

Vector Schrodinger equation: Nonlinearity, Multidimensionality, Integrability. What to Prefer?

Michail D. Todorov

For a system of nonlinear Schrödinger equations (SCNLSE) coupled both through linear and nonlinear terms, we investigate numerically the head-on and taking-over collision dynamics of polarized solitons. In the case of general elliptic polarization, analytical solutions for the shapes of steadily propagating solitons are not available, and we develop an auxiliary numerical algorithm for finding the initial shape. We use a superposition of polarized solitons as the initial condition for investigating the soliton dynamics (sech-like as well as general form). We consider the interactions with and without cross-modulation, with stationary shapes and with breathers. For general nontrivial cross-modulation, a jump in the polarization angles of the solitons takes place after the collision (polarization shock). For moderate and large values of the nonlinear coupling parameter, additional solitons are created during the collision of the initial ones.

We also find that, depending on the initial phases of the solitons, the polarizations of the system of solitons after the collision change, even for trivial cross-modulation. This sets the limits of practical validity of the celebrated Manakov solution. In the majority of cases the solitons survive the interaction, preserving approximately their phase speeds and the main effect is the change of individual polarization but the total net polarization of the system is conserved. However, in some intervals for the initial phase difference, the interaction is ostensibly inelastic: either one of the solitons virtually disappears, or additional solitons are born after the interaction.

In the case of pure linear coupling the Manakov system is enriched by a linear coupling term with complex-valued coefficient and the individual solitons are actually either blowing or dispersing breathers. The momentum of the individual quasi-particles (QPs) is conserved while the masses of the individual QPs oscillate, but the sum of the masses for the two QPs is constant. Respectively, the total energy oscillates during one period of the breathing, but the average over the period is conserved. The individual and total polarization angles for the two QPs oscillate with different periods before and after the interaction. This extends to the case of breathing our earlier results about the conservation of the total polarization for the interaction of non-breathing solitons.

The results of this work elucidate the role of the linear and nonlinear couplings, the initial phase, and the initial polarization on the interaction dynamics of soliton systems in SCNLSE. For moderate and large values of the nonlinear coupling parameter, additional solitons are created during the collision of the initial ones. Depending on the initial phases of the solitons, the polarizations of the system of solitons after the collision change, even for trivial cross-modulation. All this sets the limits of practical validity of the celebrated Manakov solution. The Manakov system loses its full integrability and the approach for its study is numerical. All the numerical experiments and computer simulations are implemented by using a fully conservative difference scheme in complex arithmetic.

Some information about the transition from Manakov model to perturbed Vector Schrodinger equation can be obtained by the so-called Complex Toda chain (CTC). We detail an asymp-

totic description of the interaction between N solitons in adiabatic approximation. These original equations are perturbed by gain/loss terms, periodic, polynomial, single and composite well (hump) external potentials. The gain and loss are taken to balance so that the solutions do not blow up or decay away. The distance between the solitons, however, is taken to be large, so that they only interact in their tails, which is the basis for the asymptotic analysis. The inverse of this large separation is the perturbation parameter. The evolution of the soliton parameters is governed by a discrete system related to the complex Toda Chain. We derive the corresponding perturbed complex Toda Chain (PCTC) models for both SCNLSE and Manakov model. We show that the soliton interactions dynamics for the PCTC models compares favorably to full numerical results of the original perturbed SCNLSE and Manakov model. Unfortunately, the cross-modulation in SCNLSE sets the limits of practical validity of the Manakov solution and corresponding CTCs.