
MODEL FOR ASTRONOMICAL DATING OF THE *CHRONICLE* OF HYDATIUS: RESULTS FOR THE INTERVAL (600-1000)

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Abstract: This article presents the details and the results of the application of a 'soft' model for astronomical dating of the seven eclipses mentioned in the *Chronicle of Hydatius*, for the time interval (600, 1000), i.e. since 600 AD till 1000 AD.

The analysis of the date shows, that in the interval (600, 1000) the following two septets (i.e. groups of seven) of eclipses are the best candidates for dating of the eclipses mentioned in the *Chronicle of Hydatius*:

I. Septet: 923-Nov-11, 939-Jul-19, 968-Dec-22, 972-Sep-25, 978-Jun-08, 983-Mar-01, 985-Jul-20.

II. Septet: 923-Nov-11, 939-Jul-19, 968-Dec-22, 991-Sep-26, 978-Jun-08, 983-Mar-01, 985-Jul-20.

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1. Introduction

For the eclipses in the past we have two main sources of data: historical chronicles and astronomical tables of the dates of the past eclipses. The dating of a certain eclipse, mentioned in the historical chronicles, is in fact identifying it with a certain eclipse from the astronomical tables, which should be such that the parameters of the eclipse (place of the observation, day, month, hour, phase) mentioned in the chronicle coincide with the parameters of the eclipse from the tables. However often the dating of historical eclipses is problematic.

One of the most famous chronicles, containing information about historical eclipses (and sometimes data about their basic parameters), is that of Hydatius ([Idatii, 1619], [Idatii, 1634], [Idatii, 1861], [Hydatii, 1894]), in which seven eclipses are mentioned: five solar and two lunar.

A soft model for astronomical dating of these seven eclipses is suggested in the paper [Tabov, 2013]; details and results of its application for the time interval (300, 600) are presented in the paper [Tabov & Umlenski, to appear]. Here we give the similar details and results for the time interval (600, 1000).

In the framework of this model the information about the eclipses is systematized in two main parts of the model:

- “Template” (“image” of the initial described by the author “septet” of eclipses), including: 1) date (day and month) and day of the week – for every one of the eclipses – and 2) intervals (in years) between the eclipses, according to the text of the *Chronicle*, and
- “Distance”.

What does mean in this case the term “soft model”?

It is natural to expect, that some of the data in the *Chronicle* may be incorrect. Therefore we consider the Template **not as an exact image** of the initial septet of eclipses described by the author, but as a fuzzy image of those seven real eclipses described by Hydatius. We assume that the inaccuracy of the data is small, i.e. that the parameters (dates, days of the week, etc.) in the Template do not differ much from the corresponding parameters of the initial (real) septet of eclipses, but are in some sense “close” to them.

For a more precise definition of this "closeness" in the paper [Tabov, 2013] is proposed a formula for the "distance" from the Template to any septet of real (happened in the past) eclipses. It allows us to calculate and compare the distances from the Template to all septets of real eclipses and to choose several "closest" to the Template (at the shortest distance to it) septets of real eclipses.

These septets are the target of the procedure of dating in the proposed model; they should be subject to further analysis and individual comparisons to determine which one of them is the most likely prototype of the Template, and thus its most probable dating.

2. Template of the eclipses in the Chronicle of Hydatius

Template

(This Template is built up from the parameters of the seven eclipses **H1**, **H2**, **H3**, **H4**, **H5**, **H6** and **H7**, described by the text in the chronicle of Hydatius, with notation and representation according to [Tabov, 2013]).

H1 Solar eclipse on November 11, Monday.

H2 Solar eclipse on July 19, Thursday.

H3 Solar eclipse on December 24, Tuesday.

H4 Lunar eclipse on September 27. It was seen in the East (Eastern parts of the Empire) and was not seen in the West.

H5 Solar eclipse on June 9, Wednesday. Time – from the 4th hour till the 6th hour. Phase – about 0.4 – 0.5 (like 5- or 6- day moon).

H6 Lunar eclipse on March 2, Friday.

H7 Solar eclipse on July 20, Monday. Time – from the 3^d hour till the 6th hour. Phase – about 0.4 (like 5-day moon).

Here the eclipses **H1**, **H2**, **H3**, **H4**, **H5**, **H6** and **H7** are visible in the region of Mediterranean (Jerusalem, Constantinople and Caves).

The Template includes also the interval between the eclipses. Let t_i be the length of the interval (in years) between H_i and H_{i+1} . It is clear from the text of the *Chronicle*, that $t_1 = 16$, $t_3 = 5$ and $t_6 = 1$, and that the approximate values of t_2 , t_4 and t_5 are **29**, **7** and 5 years, respectively.

In the *Chronicle* there are some additional calendar- and astronomical data, which also should be included in the Template: 1) According to the *Chronicle*, the day of Easter in the year of **E₅** was on March 28; 2) The eclipse **E₄** was seen only in the Eastern parts of the Roman Empire and was not seen in the Western parts.

3. The problem for dating the eclipses in the Chronicle

Our target is: to determine several most appropriate septets of eclipses from the astronomical tables of the eclipses from AD 600 till AD 1000, from which after additional analyses should be selected the best candidate for identification with the real eclipses mentioned in the *Chronicle*. These septets should be at a least possible "distance" from the Template; the "distances" from the Template can be calculated by means of the following rules, suggested in [Tabov, 2013] (they will be used further).

Let $G_E = \{E_1, E_2, \dots, E_7\}$ be a set of seven (or a septet of) eclipses, which occurred in the past.

How much, or at what extent this septet differs from the set of eclipses $G_H = \{H1, H2, \dots, H7\}$ – differs in the astronomical parameters, described above in the Template?

For searching the answer of this question we use the "metric", suggested in [Tabov, 2013], which "models" the "closeness" of particular eclipses or of a group of eclipses respectively to H_1, H_2, \dots, H_7 и G_H . This metric is important for the application of the Template described above for dating ancient eclipses and especially for the "measuring" the "closeness" of G_E to G_H .

Let E be a certain eclipse from the List of the eclipses in the past.

Recall the rules for "scores" for the "closeness" of E respectively to each one of the eclipses $H_1 - H_7$, as they given in [Tabov, 2013] with the minor changes suggested in [Tabov & Umlenski, to appear].

Let m be a fixed positive number.

Scores for evaluation of the closeness of E to the eclipse H_1 :

The total score e_1 for the closeness of an eclipse E to the eclipse H_1 is the sum of the scores for 1-1 and 1-2:

1-1 The date of the eclipse H_1 is November 11th. If the date of E is:

- November 11 => score for **1-1**: m points;
- November 10 or 12 => score for **1-1**: $0.9m$ points;
- November 9 or 13=> score for **1-1**: $0.5m$ points;
- Another day => score for **1-1**: 0 points.

1-2 If the score for **1-1** is 0 points, the score for **1-2** is also 0 points; if the score for **1-1** is different from 0, the score for **1-2** is determined by the following rules. Taking into account, that the day of the week on which the eclipse H_1 occurred is Monday, if the day of the week on which occurred E is:

- Monday => score for **1-2**: $0.5m$ points;
- Tuesday or Sunday => score for **1-2**: $0.4m$ points;
- Wednesday or Saturday => score for **1-2**: $0.3m$ points;
- Another day => score for **1-2**: 0 points.

The rules for the determination of the scores for the closeness to the other four solar eclipses - H_2, H_3, H_5 and H_7 – are omitted, because they are completely analogous to that for the case of H_1 ; different are only the dates and the corresponding days of the week.

The rules for the determination of the scores for the closeness of a lunar eclipse E to the lunar eclipses H_4 and H_6 are different:

Scores for evaluation of the closeness of E to the eclipse H_4 :¹

The total score e_4 for the closeness of a lunar eclipse E to the lunar eclipse H_4 is the sum of the scores for 4-1, 4-2 and 4-3:

4-1 The date of the eclipse H_4 is September 27. If the date of E is:

- September 27 => score for **4-1**: m points;
- September 26 or 28 => score for **4-1**: $0.9m$ points;
- September 25 or 29 => score for **4-1**: $0.5m$ points;
- Another day => score for **4-1**: 0 points.

¹ These rules differ insignificantly from the version in [Tabov, 2013], but are identical with the version in [Tabov & Umlenski, to appear].

4-2 If E was seen in the Eastern part of the Empire (Jerusalem),

the score for **4-2** is $0.5m$ points, otherwise 0 points.

If the score for **4-1** is 0 points, the scores for **4-3** and **4-2** are also 0 points.

4-3 If E was not seen in the Western part of the Empire,

the score for **4-3** is $1.5m$ points, otherwise 0 points.

Scores for evaluation of the closeness of E to the eclipse H_6 :

The total score e_6 for the closeness of a lunar eclipse E to the lunar eclipse H_6 is the sum of the scores for **6-1** and **6-2**:

6-1 The date of the eclipse H_6 is March 2. If the date of E is:

- March 2 => score for **6-1**: m points;
- March 1 or 3 => score for **4-1**: $0.9m$ points;
- February 28/29 or March 4 => score for **6-1**: $0.5m$ points;
- Another day => score for **6-1**: 0 points.

If the score for **6-1** is 0 points, the score for **6-2** is also 0 points; if the score for **6-1** is different from 0, the score for **6-2** is determined by the following rules. Taking into account, that the day of the week on which the eclipse H_6 occurred is Friday, if the day of the week on which occurred E is:

- Friday => score for **6-2**: $0.5m$ points;
- Wednesday or Friday => score for **6-2**: $0.4m$ points;
- Tuesday or Saturday => score for **6-2**: $0.3m$ points;
- Another day => score for **6-2**: 0 points.

Scores for evaluation of the lengths of time intervals between the eclipses E_1, E_2, \dots, E_7

Let n be a fixed positive number.

The intervals are in years and are equal to the differences between the years (in the Julian calendar) in which the respective eclipses occurred.

Denote by f_i the score for the closeness of the interval between E_i and E_{i+1} to the interval between H_i and H_{i+1} .

If the interval between E_1 and E_2 is:

16 years => $f_1 = n$ points, 15 or 17 years => $f_1 = 0,9n$ points, 14 or 18 years => $f_1 = 0,4n$ points, in other cases $f_1 = 0$ points.

The rules for calculation of the scores f_3 and f_6 are similar.

If the interval between E_2 and E_3 is:

29 years => $f_1 = 0.2n$ points, 28 or 30 years => $f_1 = 0,1n$ points, in the other cases $f_1 = 0$ points.

The rules for calculation of the scores f_4 and f_5 are similar.

The uncertain length of the intervals between the successive eclipses H_2 and H_3, H_4 and H_5 , and H_5 and H_6 create additional difficulties for adequate evaluation of the "closeness" of G_E to G_H . More significant deviations of the interval between E_i and E_{i+1} from the interval between H_i and H_{i+1} in more than two cases should be subject of a special attention.

Scores for Easter in the year of E_5

Let p be a fixed positive number; by g denote the score for Easter in the year of E_5 .

If in the year of E_5 Easter was on

- March 28 => $g = p$;
- March 27 or 29 => $g = 0,9p$;
- March 26 or 30 => $g = 0,5p$;
- Another day => $g = 0$.

4. Closeness of a set G_E of 7 eclipses to the septet G_H

Let $G_E = \{E_1, E_2, \dots, E_7\}$ be a set of seven eclipses. We define a "distance" of G_E to the septet $G_H = \{H1, H2, \dots, H7\}$ ("of Hydatius") by the astronomical parameters described above for the group G_H and according to the rules for giving scores given above.

We define the "distance" d from G_E to G_H in the following way:

$$d = 12m + 3,6n + p - (e_1 + e_2 + \dots + e_7 + f_1 + f_2 + \dots + f_6 + g).$$

It is easy to check that in case of coincidence of the respective parameters for the eclipses of G_E and G_H the distance d is equal to 0. The less is d , the "closer" is the septet G_E to G_H .

5. Searching for the closest (to G_H) septet of eclipses G_E

The proposed formula for calculation of the distance from G_E to G_H is an essential part of our model for astronomical dating. It is natural to combine it with different methods for determining the closest to G_H "septets" of eclipses in a given historical period – for example, in the time interval from AD 600 to AD 1000.

A brief description of a possible approach how to "search" for suitable "septets" at shortest distance from G_H and its application are given below.

The first step is the reduction of the list of all eclipses of the period (assuming that this is the interval from AD 600 to AD 1000) to its part L , containing only the eclipses visible from the Mediterranean region (Jerusalem, Constantinople, Caves).

From this reduced list L we select seven sets of eclipses G^1, G^2, \dots, G^7 : the set G^1 contains only those eclipses of L , whose date is "around the date of $H1$ ", i.e. about November 11, and more precisely, in the framework of the proposed Template, on the days from 9 to 13 November inclusive. Similarly, we select the other sets G^2, G^3, \dots, G^7 . The result is represented in **Table 1**.

Set	O	Description of the eclipses in the set
Set G^1 (of $H1$)	H1	Solar eclipse on November 11, Monday. The set G^1 contains the solar eclipses from L which occurred on November 09, 10, 11, 12 and 13. For these eclipses we also say that they are of type H1 .
Set G^2 (of $H2$)	H2	Solar eclipse on July 19, Thursday. The set G^2 contains the solar eclipses from L which occurred on July 17, 18, 19, 20 and 21. For these eclipses we also say that they are of type H2 .
Set G^3 (of $H3$)	H3	Solar eclipse on December 24, Tuesday. The set G^3 contains the solar eclipses from L which occurred on December 22, 23, 24, 25 and 26. For these eclipses we also say that they are of type H3 .
Set G^4 (of $H4$)	H4	Lunar eclipse on September 27. It was seen in the East (Eastern parts of the Empire). The set G^4 contains the lunar eclipses from L which occurred on September 25, 26, 27, 28 and 29, seen in the East (in Jerusalem). For these eclipses we also say that they are of type H4 .

Set G^5 (of H5)	H5 Solar eclipse on June 9, Wednesday. The set G^5 contains the solar eclipses from L which occurred on December 07, 08, 09, 10 and 11. For these eclipses we also say that they are of type H5 .
Set G^6 (of H6)	H6 Lunar eclipse on March 2, Friday. The set G^6 contains the lunar eclipses from L which occurred on February 28 and 29 and on March 01, 02, 03, and 04. For these eclipses we also say that they are of type H6 .
Set G^7 (of H7)	H7 Solar eclipse on July 20, Monday. The set G^7 contains the solar eclipses from L which occurred on July 18, 19, 20, 21 and 22. For these eclipses we also say that they are of type H7 .

Table 1. The sets G^1, G^2, \dots, G^7

In order to analyse the number and the distribution of the solar eclipses of the types **H1, H2, H3, H5** and **H7** in the interval AD 600-1000, we arrange them in chronological order in **Table 2**:

Type H1	Type H2	Type H3	Type H5	Type H7
		604-Dec-26		
			606-Jun-11	
			625-Jun-10	
	836-Jul-17			
			894-Jun-07	
			913-Jun-07	
	920-Jul-18			920-Jul-18
923-Nov-11				
	939-Jul-19			939-Jul-19
942-Nov-11				
		949-Dec-22		
	966-Jul-20			966-Jul-20
		968-Dec-22		
			978-Jun-08	
	985-Jul-20			985-Jul-20

Table 2. The eclipses of the types **H1 – H3, H5** and **H7** in the interval AD 600 – 1000

From the septets of eclipses we are interested in (7 eclipses, one of each type **H1 – H7**) we should choose several with highest scores according to the scheme of the model. Every such septet should be in a certain interval of length 100 years. Hence, in order to have in a certain interval of length 100 a septet with a high score, in this interval should present eclipses of at least five of the types **H1 – H7**. Then among them there should be at least 3 solar ones.

From **Table 2** it is clear that:

In the interval 600-800 there are solar eclipses of two types: **H3** and **H5**, and there is no solar eclipse of the types **H1, H2**, and **H7**;

In the interval 800-1000 there are solar eclipses of two types: **H2** and **H5**, and there is no solar eclipse of the types **H1, H2**, and **H7**.

Consequently in order to select septets with high scores, it is sufficient to consider only the eclipses from the interval AD 890-1000.

From the list **L** we find successively *in the interval AD 890-1000*:

Eclipses from the set **G¹** (of type **H1**): 923-Nov-11, 942-Nov-11.

Eclipses from the set **G²** (of type **H2**): 939-Jul-19, 966-Jul-20, 985-Jul-20.

Eclipses from the set **G³** (of type **H3**): 949-Dec-22, 968-Dec-22.

Eclipses from the set **G⁴** (of type **H4**): 953-Sep-25, 972-Sep-25, 991-Sep-26 (visible in Jerusalem, but not visible in Constantinople and Caves).

Eclipses from the set **G⁵** (of type **H5**): 894-Jun-07, 913-Jun-07, 978-Jun-08.

Eclipses from the set **G⁶** (of type **H6**): 918-Feb-28, 937-Feb-28, 956-Feb-28, 964-Mar-01, 983-Mar-01.

Eclipses from the set **G⁷** (of type **H7**): 920-Jul-18, 939-Jul-19, 966-Jul-20, 985-Jul-20.

Now for every one of the listed above eclipses from the sets **G¹** – **G⁷** in the interval AD 890-1000 we find the score for its "closeness" to the respective element of the Template: for the eclipses from **G¹** – to **H1**, from **G²** – to **H2**, and so on.

According to the proposed model, if **E** is an arbitrary solar eclipse, the score for its closeness to **H1** should be calculated in the following way:

Scores for evaluation of the closeness of **E** to the eclipse **H1**:

The total score **e₁** for the closeness of an eclipse **E** to the eclipse **H1** is the sum of the scores for **1-1** and **1-2**:

1-1 The date of the eclipse **H1** is November 11th. If the date of **E** is

- November 11 => score for **1-1**: **m** points;
- November 10 or 12 => score for **1-1**: 0.9**m** points;
- November 9 or 13=> score for **1-1**: 0.5**m** points;
- Another day => score for **1-1**: 0 points.

1-2 If the score for **1-1** is 0 points, the score for **1-2** is also 0 points; if the score for **1-1** is different from 0, the score for **1-2** is determined by the following rules. Taking into account, that the day of the week on which the eclipse **H1** occurred is Monday, if the day of the week on which occurred **E** is

- Monday => score for **1-2**: 0.5**m** points;
- Tuesday or Sunday => score for **1-2**: 0.4**m** points;
- Wednesday or Saturday => score for **1-2**: 0.3**m** points;
- Another day => score for **1-2**: 0 points.

Applying these rules for **E = 923-Nov-11** (November 11, 923 was in Tuesday) we find: Score **1-1**: **m**; for **1-2**: 0.4**m**; **Total**: 1.4 **m**.

Similarly:

E = 942-Nov-11 (Friday). Score for **1-1**: **m**; for **1-2**: 0; **Total**: **m**.

Closeness to **H2**, **H3** and so on:

According to the proposed model, if **E** is an arbitrary solar eclipse, the rules for calculating the score for its closeness to **H2** are similar to that for the closeness to **H1**. Applying them to the eclipses of the set **G²** in the interval AD 890-1000, we find the scores for their closeness to the respective element of the Template – **H2**:

E = 920-Jul-18 (Tuesday). Score for **2-1**: 0.9**m**; for **2-2**: 0.3**m**; **Total**: 1.2**m**.

E = 939-Jul-19 (Friday). Score for **2-1**: m ; for **2-2**: $0.4m$; **Total**: $1.4m$.

E = 966-Jul-20 (Friday). Score for **2-1**: $0.9m$; for **2-2**: $0.4m$; **Total**: $1.3m$.

E = 985-Jul-20 (Monday). Score for **2-1**: $0.9m$; for **2-2**: $0m$; **Total**: $0.9m$.

For the eclipses of the set G^3 we obtain:

E = 949-Dec-22 (Saturday). Score for **3-1**: $0.5m$; for **3-2**: 0 ; **Total**: $0.5m$.

E = 968-Dec-22 (Tuesday). Score for **3-1**: $0.5m$; for **3-2**: $0.5m$; **Total**: m .

For the eclipses of the set G^5 we obtain:

913-Jun-07 (Monday, Easter on March 28). Score for **5-1**: $0.5m$; for **5-2**: $0.3m$;

Score for Easter in the year of E_5 : p ; **Total**: $0.8m + p$.

978-Jun-08 (Saturday, Easter on April 11). Score for **5-1**: $0.9m$; for **5-2**: 0 ;

Score for Easter in the year of E_5 : 0 ; **Total**: $0.9m$.

For the eclipses of the set G^7 we obtain:

920-Jul-18 (Tuesday). Score for **7-1**: $0.5m$; for **7-2**: 0.4 ; **Total**: $0.9m$.

939-Jul-19 (Friday). Score for **7-1**: $0.9m$; for **7-2**: 0 ; **Total**: $0.9m$.

966-Jul-20 (Friday). Score for **7-1**: m ; for **7-2**: 0 ; **Total**: m .

985-Jul-20 (Monday). Score for **7-1**: m ; for **7-2**: $0.5m$; **Total**: $1.5m$.

Now for the lunar eclipses H4 and H6

According to the proposed model, if E is an arbitrary lunar eclipse, the score for its closeness to $H4$ should be calculated in the following way:

Scores for evaluation of the closeness of E to the eclipse $H4$:¹

The total score e_4 for the closeness of a lunar eclipse E to the lunar eclipse $H4$ is the sum of the scores for **4-1**, **4-2** and **4-3**:

4-1 The date of the eclipse $H4$ is September 27. If the date of E is

- September 27 => score for **4-1**: m points;
- September 26 or 28 => score for **4-1**: $0.9m$ points;
- September 25 or 29 => score for **4-1**: $0.5m$ points;
- Another day => score for **4-1**: 0 points.

4-2 If E was seen in the Eastern part of the Empire (Jerusalem), the score for **4-2** is $0.5m$ points, otherwise 0 points.

4-3 If E was not seen in the Western part of the Empire, the score for **4-3** is $1.5m$ points, otherwise 0 points.²

If the score for **4-1** is 0 points, the scores for **4-3** and **4-2** are also 0 points.

Applying them to the eclipses of the set G^4 in the interval AD 890-1000, we find the scores for their closeness to the respective element of the Temple – $H4$:

¹ These rules are slightly different from the respective version in the paper [Tabov, 2013], but are identical with the version in the paper [Tabov & Umlenski, to appear].

² The above mentioned difference is in this rule,

953-Sep-25 Score for 4-1: 0.5m; for 4-2: 0.5m; for 4-3: 0; Total: m.

972-Sep-25 Score for 4-1: 0.5m; for 4-2: 0.5m; for 4-3: 0; Total: m.

991-Sep-26 Score for 4-1: 0.9m; for 4-2: 0.5m; for 4-3: 1.5m; Total: 2.9m.

Similarly for the eclipses of the set G^6 in the interval AD 890-1000 we find the scores for their closeness to the respective element of the Template – H6:

918-Feb-28 Score for 6-1: 0.5m; for 6-2: 0.4m; Total: 0.9m.

937-Feb-28 Score for 6-1: 0.5m; for 6-2: 0; Total: 0.5m.

956-Feb-28 Score for 6-1: 0.5m; for 6-2: 0.3m; Total: 0.8m.

964-Mar-01 Score for 6-1: 0.9m; for 6-2: 0; Total: 0.9m.

983-Mar-01 Score for 6-1: 0.9m; for 6-2: 0.4m; Total: 1.3m.

According to our analysis of the data in the *Chronicle*, three of the intervals between the seven eclipses are most probably exact, or almost exact – between H1 and H2, between H3 and H4 and between H6 and H7. In accordance with this vision the model suggests higher scores for closeness of the respective intervals in a septet and in the Template. Therefore we will form preliminary suitable pairs of eclipses among the chosen above, so that the interval between the eclipses in every such pair is close (in length) to the respective interval in the Template; this procedure will make easier the further choice of septets of eclipses with highest scores.

We start with pairs of eclipses from $G^1 - G^2$ (here the points for the interval are determined by the following rule: If the interval between E_1 and E_2 is: 16 years $\Rightarrow f_1 = n$ points, 15 or 17 years $\Rightarrow f_1 = 0,9n$ points, 14 or 18 years $\Rightarrow f_1 = 0,4n$ points, in other cases $f_1 = 0$ points.):

$E_1 = 923\text{-Nov-11}$ (Score 1.4 m) & $E_2 = 939\text{-Jul-19}$ (Score 1.4 m)

The interval between **923-Nov-11 & 939-Jul-19** equals **16 years** \Rightarrow The score for this interval equals **n**.

Total score for the pair **923-Nov-11 & 939-Jul-19** $\Rightarrow 2.8m + n$.

Similarly:

$E_1 = 942\text{-Nov-11}$ (Score m) & $E_2 = 966\text{-Jul-20}$ (Score 1.3 m)

The interval between **942-Nov-11 & 966-Jul-20** equals **24 years** \Rightarrow the score for this interval equals **0**.

Total score for the pair **942-Nov-11 & 966-Jul-2** $\Rightarrow 2.3m$.

Similarly for the pairs of eclipses from $G^3 - G^4$ (here the points for the interval are determined by the following rule: If the interval between E_3 and E_4 is: 5 years $\Rightarrow f_3 = n$ points, 4 or 6 years $\Rightarrow f_3 = 0,9n$ points, 3 or 7 years $\Rightarrow f_3 = 0,4n$ points, in the other cases $f_3 = 0$ points.):

$E_3 = 428\text{-Dec-22}$ (Score 1.3 m) & $E_4 = 432\text{-Sep-25}$ (Score m)

The interval between **428-Dec-22 & 432-Sep-25** equals **4 r.** \Rightarrow The score for this interval equals **0.9n**.

Total score for the pair **428-Dec-22 & 432-Sep-25** $\Rightarrow 2.3m + 0.9n$.

$E_3 = 447\text{-Dec-23}$ (Score 1.4 m) & $E_4 = 451\text{-Sep-26}$ (Score 1.4 m)

The interval between **447-Dec-23 & 451-Sep-26** equals **4 r.** \Rightarrow The score for this interval equals **0.9n**.

Total score for the pair **447-Dec-23 & 451-Sep-26** $\Rightarrow 2.8m + 0.9n$.

$E_3 = 447\text{-Dec-23}$ (Score 1.4 m) & $E_4 = 489\text{-Sep-25}$ (Score m)

The interval between **447-Dec-23 & 489-Sep-25** equals **34 r.** \Rightarrow The score for this interval equals **0**.

Total score for the pair **447-Dec-23 & 451-Sep-26** $\Rightarrow 2.4m$.

$E_3 = 949\text{-Dec-22}$ (Score 0.5 m) & $E_4 = 953\text{-Sep-25}$ (Score m)

The interval between **949-Dec-22 & 953-Sep-25** equals 4 r. => The score for this interval equals **0.9n**.

Total score for the pair **949-Dec-22 & 953-Sep-25** => $1.5m + 0.9n$.

E₃ = 968-Dec-22 (Score m) & E₄ = 972-Sep-25 (Score m)

The interval between **968-Dec-22 & 972-Sep-25** equals 4 r. => The score for this interval equals **0.9n**.

Total score for the pair **968-Dec-22 & 972-Sep-25** => $2m + 0.9n$.

E₃ = 968-Dec-22 (Score m) & E₄ = 991-Sep-26 (Score 2.4 m)

The interval between **968-Dec-22 & 991-Sep-26** equals 23 r. => The score for this interval equals **0**.

Total score for the pair **968-Dec-22 & 991-Sep-26** => $3.4m$.

Pairs of eclipses from **G⁶ – G⁷** (here the points for the interval are determined by the following rule: If the interval between **E₆** and **E₇** is: 1 year => $f_3 = n$ points, 0 or 2 years => $f_3 = 0,9n$ points, 3 years => $f_3 = 0,4n$ points, in other cases $f_7 = 0$ points):

E₆ = 918-Feb-28 (Score 0.9 m) & E₇ = 920-Jul-18 (Score 0.9 m)

The interval between **918-Feb-28 & 920-Jul-18** equals 4 r. => The score for this interval equals **0.9n**.

Total score for the pair **918-Feb-28 & 920-Jul-18** => $1.8m + 0.9n$.

E₆ = 937-Feb-28 (Score 0.5 m) & E₇ = 939-Jul-19 (Score 0.9 m)

The interval between **937-Feb-28 & 939-Jul-19** equals 2 r. => The score for this interval equals **0.9n**.

Total score for the pair **937-Feb-28 & 939-Jul-19** => $1.4m + 0.9n$.

E₆ = 964-Mar-01 (Score 0.9 m) & E₇ = 966-Jul-20 (Score m)

The interval between **964-Mar-01 & 966-Jul-20** equals 2 r. => The score for this interval equals **0.9n**.

Total score for the pair **964-Mar-01 & 966-Jul-20** => $1.9m + 0.9n$.

E₆ = 983-Mar-01 (Score 1.3 m) & E₇ = 985-Jul-20 (Score 1.5m)

The interval between **983-Mar-01 & 985-Jul-20** equals 2 r. => The score for this interval equals **0.9n**.

Total score for the pair **983-Mar-01 & 985-Jul-20** => $2.8m + 0.9n$.

For calculating the scores of the septets we have to take into account the following rules for giving scores for the intervals:

If the interval between **E₂** and **E₃** is: 29 years => $f_2 = 0.2n$ points, 28 or 30 years => $f_2 = 0,1n$ points, in the other cases $f_2 = 0$ points.

If the interval between **E₄** and **E₅** is: 7 years => $f_4 = 0.2n$ points, 6 or 8 years => $f_4 = 0.1n$ points, in the other cases $f_4 = 0$ points.

If the interval between **E₅** and **E₆** is: 5 years => $f_5 = 0.2n$ points, 4 or 6 years => $f_5 = 0.1n$ points, in the other cases $f_5 = 0$ points.

6. Conclusion

The analysis of the above results shows that highest scores have (and consequently are most perspective) the following two septets:

I. Septet 923 & 939 (Score $2.8m + n$), 968 & 972 (Score $2m + 0.9n$), 978 (Score $0.9m$), 983 & 985 (Score $2.8m + 0.9n$).

If the interval between **E₂** and **E₃** is 29 years => $f_2 = 0.2n$.

If the interval between **E₄** and **E₅** is 6 years => $f_4 = 0.1n$.

If the interval between E_5 and E_6 is 5 years $\Rightarrow f_5 = 0.2n$.

Total score: $2.8m + n + 2m + 0.9n + 0.9m + 2.8m + 0.9n + 0.2n + 0.1n + 0.2n = 8.5m + 3.3n$.

II. Septet. 923 & 939 (Score $2.8m + n$), **968 & 991** (Score $3.4m$), **978** (Score $0.9m$) **983 & 985** (Score $2.8m + 0.9n$).

If the interval between E_2 and E_3 is 29 years $\Rightarrow f_2 = 0.2n$.

If the interval between E_4 and E_5 is -14 years $\Rightarrow f_4 = 0$.

If the interval between E_5 and E_6 is 6 years $\Rightarrow f_5 = 0.1n$.

Total score: $2.8m + n + 3.4m + 0.9m + 2.8m + 0.9n + 0.2n + 0.1n = 9.9m + 2.2n$.

For $m = n = p = 10$ we have:

$$12m + 3.6n + p = 166;$$

the distance from **I. Septet** to G_H equals

$$\begin{aligned} d &= 12m + 3.6n + p - (e_1 + e_2 + \dots + e_7 + f_1 + f_2 + \dots + f_6 + g) \\ &= 166 - (8.5m + 3.3n) = 166 - 118 = 48, \end{aligned}$$

and the distance from **II. Septet** to G_H equals

$$\begin{aligned} d &= 12m + 3.6n + p - (e_1 + e_2 + \dots + e_7 + f_1 + f_2 + \dots + f_6 + g) \\ &= 166 - (9.9m + 2.2n) = 166 - 121 = 45. \end{aligned}$$

Hence, according to our model, **II. Septet** has advantage before the **I. Septet** and the other septets; for more precise results and conclusions further investigations are necessary; they should involve in particular analysis of the time and the phases of the eclipses.

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