Review

On DrSci dissertation
Extremal problems in Euclidean Combinatorial Geometry
For the scientific degree "Doctor of Sciences" in
Area of higher education: 4. „Natural Sciences, mathematics and informatics"
Professional area: 4.5 „Mathematics"
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This review is prepared and presented on the basis of order № 68/25.03.2024 of the Director of Institute of Mathematics and Informatics, Bulgarian Academy of Sciences, for appointment of Scientific Jury and the minutes of the first meeting of the Jury conducted on 27.03.2024.

1. **Presentation of the candidate.** Danila Cherkashin has completed secondary education in 2009 in St. Petersburg and higher education (MS) in 2015 in State University of St. Petersburg. He has defended PhD dissertation in professional area 4.5 (Mathematics) in 2018 entitled "Extremal problems in hypergraph coloring" after a PhD procedure at Petersburg Department of Steklov Mathematical Institute of Russian Academy of Sciences (PDMI-RAS). In 2018-2022 he worked at PDMI-RAS and since 2022 he is with Institute of Mathematics abd Informatics of the Bulgarian Academy of Sciences (IMI-BAS) as a posdoc (2022-2023) and associate professor since July 2023.

2. **Materials presented and preliminary defense.** All required for preliminary defense admission and defense admission materials are presented in IMI-BAS in time and correspondingly to the requirements of the Regulations of IMI. The preliminary defense is conducted on 12.03.2024, followed by a positive evaluation for the defense preparadness.

3. **Actuality of the topic and approprieteness of the set goals and objectives.** The dissertation consists of three main parts. The first part (Chapters 2-4) considers certain minimization problems of distance in the Euclidean space. Chapter 5 determines the independence and chromatic numbers of some

Johnson-type graphs. Finally, the third part (Chapters 6-7) investigates the chromatic numbers of some geometric objects. At first sight these areas sound quite different, but in reality they are quite close to each other in nature: they all optimize some parameters related to elementary objects.

Since the objects considered are simple, the solutions of these optimization problems need strong problem solving skills. On the other hand their simplicity makes them mathematical problems to be ``one step from practical applications". The fact that these problems are in focus of international interest is demonstrated with the recent papers in the areas written by famous authors like Peter Frankl, Daniela Kühn and Deryk Osthus.

Summarizing, I am convinced that the topic of the dissertation is sufficiently relevant on an international scale.

4. Characterization, evaluation and contributions of the dissertation work. The dissertation consists of 7 chapters. Chapter 1 is just a concise introduction.

Chapter 2 considers problems concerning the Steiner trees. The basiuc question here is to find the minimum length of a set making a given (finite or infinite) set connected in the Euclidean space. The chapter contains 3 three interesting new results.

Edelbrunner and Strelkova have asked whether the measure of n-point ambigouos configurations is zero or not.Theorem 2.1.1- gives the answer in a much stronger form: its Hausdorff measure is 2n-1.

Theorem 2.1.4 is a far reaching improvement of Edelsbrunner's and Strelkova's result, claiming that the n-point configurations in 2-dimension having a unique Steiner tree are path-connected. The proof of this theorem is mainly based on some clever transformations.

Theorem 2.2.1 gives an improved construction of an infinite Steiner tree connecting an uncountable set. The fact that this problem is in the focus of interest is proved by the recent fast answer of other authors slightly improving this construction.

Conjecture 2.5.1 shows the way toward further research.

Chapter 3 is dealing with the Gilbert-Steiner problem. This is strongly related to to the problem of Steiner trees, but here some masses are transported through the connections. The main result of this chapter is

Theorem 3.3.1 claiming that if a planar flow has a branching point of degree at least 4 then there is a flow with strictly smaller Gilbert functional. A consequence is Theorem 3.1.2 stating that the planar Gilbert-Steiner problem has no branching point of degree 4 or more. Section 3.5 gives many good open problems for the future generations.

The longest (54 pages) chapter is Chapter 4. Its basic problem is to minimize the largest distance from a given set M of points. In the dual problem we are looking for a set of minimal length which has distance at most r from M.

Here Theorem 4.2.3 gives a construction M with an infinite number of corner points such that it has a unique maximal minimizer for M, which is also determined. Theorem 4.2.4 gives a class of simple curves which are maximal distance minimizers.

The main result of this chapter is Theorem 4.3.2. Earlier Miranda, Paolini and Stepanov conjectured that a certain nice construction (horseshoe) is the solution for a class of problems where M has a bound on the radius of minimal curvature. Theorem 4.3.2 proves this conjecture for a large class, this is why it can be called the ``Horseshoe Theorem". Its proof contains many steps with many ideas.

At the end of the chapter there is a long list (2 pages) of interesting open questions.

The Johnson-type graphs in Chapter 5 are closely related to Extremal Set Theory. The main results in this chapter are on the one hand Theorems 5.1.7, 5.1.8 and 5.1.9 exactly determining the independence numbers of some natural Johnson-type graphs, on the other hand Theorems 5.1.10, 5.1.11 and 5.1.12 giving quite tight lower and upper estimates on the chromatic numbers of some other Johnson-type graphs. A pleasant surprise for the reviewer is that his method was used in the proofs. The constructions are based on Reed-Solomon codes.

Chapter 6 investigates the chromatic number of the infinite graph whose vertex set is a two-dimensional sphere of radius *r* and two vertices are adjacent if their Euclidean distance is exactly 1. It is trivial that this chromatic number is 2 if *r* is ½, but it was unclear what happens if *r* is a little bit larger. Simmons conjectured in 1976 that the chromatic number becomes 4. Now (after 48 years!) Cherkashin and Voronov proved the conjecture. This is a real breakthrough! The proof is, of course, long and clever.

The open questions are not missing at the end of this chapter either.

Chapter 7 studies the chromatic number of a higher-dimensional object, called slice. The main result of the chapter determined this chromatic number for the geometric object which is a product of 3 copies of the real line and 6 copies of the interval [0, epsilon] where epsilon is a small positive number. Theorem 7.1.2 claims that this chromatic number is between 10 and 15. The upper bound easily comes from known results, but the lower bound is quite difficult to prove. The proof is based on Theorem 7.1.3 a statement on the coloring of regular simplices in dimension d and is of an independent interest. Unfortunately there is only one open problem at the end of this chapter.

In conclusion, this is an excellent work by international standards, containing important results with difficult proofs.

5. Critical remarks and recommendations. The number of typos is very low, two examples, just to show that the reviewer have read the thesis: on page 8 in Theorem 2.1.1 n should be at least 4, not at most, in the last row of page116 n should be replaced by d.

6. Contributions of the dissertation. The abstract (autoreferat) presents a good reference to the scientific contributions in the dissertation. I accept this reference and completely agree with its claims. The results obtained and their number meet, in my opinion, the high standards of IMI-BAS for a DrSci dissertation.

7. Personal impressions. I have never met the author. I saw only his ``defence", online. He gave the impression of a real scientist.

CONCLUSION

The submitted dissertation, the abstract, and the related scientific works show that Associate Professor Danila Cherkashin is a strong scientist in the field of mathematics, namely in the areas of extremal problems for graphs, finite sets and geometry. He has siginificant new results in these areas.

The dissertation contains original theoretical results which are very close to practical applications. The publications meet all the requirements of the Law on the Development of the Academic Staff in the Republic of Bulgaria (ZRASRB), the Regulations for the Implementation of ZRASRB and the relevant Regulations of IMI-BAS. The presented materials and the result in the dissertation fully correspond to the specific requirements of IMI-BAS, adopted in connection with the Regulations of IMI-BAS for the application of RASRB.

The materials presented by the candidate do not repeat those from previous procedures fro aquiring a scientific title and occupying anacademic position. No plagigarizm was found in the thesis and in the related publications.

The dissertation shows that Associate Professor Danila Cherkashin has indepth theoretical knowledge and professional skills. It demonstartes qualities and skills for conducting research with original and globally significant contributions.

Due to the above I confidentally give my positive assesment of the conducted research presented in the above-reviewed dissertation, the abstract, the results achieved and the scientific contributions, and I propse the honorable scientific jury to award the scientific degree ``Doctor of Sciences" to Danila Cherkashin in area of higher education 4. „Natural sciences, mathematics and informatics", professional area 4.5 „Mathematics".

Budapest, April 22, 2024

Signature:

Gyula OH Katona

Professor, Dr.Sci.