Arcs of High Divisibility and Their Applications to Coding Theory

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(based on joint papers with Sascha Kurz, Francesco Pavese, Assia Rousseva)

The $(t \mod q)$-arcs were introduced as a tool for an unified treatment of the extension problem for linear codes [4]. An arc $\mathcal{K}$ in $\mathrm{PG}(r,q)$ is called a $(t \mod q)$-arc if $\mathcal{K}(L) \equiv t \pmod q$ for every line L from $\mathrm{PG}(r,q)$. If in addition $\mathcal{K}(P) \leq t$ for every point P then $\mathcal{K}$ is called a strong $(t \mod q)$-arc.

There exists a general lifting construction for (strong) $(t \mod q)$-arcs which given a $(t \mod q)$ arc in $\mathrm{PG}(r,q)$ produces such an arc in $\mathrm{PG}(r+1,q)$. It was conjectured that all strong indecomposable $(t \mod q)$-arcs in $\mathrm{PG}(r,q)$ for $r \geq 3$ are lifted. This conjecture turned out to be wrong. Three exceptional $(3 \mod 5)$-arcs in $\mathrm{PG}(3,5)$ of respective sizes $128, 143$ and 168 that are not lifted were constructed by computer in [2]. This result was used to fill in the gap in the non-existence proof for the putative $[104,4,82]_5$-code. A geometric (computer-free) description of the three exceptional $(3 \mod 5)$-arcs was presented in [3]. One of them uses the Abatangelo-Korchmaros-Larato cap of size 20 in $\mathrm{PG}(3,5)$ [1], while the other two are based on the elliptic and hyperbolic quadrics.

In this talk, we present a geometric description of the three exceptional $(3 \mod 5)$-arcs in $\mathrm{PG}(3,5)$ and prove that every strong $(3 \mod 5)$-arc in $\mathrm{PG}(r,5)$, $r \geq 4$, is either lifted or a quadratic arc.

References

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