI could help teachers is when improving their qualification through ITS. A research conducted by scientists at the University of South Carolina [2] shows that when middle school mathematics teachers complete an online professional program that uses artificial intelligence to improve their knowledge and teaching skills, their students' math scores improve.

4. Challenges of using AI in mathematics education. The use of artificial intelligence systems in mathematics education has great advantages, but at the same time, their successful implementation faces a number of challenges.

One of the challenges is the vast variety ITS and AI-based calculators – which ones to choose and how to use them in the learning process so that they best meet the specific learning needs.

The next challenge is related to the need for teachers and students to be familiar with the advantages and disadvantages, the possibilities and means of using artificial intelligence in mathematics education. Teachers must take training courses on using AI in the educational process in order to effectively utilize it in the classroom and improve students' learning outcomes.

Another challenge is related to the need to update mathematics curricula and introduce new strategies and techniques into the learning process so as to ensure a controlled use of AI.

As ITS collect and analyse a large amount of information about learners, another challenge is related to ensuring the privacy and protection of students' data.

Last but not least, it is a challenge to ensure equal access for all students to AI-based learning tools. The lack of that would result in ineffective mathematics learning for some students.

5. Conclusions. Having more and more capable computer-based learning tools with built-in AI for use of both teachers and students will improve and make mathematics learning more accessible and effective.

The potential of AI to greatly improve personalized learning for students is enormous. The extensive use of ITS will positively change the way students learn and engage with mathematics.

Artificial intelligence will not replace educators, but it can be used to free up some of their time and it may also help them grow as mathematics educators. The new generations of students need innovative teaching methods. This can be achieved by implementing various techniques allowing them to develop their reasoning, creative and critical thinking.

In the near future, artificial intelligence is going to support professional work in many fields, including education, so that teachers and students can achieve higher quality in less time.

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