Characterization properties of exponential distribution

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The problem of characterization of probability distributions can be described as follows. It is known that a family of distributions $\mathcal{F}$ possesses certain property $\mathcal{P}$. Is it true, conversely, that a distribution has the property $\mathcal{P}$ only if it is a member of $\mathcal{F}$? If so, then $\mathcal{P}$ characterizes the family $\mathcal{F}$. In this talk, we will discuss two recent characterizations of the exponential distribution.

Characterization 1. The hypoexponential distribution is the distribution of a sum of independent exponential random variables. We prove that the following converse result is true. If for some $n \geq 2$, $X_1, X_2, \ldots, X_n$ are independent copies of a random variable X with unknown absolutely continuous distribution F and a specific linear combination of X_j's has hypoexponential distribution, then F is exponential.

Characterization 2. Let $X_1, X_2, \ldots, X_n$ be independent copies of an exponentially distributed random variable X. It is known that for every n

$$\max\{X_1, X_2, \ldots, X_n\} = X_1 + \frac{1}{2}X_2 + \cdots + \frac{1}{n}X_n, \quad \textit{(Sukhatme-Rényi).}$$

We obtain that if Sukhatme-Rényi decomposition of maxima holds for one fixed n, then X has exponential distribution.

References

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