MODEL FOR ASTRONOMICAL DATING OF THE *CHRONICLE* OF HYDATIUS

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Abstract. *This article presents a 'soft' model for astronomical dating of the seven eclipses mentioned in the Chronicle of Hydatius. The information about them is used in the construction of the two main components of the model: 1) "Template" ("image" of the initial "septet" of eclipses, described by Hydatius), and 2) "Distance".*

We assume that some of the data in the Chronicle may be incorrect. This means that the Template is considered as a fuzzy image of those seven real eclipses described by Hydatius. We assume that the inaccuracy of the data is small, i.e. that the parameters (dates, days of the week, etc.) in the Template do not differ much from the corresponding parameters of the initial (real) septet of eclipses, but are in some sense "close" to them.

For a more precise definition of this "closeness" in the paper is proposed a formula for the "distance" from the Template to any septet of real (happened in the past) eclipses. It allows us to calculate and compare the distances from the Template to all septets of real eclipses and to choose several "closest" to the Template (at the shortest distance to it) septets of real eclipses.

These septets are the target of the procedure of dating in the proposed model; they should be subject to further analysis and individual comparisons to determine which one of them is the most likely prototype of the Template, and thus its most probable dating.

Keywords: *Chronicle* of Hydatius, astronomical dating, eclipses, soft model, fuzzy information.

ACM Classification Keywords: I.5 PATTERN RECOGNITION; I.5.1 Models

1 Hydatius, his Chronicle and the eclipses in it

According to [New Advent Catholic Encyclopedia, 2013], Hydatius lived after the middle of the V century. From 427 to about 468 he was a bishop of Lemica. His Chronicle (continuation of the Chronicle of St. Jerome) has reached us in a single almost full copy. Its first part, describing events from 379 to 427, is based on records by writers preceding him, while the second part contains his descriptions of events from 427 to 468, of which he was a contemporary.

The Chronicle of Hydatius has been published and commented many times. Two of its editions are considered as main: 1) in Migne, PL LI, 873-890 [Idatii, 1861, pp. 873-890], and LXXIV, 701-750 [Idatii 1879, pp. 701-750]; and 2) in "Mon. Ger. Hist.: Auct. Antiq. ", XI (ed. Mommsen), 13-36 [Hydatii, 1894, pp. 1-36]. Among the recent editions we should mention: [Vilar, 2004], [Muhlberger, 2006] and [Cardoso, 1995].

In the Chronicle there are brief notes about the events, ordered chronologically by year of the reign of the successive Roman emperors, starting with Theodosius I. The years are numbered and the numbers start from 1 for each of the emperors. Occasionally, year numbers appear, according to the "Era" of Eusebius and other calendars. For the eclipses, there are given the day and the month, and sometimes also the day of the week, in which they occurred.

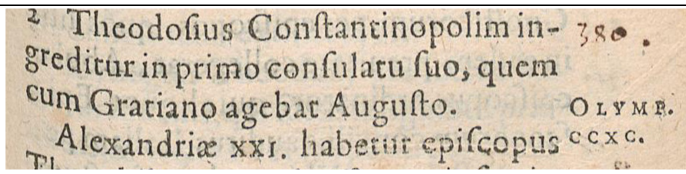
By H-1619, H-1634, H-1845, H-PL and H-MGH we denote five editions of the Chronicle of Hydatius – respectively of 1619 [Idatii, 1619], 1634 [Idatii, 1634], 1845 [Idatii, 1845], in the series *Patrologia Latina* – [Idatii, 1861], [Idatii, 1879] and *Monumenta Germaniae Historiae* [Hydatii, 1894].

Fig. 1 represents a fragment of the beginning of the Chronicle in the edition H-1619. After the title with the name of the 39th Roman Emperor Theodosius the Chronicle starts with the first year – indicated by 1 – of his reign: "1 Theodosius …". In the right margin somebody has added by hand the number 379 – it denotes the AD year. It is important to note, that the number 379 is not present in the original text of the chronicle.

IDATII EPISCOPI
CHRONICON.
ROMANORVM XXXIX.
Theodoſius per Gratianum in conſortium regni adſumptus, cum ipſo, & Valentiniano iuniore regnat annis XVII.
1 Theodoſius natione Hiſpanus, de prouincia Gallæcia, ciuitate Cauca, à 379

Figure 1. Fragment from the beginning of the Chronicle in the edition H-1619. After the title (containing the name of the 39th Roman Emperor Theodosius) the Chronicle starts with the first year – indicated by 1 – of Theodosius' reign: "1 Theodosius …". In the right margin somebody has added by hand the number 379 – it denotes the year of AD.

Another fragment from the beginning of the Chronicle in the edition H-1619 is shown in **Fig. 2.** The text is related to the second year – indicated by 2 – of the reign of Theodosius. Here, in the right margin, the year of AD 380 is written by hand, and the year CCXC is printed according to a reckoning "by Olympiads", indicating in this way the beginning of the 290th four year cycle.



2 Theodoſius Conſtantinopolim ingreditur in primo conſulatu ſuo, quem cum Gratiano agebat Auguſto. 380
OLYMP.
Alexandriæ XXI. habetur epiſcopus CCXC.

Figure 2. Fragment of the first page of the Chronicle in the edition H-1619 with a text about the second year of the reign of Theodosius. In the right margin the year of AD 380 is written by hand, and the year CCXC is printed, according to a reckoning "by Olympiads", indicating in this way the beginning of the 290th four-year cycle.

An Olympiad is a period of four years associated with the Olympic Games of the Ancient Greeks. During the Hellenistic period it was used as a calendar epoch. It was generally agreed that the first Olympic games had happened during the summer of 776 BC [Bickerman, 1980, p. 75]. Hence, by this reckoning, the first Olympiad lasted from the summer of 776 BC to that of 772 BC [Olympiad, 2013].

III. (*Olymp.* CCXC.) Athanaricus, rex Gothorum, apud Constantinopolim decimo quinto die ex quo a Theodosio fuerat susceptus, interiit.

Figure 3. Fragment of the Chronicle in H-PL with a text about the third year of the reign of Theodosius. The notation *Olymp. CCXC* is in brackets.

According to the text in H-PL (**Fig. 3**), the beginning of the CCXC (290th) four-year cycle is not in the second, but in the third (here the number 3 is written in Latin digits) year of the reign of Theodosius. The dating *Olymp. CCXC* is in brackets, which means, that it is not a part of the original text.

Also, in the third year of Theodosius is the beginning of the CCXC (290th) four-year cycle "by Olympiads" in H-MGH (**Fig. 4**). There are no brackets here and therefore the reader could be misled. The supposed respective AD year is given as a hint in the right margin.

OLYMP. CCLXXXX.

6. 'III. *Aithanaricus rex Gothorum aput Constantinopolim XV die, ex quo a Theodosio fuerat susceptus, interiit.*' 381

Figure 4. Fragment of the Chronicle in H-MGH with a text about the third year of the reign of Theodosius.

Another chronological guide is put **(Fig. 5**) in the margin of the fragment of the text of the Chronicle in [H-1619, p. 16]: for the 29th year of the reign of Arcadius and Honorius no occurrences are mentioned, but in the margin by hand is put the respective year AD. For the 30th year in the margin are printed:

- the year according to the "Era of Abraham": II.CCCCXL, i.e. 2440, and
- the year by Olympiads: Olymp. CCCI, i.e. [the beginning of] 301st Olympiad.

By hand is added the AD year: 424, and the comment that, according to the Chronicle of Marceline, the year is 423.

423 29

II. CCCCXL. 30 Honorius actis tricennalibus ſuis

424 Rauennæ obiit.

marcell. 423.

OLYMP. Paulinus nobiliſſimus, & eloquen-

CCCI. tiſſimus, dudum conuerſione ad Deum

Figure 5. Fragment of the Chronicle in H-1619 with a text about the 29th and 30th year of the reign of Arcadius and Honorius.

XXIX. (*Eus.* MMCCCCXL.)

XXX. (*Olymp.* CCCI.) Honorius actis tricennalibus suis Ravennæ obiit.

Figure 6. Two fragments of the Chronicle in H-PL with a text about the 29th and 30th year of the reign of T Arcadius and Honorius.

In H-PL 2440th year by Abraham started in the 29th year of Arcadius and Honorius (**Fig. 6**), in H-1845 – in the 30th (**Fig. 7**). In H-MGH the guiding mentioning of the 2440th year by Abraham is missing (**Fig. 8**).

Imp.		Olymp.	Christ.
XXX.	Abrahami IICCCCXL.	IV.	
	Honorius actis tricennalibus suis Ravennae obiit [1].		424

Figure 7. Fragment of the Chronicle in H-1845 with a text about the 29th and 30th year of the reign of Arcadius and Honorius.

79.	XXVIIII.	422
80.	'XXX. (*Honorius*' actis tricennalibus suis '*Ravenna obiit*'.)	423

Figure 8. Fragment of the Chronicle in H-MGH with a text about the 29th and 30th year of the reign of Arcadius and Honorius.

In the various editions of the Chronicle the text of the manuscript is "supplemented" by different pieces of information from other sources, whereas the publisher's intent was to help the readers to relate the story told by the author to the most important events of the times. Often a detailed "chronological net" is given for the years in various calendars.

However, the additional information sometimes is in contradiction with the data within the chronicle and with the astronomical dating of the eclipses mentioned there. For example, according to the construction in H-MGH it turns out that the first years of the 308th and 310th Olympiad should correspond to AD 453 and 459. This is impossible because the interval between them would not be 459-453 = 6 years (**Fig. 9**), but 8 years. The Editor of H-MGH (Momsen) has noticed this and has put a question mark (**Fig. 9**).

OLYMP. CCCVIII.
154. XXVIIII. 'Secundo regni anno principis Marciani Huni, qui Italiam praedabantur, 453

OLYMP. CCCX.
193. III. ('Theudoricus cum duce suo Sonerico exercitus sui aliquantam ad Baeticam 459?

Figure 9. Two fragments from the Chronicle of Hydatius in H-MGH; according to the Editor, the first year of the 308th Olympiad and the first year of the 310th Olympiad correspond to AD 453 and AD 459, respectively.

All these examples show that some of the details in the Chronicle – e.g. the intervals of time between eclipses – perhaps are not exact, maybe due to misinterpretation or for other reasons. But which of them exactly? The answer to this question must be consistent with the analysis of the whole "astronomical picture" in the Chronicle, and it is natural to expect it to be part of the goals set by this model and its investigation. Either way, it is appropriate to focus only on the text of the Chronicle and on the data contained in it.

2 Template of the eclipses in the Chronicle of Hydatius

Seven eclipses are described in the chronicle of Hydatius: five solar and two lunar. In the "List of Ginzel" [Ginzel, 1899] they bear numbers 59-65; here they will be numbered consecutively from 1 to 7. The respective texts in the Chronicle (as in H-PL) are:

Eclipse **H1.** “Solis facta defectio tertio idus Novembris feria secunda.”

Eclipse **H2.** “Solis facta defectio die decimo quarto kal. Augusti, qui fuit quinta feria.”

Eclipse **H3.** “Solis facta defectio die nono kal. Januarias, qui fuit tertia feria.”

Eclipse **H4.** “Quinto kal. Octobris a parte Orientis luna fuscatur.”

Eclipse **H5.** “Quinto idus Junias die, quarta feria, ab hora quarta in horam sextam, ad speciem lunae quintae vel sextae, sol de lumine orbis sui minoratus apparuit.”

Eclipse **H6.** “In provincia Gallaecia prodigiorum videntur signa diversa. Aera D, VI nonas Martias pullorum cantu, ab occasu solis luna in sanguinem plena convertitur. Idem dies sexta feria fuit.”

Eclipse **H7.** “Decimo tertio kalend. Augusti die, secunda feria, in speciem lunae quintae sol de lumine suo ab hora tertia in horam sextam cernitur minoratus.”

From these texts we obtain the following parameters of the eclipses (date = month and day, day of the week, time and phase):

H1 Solar eclipse on November 11, Monday.

H2 Solar eclipse on July 19, Thursday.

H3 Solar eclipse on December 24, Tuesday.

H4 Lunar eclipse on September 27. It was seen in the East (Eastern parts of the Empire) and was not seen in the West.

H5 Solar eclipse on June 9, Wednesday. Time – from the 4^{th} hour till the 6^{th} hour. Phase – about 0.7 – 0.8 (like 5- or 6- day moon).

H6 Lunar eclipse on March 2, Friday.

H7 Solar eclipse on July 20, Monday. Time – from the 3^{d} hour till the 6^{th} hour. Phase – about 0.7 – 0.8 (like 5-day moon).

- **Comments for the parameters – why fuzzy?**

Now the day and the night together have 24 hours and the length of the hours is constant (does not change). In the summer, the day lasts longer than 12 hours, and the night is shorter; in the winter is the opposite.

But in the past it was not so.

The day was divided into two halves, light and dark, rather than in two twelve-hour periods; the light half was divided in 12 hours, as well as the dark one [Stephenson, 1997, p. 381]. Thus, in winter, an hour would be longer at night than during the day. Their length remained constant throughout the day, but changed from day to day; it could be said that the hours were at the same time "seasonal".

The days begun with the sunrise or (as e.g. in the Jewish calendar) with the sunset [Stephenson, 1997, p. 381]. This was so probably because people measured time with sundials. The variable length of the hours is embarrassing for accurate astronomical calculations, so together with the improvement of the methods and the instruments for measuring time, the astronomers begun using fixed-length hours, which gradually entered in the daily life. The starting point for counting the hours moved at midnight or in the middle of the day, and this led to the emergence of other beginnings of the day and night.

But if the beginning of the day in one calendar is shifted with respect to the beginning of the day in another calendar, the transition from the one to the other is ambiguing, because the day in one includes parts of two days of the other. This is true also for different versions of the Julian Calendar.

May be namely the different beginnings of the days have caused the formal “shifts” in one day, which we sometimes meet in the old documents. As an example consider the following record from Braunschweig for a total solar eclipse, which occurred (according to the modern standard astronomical Julian calendar) on June 16 1406:

“1406. In this year there was an eclipse of the Sun so that the Sun stopped shining (vorgingk or schyn) before the Prime of the day (i.e. the Office held c. 6 a.m.) on St. Vitus’ day (Jun 15); it was so dark that people could not recognize one another.” (*Bothonis Chronicon Brunsvicensis picturatum*: [Stephenson, 1997, p. 405]). From this it becomes clear that if a historical source of information gives for an eclipse the date June 15, in the modern astronomical literature its date could be June 16.

Other similar examples are considered in [Stephenson, 1997, p. 406-407].They illustrate a part of the difficulties, which we meet when trying to determine the exact dates of the eclipses, mentioned or described in old chronicles and documents. The possibility that some dates mentioned in the Chronicle of Hydatius are shifted in one or two days with respect to the modern astronomical Julian calendar is taken into account in the construction of the model presented here.

- **Template Description:**

1. Parameters of the eclipses mentioned in the Chronicle of Hydatius:

Parameters of the eclipse H1 according to the Chronicle

1-1. Solar eclipse. Date of the eclipse - November 11.

1-2. Day of the week on which the eclipse occurred - Monday.

Parameters of the eclipse H2 according to the Chronicle

2-1. Solar eclipse. Date of the eclipse – July 19.

2-2. Day of the week on which the eclipse occurred - Thursday.

Parameters of the eclipse H3 according to the Chronicle

3-1. Solar eclipse. Date of the eclipse - December 24.

3-2. Day of the week on which the eclipse occurred - Tuesday.

Parameters of the eclipse H4 according to the Chronicle

4-1. Lunar Eclipse. Date of the eclipse - September 27.

4-2. Seen in the Eastern part of the Empire (Constantinople), and

4-3. Not seen in the Western parts of the empire (Spain).

Parameters of the eclipse H5 according to the Chronicle

5-1. Solar eclipse. Date of the eclipse - June 9.

5-2. Day of the week on which the eclipse occurred - Wednesday.

Parameters of the eclipse H6 according to the Chronicle

6-1. Lunar Eclipse. Date of eclipse - March 2.

6-2. Day of the week on which the eclipse occurred - Friday.

Parameters of the eclipse H7 according to the Chronicle

7-1. Solar eclipse. Date of the eclipse **-** July 20.

7-2. Day of the week on which the eclipse occurred - Monday.

2. Intervals between the eclipses

Eclipses **H1** and **H2** occurred during the reign of Emperors Arcadius and Honorius: respectively in the IX year of reign, and in the XXIV. Thus, the interval between these two eclipses is approximately 16 years.

Eclipses **H3** and **H4** occurred during the reign of Emperor Theodosius: during the XXIII year of reign, and during the XXVIII year. Thus, the interval between these two eclipses is (about) 5 years.

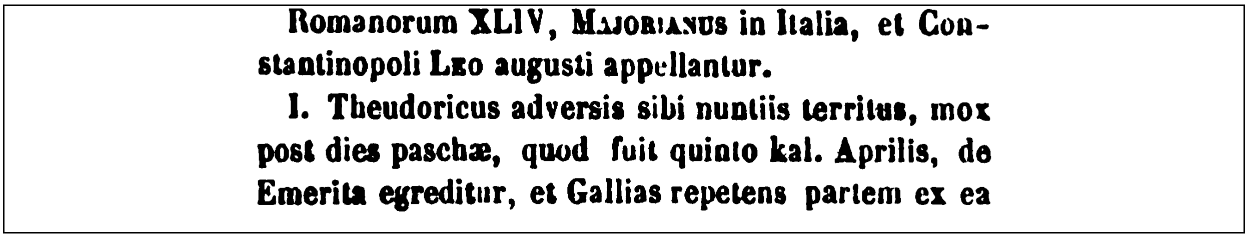
Eclipses **H6** and **H7** occurred during the reign of Emperor Norton: respectively in the I and the II year. Thus, the interval between these two eclipses is approximately 1 year.

The reigns of Emperors bearing consecutive numbers in the List of the Roman Emperors sometimes follow directly one after another with a small interval of days or months between them, but sometimes the interval is longer. "Overlapping" of the reigns also could not be excluded. So the other three intervals between successive eclipses between **H2** and **H3**, **H4** and **H5**, and H6 and H& may vary more widely. Formally the data in the Chronicle give the following very rough approximations:

- Between **H2** and **H3** - about 29 years;
- Between **H4** and **H5** - about 7 years;
- Between **H5** and **H6** - about 5 years.

3. Calendar and other astronomical data:

In the year of the eclipse **H5** Easter was on March 28 (**Fig. 10**).



Romanorum XLIV, Majorianus in Italia, et Con-
stantinopoli Leo augusti appellantur.
I. Theudoricus adversis sibi nuntiis territus, mox
post dies paschæ, quod fuit quinto kal. Aprilis, de
Emerita egreditur, et Gallias repetens partem ex ea

Figure 10. The text (in X-PL) for the first year of the reign of Emperor Majorianus, in which it is mentioned that Easter was on the fifth day of the April calends (March 28); the note about the eclipse **H5** is several lines after it. [Stephenson 1997, p. 406-407].

3 Distances from the Template to septets of real eclipses

Let $\mathbf{G_E = \{E_1, E_2, \ldots, E_7\}}$ be a set of seven eclipses which occurred in the past.

How much does this set "differ" from the set of eclipses $\mathbf{G_H}$ **= {H1**, **H2**, …, **H7}** – differ in the astronomical parameters described above for the the set $\mathbf{G_H}$?

To answer this question, we suggest a "metric" that models "closeness" of (and the "distance" between) individual eclipses and groups of eclipses respectively to **H1, H2, ..., H7** and $\mathbf{G_H}$. It is important for the application of the "template" described above in dating ancient eclipse and in particular for the evaluation of the “closeness” of $\mathbf{G_E}$ to $\mathbf{G_H}$.

Let **E** be any eclipse.

We start with defining rules for giving “scores” for the "closeness" of **E** respectively to each one of the eclipses **H1 - H7**.

Let ***m*** be a fixed positive number.

Scores for evaluation of the closeness of E to the eclipse H1:

The total score $\mathbf{e_1}$ for the closeness of an eclipse **E** to the eclipse **H1** is the sum of the scores for **1-1** and **1-2**:

1-1 The date of the eclipse H1 is November 11th. If the date of E is

- November 11 => score for **1-1:** ***m*** points;
- November 10 or 12 => score for **1-1:** 0.9***m*** points;
- November 9 or 13=> score for **1-1:** 0.5***m*** points;
- Another day => score for **1-1:** 0 points.

1-2 If the score for **1-1** is 0 points, the score for **1-2** is also 0 points; if the score for **1-1** is different from 0, the score for **1-2** is determined by the following rules. Taking into account, that the day of the week on which the eclipse **H1** occurred is Monday, if the day of the week on which occurred **E** is

- Monday => score for **1-2:** 0.5***m*** points;
- Tuesday or Sunday => score for **1-2:** 0.4***m*** points;
- Wednesday or Saturday => score for **1-2:** 0.3***m*** points;
- Another day => score for **1-2:** 0 points.

The rules for the determination of the scores for the closeness to the other four solar eclipses - **H2, H3, H5** and **H7** – are omitted, because they are completely analogous to that for the case of **H1**; different are only the dates and the corresponding days of the week.

The rules for the determination of the scores for the closeness of a lunar eclipse **E** to the lunar eclipses **H4** and **H6** are different:

Scores for evaluation of the closeness of E to the eclipse H4:

The total score $\mathbf{e_4}$ for the closeness of a lunar eclipse **E** to the lunar eclipse **H4** is the sum of the scores for **4-1, 4-2** and **4-3**:

4-1 The date of the eclipse **H4** is September 27. If the date of **E** is

- September 27 => score for **4-1:** ***m*** points;
- September 26 or 28 => score for **4-1:** 0.9***m*** points;
- September 25 or 29 => score for **4-1**: 0.5***m*** points;
- Another day => score for **4-1**: 0 points.

4-2 If ***E*** was seen in the Eastern part of the Empire (Jerusalem),

the score for **4-2** is 0.5***m*** points, otherwise 0 points.

4-3 If ***E*** was not seen in the Western part of the Empire,

the score for **4-3** is ***m*** points, otherwise 0 points.

If the score for **4-1** is 0 points, the scores for **4-3** and **4-2** are also 0 points.

Scores for evaluation of the closeness of E to the eclipse H6:

The total score e_6 for the closeness of a lunar eclipse **E** to the lunar eclipse **H5** is the sum of the scores for **6-1** and **6-2**:

6-1 The date of the eclipse **H6** is March 2. If the date of E is

- March 2 => score for **6-1:** ***m*** points;
- March 1 or 3 => score for **4-1:** 0.9***m*** points;
- February 28/29 or March 4 => score for **6-1:** 0.5***m*** points;
- Another day => score for **6-1:** 0 points.

If the score for **6-1** is 0 points, the score for **6-2** is also 0 points; if the score for **6-1** is different from 0, the score for **6-2** is determined by the following rules. Taking into account, that the day of the week on which the eclipse **H6** occurred is Friday, if the day of the week on which occurred **E** is

- Friday => score for **6-2:** 0.5***m*** points;
- Wednesday or Friday => score for **6-2:** 0.4***m*** points;
- Tuesday or Saturday => score for **6-2:** 0.3***m*** points;
- Another day => score for **6-2:** 0 points.

Scores for evaluation of the lengths of time intervals between the eclipses E_1, E_2,… , E_7

Let ***n*** be a fixed positive number.

The intervals are in years and are equal to the differences between the years (in the Julian calendar) in which the respective eclipses occurred.

Denote by f_i the score for the closeness of the interval between E_i and E_{i+1} to the interval between H_i and H_{i+1}.

If the interval between E_1 and E_2 is:

16 years => $f_1 = n$ points, 15 or 17 years => f_1 = 0,9n points, 14 or 18 years => f_1 = 0,4n points, in other cases f_1 = 0 points.

The rules for calculation of the scores f_3 and f_6 are similar.

If the interval between E_2 and E_3 is:

29 years => f_1 = 0.2n points, 28 or 30 years => f_1 = 0,1n points, in the other cases f_1 = 0 points.

The rules for calculation of the scores f_4 and f_5 are similar.

The uncertain length of the intervals between the successive eclipses H_2 and H_3, H_4 and H_5, and H_5 and H_6 create additional difficulties for adequate evaluation of the "closeness" of G_E to G_H. More significant deviations of the interval between E_i and E_{i+1} from the interval between H_i and H_{i+1} in more than two cases should be subject of a special attention.

Scores for Easter in the year of E_5

Let **p** be a fixed positive number; by g denote the score for Easter in the year of E_5.

If in the year of E_5 Easter was on

– March 28 => ***g = p*** ;
– March 27 or 29 => ***g =*** 0,9***p*** ;
– March 26 or 30 => ***g =*** 0,5***p*** ;
– Another day => ***g =*** 0.

4 Closeness of a set G_E of 7 eclipses to the septet G_H

Let $\mathbf{G_E = \{E_1, E_2, \ldots, E_7\}}$ be a set of seven eclipses. We define a "distance" of $\mathbf{G_E}$ to the septet $\mathbf{G_H}$ **= {H1, H2, …, H7}** ("of Hydatius") by the astronomical parameters described above for the group $\mathbf{G_H}$.

We define the "distance" ***d*** from G_E to G_H in the following way:

$d = 11{,}5\, m + 3{,}6\, n + p - (e_1 + e_2 + \ldots + e_7 + f_1 + f_2 + \ldots + f_6 + g).$

It is easy to check that in case of coincidence of the respective parameters for the eclipses of $\mathbf{G_E}$ and $\mathbf{G_H}$ the distance ***d*** is equal to 0. The less is ***d***, the "closer" is the septet $\mathbf{G_E}$ to $\mathbf{G_H}$.

5 Searching for the closest (to G_H) septet of eclipses G_E

The suggested formula for calculation of the distance from $\mathbf{G_E}$ to $\mathbf{G_H}$ is an essential part of our model for astronomical dating. It is natural to combine it with different methods for determination of the closest to $\mathbf{G_H}$ "septets" of eclipses in a given historical period – for example, in the time interval from AD 300 to AD 600.

A brief description of a possible approach how to "search" for suitable "septets" at shortest distance from $\mathbf{G_H}$ is given below.

1) We reduce the list of all eclipses of the period (assuming that this is the interval from AD 300 to AD 600) to its part **L,** containing only the eclipses visible from the Mediterranean region.

2) We select from this list **L** seven sets of eclipses $\mathbf{G^1}$, $\mathbf{G^2}$, … , $\mathbf{G^7}$: the set $\mathbf{G^1}$ contains only those eclipses of **L**, whose date is "around the date of **H1**", i.e. about November 11, and more precisely, in the framework of the proposed Template, on the days from 9 to 13 November inclusive. Similarly, we select the other sets $\mathbf{G^2}$, $\mathbf{G^3}$,…, $\mathbf{G^7}$.

3) We determine the number of the sets among $\mathbf{G^1}$, $\mathbf{G^2}$,… ,$\mathbf{G^7}$, having at least one element (eclipse) in the interval that starts from 300 and has a length of 100, i.e. in the interval (300, 400).

4) If the number determined in 3) is seven, we consider all possible septets of eclipses $\mathbf{G_E = \{E_1, E_2, \ldots, E_7\}}$ in the same interval (300, 400), where $\mathbf{E_i}$ is from $\mathbf{G^i}$ for i = 1,2, …, 7. For each such septet we calculate its distance ***d*** to $\mathbf{G_H}$ and choose several such septets with smallest ***d***.

5) We replace successively the interval (300, 400) by the intervals (301, 401), (302, 402), etc. and to each of them apply 3) and 4).

6) If the number of 3) is 6, we apply 4) and 5) for sextets (sets of six) of eclipses $\mathbf{G_E}$, replacing by 0's the scores of the "missing eclipse" (of one of the sets $\mathbf{G^1}$, $\mathbf{G^2}$, **…** , $\mathbf{G^7}$) in the formula for ***d***.

The determined in this way septets and sextets of eclipses are "candidates" for sets of eclipses that are closest to $\mathbf{G_H}$ and therefore are "astronomically probable" datings of $\mathbf{G_H}$ in the interval from AD 300 to AD 600.

6 Distance from the traditional dating of the eclipses in the *Chronicle* to the Template G_H

The seven eclipses of the *Chronicle* of Hydatius are contained in the famous "List of Ginzel" [Ginzel, 1899] of all "antient" eclipses mentioned and described in the major historical documents. There they are numbered successively from 59 to 65 and are dated as follows:

№ 59: **402-Nov-11** (November 11, AD 402)

№ 60: **418-Jul-19**

№ 61: **447-Dec-23**

№ 62: **451-Sep-26**

№ 63: **458-May-28**

№ 64: **462-Mar-2**

№ 65: **464-Jul-20**

At what extent does this dating satisfy the astronomical information about the seven eclipses in the Chronicle of Hydatius?

Or, in other words, how far (at what distance) is this dating from $\mathbf{G_H}$?

To give an appropriate answer to the last question we will calculate the distance $\boldsymbol{d}$ from the septet of eclipses № 59, № 60, № 61, № 62, № 63, № 64, № 65 to $\mathbf{G_H}$.

We put

E_1 = 402-Nov-11 ; E_2 = 418-Jul-19 ; E_3 = 447-Dec-23 ; E_4 = 451-Sep-26 ;

E_5 = 458-May-28; E_6 = 462-Mar-2 ; E_7 = 464-Jul-20.

First we calculate the the scores $e_1, e_2, ..., e_7, f_1, f_2, ..., f_6$ and g:

The total score e_1 of the closeness of an eclipse of **E** to **H1** equals the sum of the scores for **1-1** and **1-2**.

1-1 The date of the eclipse **H1** is November 11. Since the date of E_1 is also November 11, the score for **1-1** is m points;

1-2 The score for 1-1 is different from 0; the day of the week on which the eclipse **H1** occurred is Monday, and the day of the week on which occurred E_1 is Tuesday; therefore the score for **1-2** is 0.4m points.

Hence $e_1 = m + 0{,}4m = 1.4m$.

Similarly

$e_2 = m + 0{,}4m = 1{,}4m$

$e_3 = 0{,}9m + 0{,}5m = 1{,}4m$

$e_4 = 0{,}9m + 0{,}5m = 1{,}4m$

$e_5 = 0$

$e_6 = m + 0{,}5m = 1{,}5m$

$e_7 = m + 0 = m$,

and therefore

$$e_1 + e_2 + e_3 + e_4 + e_5 + e_6 + e_7 = 8{,}1m.$$

Further

$f_1 = n$

$f_2 = 0{,}2n$

$f_3 = 0{,}9n$

$f_4 = 0{,}2n$

$f_5 = 0{,}1n$

$f_6 = 0{,}9n$

$f_2 = 0{,}2n,$

hence

$$f_1 + f_2 + f_3 + f_4 + f_5 + f_6 = 3{,}3n.$$

Finally, since in 458 Easter was on April 20, while according to the Chronicle in the year of the eclipse **H5** Easter was on March 28,

$$g = 0.$$

Now for the distance d from G_E to G_H we obtain

$$d = 11{,}5m + 3{,}6n + p - (e_1 + e_2 + \ldots + e_7 + f_1 + f_2 + \ldots + f_6 + g)$$

$$= 11{,}5m + 3{,}6n + p - (8{,}1m + 3{,}3n + 0)$$

$$= 3{,}4m + 0{,}3n + p.$$

For $m = n = p = 20$ we have $d = 94$. Because

$$d_{max} = 11{,}5m + 3{,}6n + p = 230 + 72 + 20 = 322,$$

then $d = 94$ equals about 28.3% of d_{max}.

7 The problem H5

The zero score $e_5 = 0$ is impressive. Could this score be a result of incorrect dating of the eclipse **H5** (**№ 63** in the List of Ginzel)?

This question is interesting, since an analysis of the eclipses in the second half of the V century suggests the hypothesis that from astronomical point of view there is another preferable eclipse for the role of **H5**: the eclipse **476-Jun-07**.

Let us put E^1_5 **= 476-Jun-07** and consider the septet $G^1_E = \{E_1, E_2, E_3, E_4, E^1_5, E_6, E_7\}$ instead of $G_E = \{E_1, E_2, E_3, E_4, E_5, E_6, E_7\}$. Most of the scores involved in the formula for the distance d^1 between G^1_E and G_H coincide with the respective scores already calculated for the distance d between G_E and G_H. Different are only

$e^1_5 = 0{,}5m + 0{,}3m = 0{,}8m$

$f^1_4 = 0$

$f^1_5 = 0$

$g^1 = p.$

Substituting in the formula we get

$$d^1 = 2{,}6m + 0{,}6n.$$

The choice $m = n = p = 20$ leads to $d^1 = 64$.

The distance d^1 is significantly less than the distance d; from the point of view of the defined above distance the septet of eclipses G^1_E (which includes the eclipse **476-Jun-07** instead of **458-May-28**) is substantially closer to G_H, i.e. to the basic parameters of the seven eclipses of the *Chronicle*, than G_E.

This numerical result reflects an important substantive fact: that the date of Easter in the year of the eclipse E^1_5 **= 476-Jun-07**, coincides with the date March 28, mentioned in the *Chronicle* as the date of Easter in the year of the eclipse **H5**. It provides a strong argument in favor of the hypothesis that the E^1_5 is a better candidate than E_5 for the role of **H5**, in the sense that the $G^1_E = \{E_1, E_2, E_3, E_4, E^1_5, E_6, E_7\}$ better than G_E meets the main parameters of the eclipses of the *Chronicle*.

The coincidence of the year of an eclipse "around June 9" (i.e. in the range from June 7 to 11) with a year of "Easter on March 28" is a very rare event. Therefore, if the other six eclipses of the *Chronicle* Hidatsiy are dated correctly (in G_E), the probability that **H5** is a description of the later eclipse E^1_5 **= 476-Jun-07** is rather high.

Is such a thing possible?

At a first glance it seems that the answer is clearly negative. A reason for it is the chronological sequence of the events in the Chronicle.

But it could happen that in the extant copy of the *Chronicle* the chronological order is not correct: that it has been changed, may be occasionally, for instance, if 2-3 leaves of the manuscript of the Chronicle, which have been in the end, were moved in the middle. Having in mind the corrupted form of the preserved manuscript of the *Chronicle*, such a possibility is not excluded.

But evidently a "time shift" of $\mathbf{E_5}$ in about 18 years would also cause a similar shift (with respect to the contemporary historical events) of the rule of Emperor Majoranus.

Thus it becomes clear that to answer the question whether the year of the eclipse **H5** is 476, additional research and analysis of the sources concerning the epoch of Emperor Majoranus are necessary.

8 Concluding remarks and acknowledgments

The datings of ancient eclipses are used in history; they are usually searched for in the range of 30-40 years, and rarely - if the eclipses are mentioned in ancient Shumerian, Babylonian and Egyptian sources – in intervals of several centuries.

The narrow range facilitates the search, and, which is more important, it reduces at a great extent the number of "solution" to reasonable figures.

For building the chronology of ancient history on the basis of exact sciences - in which astronomy should play a leading role - it is necessary to investigate the dating of eclipses in a large range, covering several hundreds to several thousands of years. In this case, the number of the possible "purely astronomical solutions" for a great number of particular historical eclipses could be great.

From this perspective, the task of searching for astronomical dating of a "group of eclipses," which occurred "close" to each other (in the time), has the advantage that the resulting number of "solutions" is much smaller.

Dating of "group of eclipses" has been carried out by N. Morozov in the 20-ies of the XX century, for the so called "Triad of Thucydides" [Morozov, 1928]. Half a century later, now with the help of computer, dating of the same triad was carried out by A. Fomenko [Fomenko, 1990, pp. 91-95].

It should be noted that the approaches of Morozov and Fomenko are "harder" than the one proposed here; it treats the information from the work of Thucydides as "entirely accurate."

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