Short Report

for the award of the educational and scientific degree "Doctor" (PhD) at the Institute of Mathematics and Informatics, Bulgarian Academy of Sciences in the field of higher education 4. Natural Sciences, Mathematics and Informatics professional field 4.5 Mathematics; PhD program "Probability Theory and Mathematical Statistics", prepared by Prof. Dr. Sc. Mladen Svetoslavov Savov from the department of "Probability, Operations Research and Statistics", Faculty of Mathematics and Informatics, Sofia University "St. Kliment Ohridski" (FMI-SU) and department of "Operations Research, Probability and Statistics", Institute of Mathematics and Informatics, Bulgarian Academy of Sciences (IMI-BAS), member of the jury in accordance with Order No. 90/18.11.2025 of the Director of IMI-BAS and writing a short report according to the decision of the jury during the first meeting held on 26.11.2025.

The opinion was prepared in compliance with Order No. 90/18.11.2025 of the Director of IMI-BAS, issued on the basis of a decision of the Scientific Council of IMI-BAS (Minutes No.13/14.11.2025) in accordance with the requirements of IMI-BAS for the awarding of the educational and scientific degree "Doctor" (PhD), which comply with the minimum national requirements for this degree.

As a member of the jury, I received all the documents submitted by the candidate Dragomir Kolev Nedeltchev (IMI-BAS).

1. Applicant Curriculum Vitae

Dragomir Nedeltchev was born in 1965 and graduated in 1992 from the Kiev Polytechnic Institute with a master's degree in engineering. Over the next almost thirty years, the candidate held various positions in the financial sector. His last position was as a market risk assessor at Ernst & Young. Dragomir Nedeltchev is currently a PhD student under the supervision of Assoc. Dr. Tsvetelin Zaevski (IMI-BAS).

2. Fulfillment of minimal requirements

Dragomir Nedeltchev satisfies all the minimum requirements for the acquisition of the educational and scientific degree "Doctor."

3. Scientific contributions of the dissertation

The dissertation is devoted to the study of complex risk measures on the log-returns of assets, with the aim to better capture the realities of the market, the need of practitioners and possibly to meet a change driven by the regulators who are looking for better ways to assess risk and therefore prevent financial crises. In fact, the latter is the basis for the introduction of new risk measures.

It is interesting to note that the dissertation systematizes the main properties that a risk measure should satisfy and which arise from natural practical considerations. However, these properties are generally not inherent in the indicators widely used by regulators, such as Value-at-Risk (VaR). To overcome this shortcoming, Dragomir Nedeltchev proposes the use of the ERM (expectile VaR) and EVaR (entropic VaR) measures, examining in detail their fundamental characteristics. It turns out that it is precisely these measures that satisfy the basic axiomatic requirements for risk measures to a greater extent. The implementation of the measures under consideration is not carried out directly on empirical financial data (S&P 500), but on a set of five carefully selected and calibrated stochastic processes that are supposed to adequately model the dynamics of price changes. This approach allows key market characteristics to be captured, which are often lost in analysis based solely on empirical observations. With the right choice of stochastic process, this gives a much richer structure for analysis and forecasting. For these processes, Dragomir Nedeltchev has calculated the new risk measures.

I will examine the content and contributions of the relevant chapters of Dragomir Nedeltchev 's dissertation. I will note that the main contributions are in chapters 5-9, while the first four chapters are generally introductory.

Chapter 1 contains an introduction that convincingly motivates the need for introducing new risk measures through practically grounded criticisms of existing approaches. It clearly highlights that many widely used risk measures fail to satisfy several basic and natural properties that an adequate risk measure should possess. Furthermore, the applicant argues in detail that such measures should be applied to calibrated models of financial asset price dynamics rather than directly to raw empirical data. The chapter also includes a detailed, well-structured, and comprehensive review of the existing scientific literature.

Chapter 2 contains information about the main objects and their notation in the dissertation and introduces and justifies the main properties that the risk measures should satisfy.

Chapter 3 describes the main five stochastic models that are used in the dissertation to model the logarithm of market returns, namely the classical Black-Scholes model; a model based on a stable Levy processes with exponentially tampered tails; the Heston model (with stochastic volatility); the Bates model (Heston model with jumps); a pattern with stochastic volatility and endless activity of the jumps.

Chapter 4 focuses on the two principal risk measures employed by regulatory authorities, namely Value at Risk (VaR) and Expected Shortfall (ES). It is shown that VaR, which essentially corresponds to a quantile of the return distribution, fails to be subadditive unless additional assumptions are imposed on a specific class of distributions for log-returns. In contrast, ES satisfies the fundamental axioms of a coherent risk measure, but it cannot be directly subjected to back-testing using historical data. The applicant derives the key analytical expressions required for the computation of these risk measures, presents the dual representation in the case of ES, and characterizes the associated acceptance set.

Chapter 5 discusses one of the two main risk measures that are proposed in this work, namely Expectation-Based VaR (ERM). A brief literature review of the properties of ERM shows that this measure is coherent and subject to back-testing, which is a serious advantage over the measures used so far. The main purpose of Chapter 5 is to provide a formula for calculating (amenable to numerical calculation) the ERM. Therefore, in Proposition 5.1 it is shown that an expectile solves a specific equation between truncated moments of the original random variable, and therefore ERM is the inverse function of that solution. Proposition 5.2 shows its duality. After a brief introduction to the theory of Fourier transforms, in Proposition 5.3, the basic formulas for finding cumulative distribution functions by inverting the Fourier transform are obtained. Given that random variables will originate from random processes characterized by their characteristic exponents, Proposition 5.3 is necessary. Proposition 5.4 gives the truncated moments by Fourier inversion, and Theorem 5.2. provides, in the same context, the value of the risk measure.

Chapter 6 serves as a preparatory chapter for the study of entropic Value at Risk (EVaR), the second risk measure studied in the dissertation. Since EVaR is defined in terms of the moment generating function, in a manner closely related to large deviation principles, the candidate analyzes the moment generating function of the Heston model under the assumption that the initial volatility is distributed according to its stationary distribution. A key step in this analysis is the identification of the fundamental strip of analyticity of this function. This is accomplished in full generality, and the result has been published separately as an independent research article.

Chapter 7 is devoted to the study of entropic Value at Risk (EVaR). When negative exponential moments exist, the use of the Chernoff inequality yields a family of upper bounds for the classical Value at Risk, with the smallest such bound defining EVaR. This approach establishes the coherence of EVaR, and the corresponding acceptance set is explicitly characterized. Subsequently, Theorem 7.1 provides a general formula for computing this risk measure, naturally expressed in terms of the moment generating function. Finally, explicit expressions for EVaR are derived for the stochastic processes investigated in the dissertation. Since these processes are specified via their characteristic exponents, the resulting formulas for EVaR admit closed-form representations.

Chapter 8 provides a very detailed practical study of the new risk measures applied to the five main stochastic processes that are used to model price behaviour in the market. I do not have good

expertise, but it seems to me that the research was done competently and conscientiously, discussing problems and shortcomings.

Chapter 9 contains a study of the calculation of the Hurst index for fractional Brownian motion. This analysis is relevant under the assumption that financial markets may be driven by such a process, an assumption adopted by a few practitioners due to the presence of long-range dependence in this model. I cannot assess the substantive correctness of these results in detail, and, to some extent, this chapter appears to be somewhat peripheral to the main

Chapter 10 contains concluding remarks and directions for future research on the topic of the dissertation.

4. Recommendations and Comprehensive Evaluation

I have been observing the academic development of the candidate Dragomir Nedeltchev for some time. Given his professional commitments and the fact that he has been largely away from active scientific research for more than twenty years, it is particularly impressive that he has succeeded in entering a demanding mathematical field, mastering a substantial body of analytical techniques, and formulating—together with Assoc. Prof. Zaevski—a clear and meaningful research direction. This line of research presents challenges of both theoretical and practical nature. As a result, a coherent and self-contained dissertation has been produced, whose results have already attracted the attention of researchers in the field. I sincerely hope that Dragomir Nedeltchev will continue his collaboration with our academic community.

The dissertation is well written and motivated. The proofs are correct. I have the following minor remarks. On p. 17 Theory of Measure should be changed to Measure theory. Section 2.2.3 – Levy processes are generally used in the plural in the literature and in Definition 2.5 there are many conditions, i.e. 4 and 5 are equivalent, and only one should be left; a book on Levy processes should be cited both for the definition and, for example, for stable Levy processes. The latter also applies to Proposition 5.3, which is a standard result. It is good to mention that at least one of the two a_1, a_2, is not zero, otherwise there may be a problem in the theorem itself. On p. 41 there is beta which must be b. In the proof of Proposition 7.4, for clarity, reference should be made to Theorem 7.1 (a_ETS<c_ETS); also, in (7.23) coexist lambda_1 and x; the proof where the poles of the gamma function are used should be made in more detail with the necessary citations. In Proposition 7.5 there is f, and above is g. In general, this part is hastily written and in addition to the mentioned improvement, one can also discuss the prominence of the function of moments, which is the basis of these results.

5. Conclusion

According to the submitted documents, the candidate Dragomir Kolev Nedeltchev meets all the requirements set by the Law on the Development of the Academic Staff in the Republic of Bulgaria, the Regulations for its implementation and the Regulations of the Institute of Mathematics and Informatics, Bulgarian Academy of Sciences, defining the specific additional requirements for awarding scientific titles and academic positions. My opinion regarding the dissertation of Dragomir Nedeltchev is positive and I believe that the candidate is already an established researcher and practitioner in the field of financial mathematics.

That is why I give an overall positive assessment of the dissertation and the work of Dragomir Nedeltchev and **recommend** to the scientific jury to award Dragomir Nedeltchev the educational and scientific degree of "Doctor" in the scientific field 4. Natural Sciences, Mathematics and Informatics professional field 4.5. Mathematics.

……..

Prof. DSc. Mladen Savov

Sofia. Sofia

29.12.2025