Review

on the competition for the academic position of "Professor" in the field of higher education 4. "Natural Sciences, Mathematics, and Informatics", professional direction 4.5 "Mathematics", scientific specialty "Mathematical Analysis" (Applications of Fractional Calculus), announced in the *State Gazette*, issue 43/17.05.2024, with the sole applicant being Associate Professor DSc Emilia Grigorova Bazhlekova from the Institute of Mathematics and Informatics at the Bulgarian Academy of Sciences (IMI-BAS).

Prepared by:

Prof. Dr. Maria Dimitrova Datcheva, Institute of Mechanics at the Bulgarian Academy of Sciences (IMech-BAS), member of the Scientific Jury, approved by the Scientific Council of IMI-BAS on May 31, 2024 (Protocol No. 7) and appointed by Order No. 206/16.07.2024 of the Director of IMI-BAS.

Brief Biography

Emilia Bazhlekova completed her high education in 1981 at the "Geo Milev" Mathematical High School in Pleven. She graduated with a master's degree from the Faculty of Mathematics and Informatics at Sofia University "St. Kliment Ohridski" in 1986, specializing in Complex Analysis. From 1989 to 1993, she was a PhD student at the Institute of Mathematics and Informatics at the Bulgarian Academy of Sciences (IMI-BAS) under the supervision of Corresponding Member Ivan Dimovski. From 1997 to 2001, she participated in a research specialization in the Applied Analysis group at Eindhoven University of Technology, the Netherlands. In 2001, she defended her PhD thesis titled "Fractional Evolution Equations in Banach Spaces" at Eindhoven University of Technology, and in 2022, she was awarded the degree of Doctor of Science (professional field 4.5 "Mathematics") after defending a dissertation titled "Subordination Principle for Generalized Fractional Evolution Equations" at IMI-BAS (Diploma No. 001504 / 30.11.2022). Since 2014, Emilia Bazhlekova has held the academic position of Associate Professor at IMI-BAS.

Materials Provided for the Procedure:

The submitted documents include all administratively required items in accordance with the list of 21 requisites, including the necessary diplomas and administrative reports; a professional CV; a complete list of publications and a separate list of publications submitted for the competition; a list of all recorded independent citations and a list of independent citations of the publications submitted for the competition (with full bibliographic data); a list of research projects in which the candidate has participated; a report on meeting the minimum requirements of IMI-BAS for the academic position of "Professor"; an author’s report; electronic copies of the publications with which Assoc. Prof. Bazhlekova is participating in the competition, as well as the Bulgarian translations of the abstracts of these publications.

Publication Activity

The full list of publications provided by the candidate includes 57 articles in specialized scientific journals. Ten of these articles are indexed in *ZentralBlatt*, and 44 are indexed in *Scopus*, with 41 of them appearing in journals with an impact factor and/or impact ranking. All publications are thematically related to the current competition. The list also includes the dissertation for the awarding of the PhD degree.

Of the 22 publications presented by Assoc. Prof. Bazhlekova for this competition, 9 are published in journals with JIF and ranked in Q1 (Fractional Calculus and Applied Analysis [1, 21, 22], Fractal and Fractional [19, 16, 20], Journal of Computational and Applied Mathematics [15], Computers and Mathematics with Applications [10], Numerische Mathematik [4]). Two are published in Q4 journals (Journal of Theoretical and Applied Mechanics [18] and Comptes rendus de l'Académie bulgare des Sciences [7]).

Six of these publications [1, 2, 3, 9, 12, 13] are presented as equivalent to a monograph in the habilitation extended report. Of the 22 publications presented for the competition, four are single-authored, while the others are co-authored, with 16 papers having one co-author and the rest involving multiple co-authors. In 17 of the publications, the candidate is the first author. The published results are directly related to the topic of this competition—applications of fractional calculus.

The publications have not been used by the candidate in previous competitions for academic positions and/or the acquisition of scientific degrees.

A review of the *Scopus* database shows that 45 of Emilia Bazhlekova's works are indexed in *Scopus* (from 2003 to 2024), with 470 independent citations recorded for these articles during this period. The candidate's Hirsch index based on independent citations is 12 (*Scopus*) and 13 (*WoS*). The publications presented in this competition, which are indexed in *Scopus*, have received 224 independent citations to date.

Fulfillment of the Minimum Requirements for Holding the Academic Position of "Professor" according to the criteria in the relevant regulations for the acquisition of scientific degrees and for holding academic positions at the Bulgarian Academy of Sciences (BAS) and at the Institute of Mathematics and Informatics at BAS (IMI-BAS).

The requirements in the different groups of criteria for holding the academic position of "Professor" in the professional field 4.5 Mathematics at the Institute of Mathematics and Informatics at BAS (IMI-BAS) coincide with or are higher than those for BAS and those specified in the Regulations for the Implementation of the Law on the Development of the Academic Staff in the Republic of Bulgaria (ZRASRB). The analysis is based on these higher criteria.

Group A: A certificate from IMI-BAS has been presented, diploma No. 000001/16.12.2011 for the awarded scientific degree "Doctor" in Mathematical Analysis in 2001 at the Eindhoven University of Technology – 50 points.

Group B: 4. Habilitation work – 6 scientific publications in journals that are referenced and indexed in world-renowned scientific databases (Web of Science, Scopus, Zentralblatt). Instead of an extended habilitation report based on these 6 publications, an extended summary is provided in the Author's Report, Chapter 1. "Study of the solutions of fractional evolution equations and obtaining Duhamel representations through Dimovski convolution calculus" – 134 points (minimum required 100 points), articles [1, 2, 3, 9, 12, 13].

Group C: In the report prepared by the candidate, the following are provided:Indicator 7: 8 publications in Q1 journals and 2 publications in Q4 journals; 4 publications in journals with impact rank but not falling into quartiles; 1 publication indexed in ZentralBlatt; 1 publication indexed in Scopus without impact rank or impact factor – a total of 552 points (minimum required 220 points).

Group D: 852 independent citations in scientific journals, monographs, collective volumes have been presented. The publications submitted for the competition have 224 citations to date of this review in journals referenced and indexed in Web of Science and Scopus – 1344 points (minimum required 140 points).

Group E: For the awarded scientific degree "Doctor of Sciences," diploma from BAS No. 001504/30.11.2022 – indicator 12 (75 points); indicator 13 – there was no a supervised successfully defended PhD student; indicator 14 – participation in 3 nationally funded projects (30 points); indicator 15 – participation in 5 international scientific projects (100 points); for indicators 16 to 20 no results are reported. Total for this group – 205 points (minimum required 150 points).

The Regulations of IMI-BAS impose additional requirements on candidates applying for the academic position of "Professor" in professional field 4.5 Mathematics (Article 3, Paragraph 1, Item 4). According to these regulations, candidates must have at least 12 publications in journals with JIF or SJR for participation in a competition for the academic position of "Professor" (amendment from 25.03.2022). For professional field 4.5, it is required that at least 6 of these publications are in journals with JIF, and at least 6 of the required publications under Article 3, Paragraph 1 must be published or accepted for publication after the date of the most recent procedure under the ZRASRB. The candidate has a total of 18 publications in journals with JIF/SJR (minimum required is 12), 11 of which have JIF, and 6 of these were published after the most recent procedure for the acquisition of the scientific degree "Doctor of Sciences" – [17-22]. The conclusion is that Associate Professor Bazhlekova meets and in some criteria significantly exceeds the minimum requirements for holding the academic position of "Professor" in accordance with ZRASRB and the regulations for its implementation.

Scientific and Applied Research According to the Materials Submitted for the Competition

The scientific research and achievements of Emilia Bazhlekova are of a fundamental nature and primarily consist of theoretical and numerical results in the field of the development and application of fractional calculus. The relevance of research in this field is indisputable, given the importance of physical and biological processes and phenomena, for the modeling of which fractional derivatives and fractional calculus are used.

Six publications from the list of publications submitted for the competition, [1,2,3,9,12,13], are presented by the candidate as a cycle equivalent to a monographic work on the topic: "Investigation of solutions to fractional evolutionary equations and obtaining Duhamel representations through Dimovski convolution calculus." The focus of the research is on the solutions to fractional evolutionary equations over time [1,3,9,12], and simultaneously over time and space [12], which describe various physical processes, such as diffusion [9] and the flow of a viscoelastic fluid [1,3,13]. In this group of works, Dimovski's method of convolution calculus is applied.

The primary objective is to derive Duhamel representations of the solutions to the considered equations, which can be used for numerical experiments [1,2,9], to investigate analytical solutions [3,12,13], to obtain integral representations of the fundamental solution [13], and to apply the subordination principle to find conditions for the solution operator to be a bounded analytic operator [12].

In another group of publications [7,8,10,19,20], constitutive models formulated through equations with fractional derivatives are considered—viscoelastic models with various relaxation kernels—focusing on the conditions under which these models comply with, and do not violate, the fundamental laws of thermodynamics, which is a key criterion in constructing physical and mathematical models of thermodynamic processes. The specific models examined in these works include the fractional Maxwell model [8] and the Jeffrey model [10] for wave propagation in a viscoelastic medium of fluid type, the generalized Zener model—one of the most widely used fractional derivative models for viscoelastic deformable bodies [19,20], and the fractional generalization of the Burgers model.

Inverse problems for equations with fractional derivatives are discussed in works [15,16,22], where the problem is considered in the space of continuous functions (classical sense) [15] and in Sobolev space [16]. The aim is to investigate the existence and uniqueness for inverse problems involving fractional-order equations.

The group of publications [4,5,17] is devoted to the numerical solution of certain equations with fractional derivatives. In another set of works [6,11,14,18,21], various applications of fractional calculus in modelling complex and coupled processes are discussed: the application of the fractional Oldroyd-B model for modelling peristaltic flow of a viscoelastic fluid [6]; modelling diffusion-controlled adsorption of a surfactant using a generalized Ward-Tordai model [11]; modelling bioreaction-diffusion process [14] and axisymmetric flow of a viscoelastic liquid film between two interacting drops [18]. In [21], a class of equations, representing a generalization of the fractional Jeffrey's model, is studied with the aim of determining the conditions under which these equations form a physically meaningful model.

Key Results (Scientific and Applied Contributions):

I accept the contributions outlined by the candidate (although they are not clearly separated in the report, for example, not systematized and listed), as I believe they correspond to the results achieved in the publications submitted for the competition. A brief summary of the most significant contributions, in my opinion, can be provided by following the thematic division in the candidate's report.

I. Investigation of solutions to fractional evolutionary equations and obtaining Duhamel representations via Dimovski's convolution calculus

Using Dimovski's method of convolution calculus, a compact representation of the solutions to boundary value problems in mathematical physics with fractional derivatives has been obtained. This representation is convenient for numerical computations. For the fractional Oldroyd-B model, convergence estimates for the series representing the solution of the Dirichlet boundary problem have been derived [1]. For a generalized abstract form of this fractional equation in [3], a sector in the complex plane has been identified where the solution operator is a bounded analytic operator. In [13], the fundamental solution to the fractional equation of the Oldroyd-B model has been found and studied under certain conditions, and an integral representation of this solution has been derived. A similar approach is applied in [9] for the one-dimensional case of a time-fractional diffusion equation with Dirichlet boundary conditions. For the Cauchy problem of a time- and space-fractional evolutionary equation, a sector in the complex plane has been identified where the solution operator of the problem is bounded and analytic [12].

II. Analysis of Linear Viscoelastic Models with Fractional Derivatives

This group includes studies that analyze constitutive models with fractional derivatives to determine the conditions for their compatibility with the principles of thermodynamics, which is a necessary condition for these models to have physical meaning [8,10,19]. For the fractional Maxwell and Jeffrey's models, two equivalent constraints on the relaxation kernels have been defined, and the necessary and sufficient conditions that the parameters of the fractional operators must satisfy in order to meet both constraints on the relaxation moduli have been found [8,10]. It has been shown that for the fractional Burgers model, the defined constraints on the relaxation kernels are not equivalent [7]. In [19], it was demonstrated that the condition a<b is necessary for satisfying the weaker constraint on the relaxation kernel and sufficient for satisfying the stronger constraint. Additionally, the subordination principle for the Zener model with respect to the classical wave equation was established. For the generalized Zener model in [20], two integral representations of the relaxation kernel were derived: one through the application of the Laplace transform and the other by applying the subordination principle.

III. Uniqueness and Existence for Inverse Problems with Fractional Derivative Equations

For the studied inverse nonlocal boundary problem involving an equation with a convolution-type fractional derivative of the Caputo type in works [15] and [16], by estimating the time components of the expansions used in a bi-orthonormal basis of functions corresponding to the nonlocal operator, the existence and uniqueness of the defined inverse problem are proven in the classical sense [15] and in the Sobolev space [16]. In [15], the so-called multinomial Prabhakar functions were defined for the first time based on a generalization of the Caputo derivative to a multi-term derivative. In [22], the uniqueness of the solution for the inverse problem was proven by applying the subordination principle between the inverse problem for finding the potential function in a fractional diffusion equation and a simpler equation of a similar type, applying the proven injectivity of the subordination equality.

IV. Application and Analysis of Numerical Methods for Fractional Evolution Equations

The main results related to numerical methods for solving equations with fractional derivatives are presented in [4,5,17]. The candidate's contributions include obtaining estimates in Sobolev spaces, which are extensively used for detailed analysis of the numerical solutions. The numerical solutions in [4,5] concern the Dirichlet problem for viscoelastic flow based on the Oldroyd-B model, while in [17] a generalization of an Adams-type method for equations with a convolution-type fractional Caputo derivative is provided. The numerical procedure has been verified by comparison with analytical solutions, where such solutions are obtainable, demonstrating the correctness of the numerical results.

V. Application of Fractional Calculus for Modelling Complex Coupled Processes

An analysis of the time evolution of the pressure gradient for a model of peristaltic flow of a viscoelastic fluid governed by the fractional Oldroyd-B model is provided. The analysis is based on both analytical and numerical solutions. Restrictions on the model parameters are derived to ensure the model's compatibility with thermodynamic principles [6]. In [11], a generalization of the Ward-Tordai equation for diffusion-controlled adsorption of a surfactant is presented. Fractional time derivatives are introduced to describe anomalous diffusion in bulk phases. The applicability of fractional derivatives for generalizing classical mathematical models of bioreaction-diffusion processes is explored in [14], where

relationships (based on hypotheses) between the parameters of the derived models are established to ensure key characteristics of the modelled processes. In [18], equations for the flow velocity of a film between two interacting drops are derived using the classical Maxwell model of fractional order. For a generalized fractional Jeffrey's model in [21], thermodynamic constraints on the parameters are established to ensure that the fundamental solution of the corresponding one-dimensional Cauchy problem has physical meaning. Appropriate subordination relations with respect to a suitable integer-order evolution equation are also derived.

Critical Remarks and Recommendations

I have no critical remarks regarding the works and scientific results presented in the competition. The candidate is an accomplished scholar with a defined research focus and a clear vision for the development and future tasks in the scientific field in which she works. I recommend that Assoc. Prof. Bazhlekova train at least one student, trainee, doctoral student, or postdoctoral fellow to ensure continuity in the highly relevant, interesting, and promising field of fractional calculus.

Personal Impressions

I have known Assoc. Prof. Emilia Bazhlekova for a long time. I have excellent impressions of Emilia as an erudite scholar with international experience and authority, which is also demonstrated by the large number of citations of her scientific results. She possesses an exceptional sense for significant and current problems in her field of scientific interest and the capacity for an innovative and pioneering approach to solving scientific challenges. I have worked with her within the framework of a project funded by the National Science Fund (NSF) under the Ministry of Education and Science, which received a "very good" performance evaluation. Assoc. Prof. Bazhlekova's participation and scientific results within the project made a significant contribution to its successful completion. In my opinion, Assoc. Prof. Bazhlekova has a clear and constructive vision for her development as a scientist, as well as for the research related to fractional calculus and its applications in various scientific fields.

Conclusion

The research activity of the candidate, both in volume and quality, fully meets the requirements of the Act for the Development of the Academic Staff in the Republic of Bulgaria, as well as the regulations governing the acquisition of academic degrees and titles, and the appointment to academic positions at the Bulgarian Academy of Sciences (BAS) and the Institute of Mathematics and Informatics (IMI-BAS). Therefore, I am fully confident in recommending that the esteemed Scientific Jury **present Assoc. Prof. DSc Emilia Grigorova Bazhlekova for election by the Scientific Council of IMI-BAS to the academic position of 'Professor' in the field of higher education 4. 'Natural Sciences, Mathematics, and Informatics,' professional area 4.5 'Mathematics,' specialty 'Mathematical Analysis' (Applications of Fractional Calculus), for the needs of the Institute of Mathematics and Informatics at BAS.**

9.09.2024
Sofia

(Prof. Maria Datcheva, member of the Scientific Jury)