R E P O R T

for a Dissertation

for obtaining the scientific degree "Doctor of Science"

Research area: 4. Natural Sciences, Mathematics and Informatics,

Professional field: 4.5 Mathematics

Author: Danila Dmitrievich Cherkashin
Title: Extremal problems in Euclidean combinatorial geometry

Referee: Prof. Stefka Hristova Bouyuklieva
Faculty of Mathematics and Informatics,
St. Cyril and St. Methodius University of Veliko Tarnovo

I am a member of the scientific panel for this procedure according to order № 68/25.03.2024 of the Director of the Institute of Mathematics and Informatics. As a member of the scientific panel, I have received all the administrative and scientific documents required by the Act on the Development of the Academic Staff in the Republic of Bulgaria (ADASRB), the Rules for its implementation and the Rules on the terms and conditions for awarding of academic degrees and occupying of academic positions at the Institute of Mathematics and Informatics and Bulgarian Academy of Sciences.

1. Content analysis of the scientific and applied achievements in the dissertation. Characterization of the main achievements.

The dissertation is dedicated to research on some important extremal problems that fall simultaneously in the field of Euclidean geometry and combinatorics. The author focuses on the following classic problems: Steiner tree, finding the maximal independent set, finding the chromatic number. The considered problems belong to the class of NP-complete problems.

The dissertation consists of seven chapters, the first of which is an introduction, and a bibliography, presented in 147 pages. The topic is relevant, as evidenced by the publications and their citations.

Chapter 2 is devoted to Euclidean Steiner tree problem. Both finite and infinite versions of the problem are considered, which is formulated as follows: Given a finite (compact in the infinite case) set $P \subset \mathbb{R}^d$, find a connected set *St* of minimum length (one-dimensional Hausdorff measure $\mathcal{H}^1$) containing *P* (in the infinite case we want $P \cup St$ to be a connected set). The results are published in the papers [1] and [7] from the list of publications included in the dissertation.

Chapter 3 is devoted to the Gilbert–Steiner problem, which gives a generalization of the Steiner tree problem on a specific optimal mass transport. The chapter ends with several open questions on the topic.

In Chapter 4, the problem of minimizing the maximal distance to a given compact set *M* among the sets of a given length ℓ is considered. This chapter occupies 50 pages and also contains many illustrations that are part of the proof.

Chapter 5 deals with independence numbers and chromatic numbers of Johnson-type graphs. In these graphs (denoted by $J_{\pm}(d,k,t)$), vertices are all vectors with *d* coordinates (coordinates are from the set $\{-1,0,1\}$) that have Hamming weight *k*, with two vertices connected by edge if and only if their scalar product is equal to a given integer *t*. In the dissertation, the exact values of the independence numbers of Johnson-type graphs are obtained in several important cases. Chromatic numbers are related to the colorings of a given object *M* (in this chapter, a graph) and represent the minimum number of colors that allows proper coloring of *M*. In this chapter, the author presented the asymptotics of the chromatic number of Johnson-type graphs for $k = 3$ and $t = -2$.

In Chapter 6, a conjecture from 1976 is proved, which states that every coloring of a 2-dimensional sphere of radius strictly greater than 1/2 in three colors has a pair of monochromatic points at distance 1 apart. In the last seventh chapter, problems on the chromatic number of 3-dimensional slices are considered.

The dissertation is very well structured. Each of the chapters begins with a brief introduction to the problem (or problems) under consideration, then moves to the essence of the development, and finally ends with a presentation of opportunities for further work on open questions in the subject. The proofs are complete and very well illustrated. I will use Chapter 3 as an example because it is shorter than the others and easier to support my observation. This chapter consists of 5 parts - introduction (basics), preliminary results (preliminaries), main result, examples, open problems. Each of the chapters is structured in a similar way. The bibliography contains 128 titles that are cited very appropriately in the text.

2. Publications and citations.

The dissertation is based on 10 publications. Eight of the papers are in English and the remaining two are in Russian, with an English version to be published in the *Journal of Mathematical Sciences*. Papers [8] and [10] have been submitted for publication, and [9] has been accepted for publication, article [7] is cited in the list of publications as to appear but already published, and the remaining six papers have already been published in scientific journals in 2018, 2022, 2023 and 2024.

Danila Cherkashin has presented a list of 24 citations. The most cited paper is published in 2019, which, however, is not included in the list of publications on the dissertation, because it was included in the list of publications on the procedure for associate professor, but the publications that cite this paper are not included in the list with citations on that procedure. This confusion occurs because of the short time frame in which both procedures fit. I am not quite sure if it is appropriate to cite a publication that is not presented in the thesis. The paper concerns the coloring of Knesner graphs and hypergraphs, and as a topic is close to the content of the presented thesis, therefore I will acknowledge the points in the scientometrics table.

There are other inaccuracies in the presented reference to scientometric indicators. I only award 150 of the publication points because publications [5] and [6] do not actually appear in Cherkashin's Scopus profile, publication [7] is not yet indexed in WoS, and paper [9] is not yet published. However, the minimum requirement of the Regulations for the Implementation of the Law on the Development of the Academic Staff in the Republic of Bulgaria is that the points from publications on the dissertation must be at least 100, so these 150 points are sufficient. Citation scores are also more than 100. Scores on these metrics exceed the minimum requirements of law and regulation.

Only one of the publications is an independent work, and of the remaining nine articles, three are co-authored by Yana Teplitskaya, another three have one co-author each (respectively Kiselev, Voronov and Petrov) and three have three co-authors each (Voronov, Kanel-Belov, Stryukov, Basok, Rastegaev, Teplitskaya). I believe that Cherkashin's contribution to the joint publications is substantial.

3. The author’s summary

The author's summary is made in accordance with the rules and accurately and completely assesses the main results obtained in the thesis.

4. Conclusion

The presented dissertation satisfies all the criteria and indicators of the law and the regulations in Bulgaria. After I familiarized myself with the presented dissertation, the importance of the research and the scientific and applied contributions contained therein, I give an overall positive assessment to the applicant Danila Cherkashin to obtain the scientific degree "Doctor of Sciences" in

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25.04.2024 г. Jury member:

/Prof. Stefka Bouyuklieva/