Monadic Predicate Calculus Arithmetized *à la* Leibniz

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Leibniz had undertaken a few unsuccessful attempts to translate syllogistic into arithmetic and so to embody his winged "Calculemus!". In [1], Leibniz's idea was realized in two models using divisibility of integers, l.c.m., g.c.d., reciprocals, etc., as arithmetical interpretations of syllogistic relations, term conjunction, term negation, etc. In this paper, Leibniz style translation is extended on the full monadic predicate calculus, i.e., on the logic of properties.

The language of the monadic calculus contains *individual variables* $x, y, z, \ldots$, one-place *predicate symbols* $P_1, P_2, \ldots$, quantifiers, and the usual proposiotional connectives with brackets. A *monadic proposition* is a formula without free variables. For such formulae we adopt Theorem 25.4 from [2]:

Lemma 1 *Any monadic proposition is equivalent to a monadic proposition with the same predicate symbols and one variable only.*

Let the sole variable be x. In addition, we may suppose it is not bound "twice" anywhere. So, no formula which we will consider is a Boolean combination of two subformulae $A(x)$ and B, one of them containing a *free* x and the other containing x *bound*; in $(Qx)A(x)$, where Q is a quantificator, a free x does occur in A.

To build up an arithmetical model for monadic propositions, let an arbitrary integer $u > 1$ *without multiple factors* be taken, and let its divisor d_i be associated with the predicate $P_i(x)$. Further, following the construction of the formula, a *divisor* of u will be associated with any subformula containing a free x, and a *statement* about divisors will be associated with the subformula when it does not contain a free x: if a and b are associated with $A(x)$ and $B(x)$, then g.c.d.(a, b) is associated with $A(x)\&B(x)$, $\frac{u}{a}$ with $\neg A(x)$, and so on for other Boolean connectives; the statements $a = u$ and $a > 1$ are associated with $(\forall x)A(x)$ and $(\exists x)A(x)$, respectively; if statements p and q are associated with subformulae A and B, then "p and q" and "not p" will be associated with $A\&B$ and $\neg A$, respectively. Finally, a certain statement comparing divisors of u with u and 1 will model the initial monadic proposition. If this statement is an arithmetical truth for an arbitrary integer u, the proposition is called *arithmetically true*. Using that any predicate tautology is equivalent to a closed formula, we obtain the main

Theorem 1 *Any monadic formula is a predicate tautology iff its corresponding monadic proposition is arithmetically true.*

If this model is applied to the traditional predicate translations of the Aristotelian syllogisms, the *Scholastic* arithmetical semantics from [1] will be obtained. A dual model corresponding to the *Leibnizian* semantics is possible, too.

References

[1] V. Sotirov, "Arithmetizations of syllogistic à la Leibniz", *J. Appl. Non-Class. Logics* (*to appear*).

[2] G. Boolos, R. Jeffrey, "Computability and Logic", Cambridge Univ. Press, 3rd ed. 1989.