REPORT

by **Assoc. Prof. Andrey Andreev Vassilev, PhD**,

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on the procedure for a public defence of a PhD thesis

of **Dragomir Colev Nedeltchev**

entitled:

"Application of stochastic and optimization methods for risk management and pricing of financial instruments"

for conferring the educational and scientific degree "Doctor"

Scientific Advisor: Assoc. Prof. Tsvetelin Zaevski, DSc

Scientific Field: 4. "Natural Sciences, Mathematics, and Informatics",
Professional Field: 4.5 "Mathematics",
Doctoral Program: "Probability Theory and Mathematical Statistics",
Department: Operations Research, Probability and Statistics (ORPS),
Institute of Mathematics and Informatics (IMI),
Bulgarian Academy of Sciences (BAS).

This report has been prepared in my capacity as member of the Scientific Jury according to Order № 90/18.11.2025 of the Director of IMI-BAS. This report has been prepared in accordance with the requirements of the Act on Development of the Academic Staff in the Republic of Bulgaria (ADASRB), The Rules for Implementation of the ADASRB, the Rules on the Terms and Requirements for Acquisition of Scientific Degrees and Occupation of Academic Positions of BAS and the corresponding Rules of IMI-BAS.

1 General characterisation of the PhD thesis and the presented materials

The main text of the thesis consists of 124 pages in English and is structured in 11 chapters and a bibliography of 196 titles. The materials accompanying the thesis contain the necessary information about the scientific contributions and the approbation of the results

obtained. The required documents are presented, certifying that the minimum national requirements of the ADASRB have been met.

The thesis is devoted to current issues in the field of risk management and financial instrument valuation and combines scientific contributions of both a theoretical and applied nature. The structure and content of the thesis meet the generally accepted standards for a work on the basis of which the educational and scientific degree "Doctor" may be awarded.

2 Main scientific and applied scientific achievements

The first chapter of the thesis is introductory in nature and presents the objectives and methodology of the thesis, as well as motivates the relevance of the topic. The chapter also provides a detailed review of the scientific literature related to the study, which testifies to the doctoral student's extensive knowledge in the field.

The second chapter provides preliminary information in the form of notation, definitions, and key results on risk measures and the random processes used in the thesis.

The third chapter presents the main models of logarithmic asset returns used in the thesis. The constant volatility model (Black-Scholes), exponential tempered stable model, Heston's stochastic volatility model, Bates' stochastic volatility jump model, and a stochastic volatility/tempered stable model are succinctly formulated.

The fourth chapter defines two quantile risk measures – value at risk (VaR) and expected shortfall – and states selected results characterizing these measures.

The fifth chapter is devoted to expectile-based value at risk. The chapter introduces the concept of expectile and defines an expectile risk measure based on it. The chapter formulates and proves, under specific assumptions, results on the relationship between cumulative distribution functions and characteristic functions, as well as representations of truncated expectations using the characteristic function. On this basis, a result is proven that offers a way to calculate the expected shortfall and the expectile risk measure from the characteristic function of the corresponding distribution.

The sixth chapter proposes an approach to overcome the problem of unobservable initial volatility for the Cox-Ingersoll-Ross (CIR) process, which is used to model volatility in the Heston model. The approach is based on averaging over the volatility using the stationary (Γ-)distribution of the CIR process. It is shown that the procedure for averaging the characteristic function in the Heston model is well defined and leads to a new characteristic function. On this basis, the generating function obtained in the averaging procedure is considered, and it is shown that its domain of convergence is a subset of the domain of convergence of the original generating function for the Heston model. A result is formulated showing the calculation of the limits of the convergence interval and giving a formula for the generating function in this interval. A parametric analysis of the positions of the abscissas of convergence is performed over the coefficient ρ .

The seventh chapter defines a version of the concept of entropic VaR (EVaR) adapted for applications in finance. The chapter obtains some of the EVaR properties and proposes a method for its derivation through the generating function. On this basis, the generating

functions of the models from Chapter 3 are examined in sequence and methods for calculating the corresponding EVaR are provided. The EVaR measure is also derived for the case of averaging over the volatility in the Heston model.

The eighth chapter of the PhD thesis is empirical in nature and is based on daily data from the S&P 500 index. Empirical density functions are obtained for this data using a rolling window of 1000 days. The chapter proposes an approach for calibrating the parameters of the theoretical probability densities for the models considered in Chapter 3. The results of applying this approach to the sample data from the S&P 500 index are tested in terms of predictive accuracy, leading to the conclusion that most models (with the exception of the Black-Scholes model) have similar predictive power. The chapter employs the S&P 500 data to calculate and analyse the risk measures discussed above: value at risk, expected shortfall, and expectile risk measure. EVaR is also calculated for the models discussed, and the results are analysed.

The ninth chapter discusses a new "stylized fact" from the financial literature called *rough volatility*. It examines the possibility of modelling this phenomenon using fractional Brownian motion with an appropriate Hurst index. The chapter uses public data on realized variance from 31 stock market indices. The empirical work in the chapter confirms the monofractal property established by previous studies and finds that for major indices, the relationship between the scaling factor and the Hurst index is nonlinear. It is shown that the Hurst index increases during the COVID-19 pandemic.

The tenth and eleventh chapters are summary in nature and present a synthesis of the results obtained, together with directions for future work, as well as the main contributions of the thesis.

3 Approbation of the results

Some of the results in the thesis are published in the following articles:

1. T.S. Zaevski and D. Nedeltchev. From BASEL III to BASEL IV and beyond: Expected shortfall and expectile risk measures. International Review of Financial Analysis, 87:102645, 2023. ISSN 1057-5219. doi: https://doi.org/10.1016/j.irfa.2023.102645. **JCR-IF (Web of Science): 7.5 (2023), Q1 (Finance)**

2. T.S. Zaevski and D. Nedeltchev. Moment generating function of the averaged log-returns in the Heston's stochastic volatility model. Proceedings of the Bulgarian Academy of Sciences, volume 78, pp. 321–330, 2025. **JCR-IF (Web of Science): 0.3 (2024) Q4 (Mathematics)**

3. D. Nedeltchev and T. Zaevski. Volatility structure during financial turbulence. In Proceedings of the International Scientific Conference IMEA'2024, pp. 47-60, 2024.

Some of the results in the thesis have been presented at:

1. "Seminar on Advances in Statistics" – 12.03.2023

2. Scientific conference "Recent Developments in Stochastic Processes" – 29.03.2023

3. SUMMIT Annual Conference of Sofia University "St. Kliment Ohridski" – 24.04.2024

4. Scientific conference "Informatics, Mathematics, Education and Their Application – IMEA'2024" – 14.11.2024

5. Scientific conference "November Days Of Mathematics And Informatics" – 29-30.11.2024

6. SUMMIT Second Annual Conference of Sofia University "St. Kliment Ohridski" – 02.04.2025

7. 19$^{\text{th}}$ International Summer Conference on Probability and Statistics – 26.07.2025

Publication 1 from the above list has been cited 22 times, including 11 citations in outlets indexed in Scopus and Web of Science. Publication 3 from the above list has one citation in an indexed journal. The number of citations of these articles in various publications within a short period of time testifies to the significance of the respective results in both academic and applied aspects.

4 Critical comments and recommendations

I have no significant critical comments on the thesis and the materials presented. As remarks of a secondary nature, I would note that there are occasional minor linguistic omissions in the text of the thesis, and that the work would benefit from more details on the computational implementation of the empirical work presented. This in no way detracts from the merits of the thesis.

5 Qualities of the thesis abstract

The abstract in English is 32 pages long and introduces the topic and objectives of the dissertation, presents the main results, summarizes the scientific contributions made, and provides directions for future work. The abstract accurately reflects the content and achievements of the doctoral thesis. The 34-page abstract in Bulgarian corresponds to the English version.

6 Conclusion

The submitted doctoral thesis and accompanying scientific publications contain original results of both theoretical and applied value. The materials related to the procedure meet

the minimum national requirements in the professional field, and no plagiarism has been detected in the thesis and scientific publications presented. The above analysis shows that the submitted thesis meets the requirements of ADASRB, The Rules for Implementation of the ADASRB, the Rules on the Terms and Requirements for Acquisition of Scientific Degrees and Occupation of Academic Positions of BAS and the corresponding Rules of IMI-BAS.

Based on the above, I give a positive assessment of the doctoral thesis *"Application of stochastic and optimization methods for risk management and pricing of financial instruments"* and strongly recommend that the Scientific Jury award **Dragomir Colev Nedeltchev** the educational and scientific degree "Doctor" in Scientific Field 4. "Natural Sciences, Mathematics, and Informatics", Professional Field 4.5 "Mathematics", Doctoral Program "Probability Theory and Mathematical Statistics".

05.01.2026

Member of the Scientific Jury:

/Assoc. Prof. Andrey Vassilev, PhD/