

**Review of PhD Thesis of
Marin Vladimirov Genov
“Functions Holomorphic over Finite-Dimensional Commutative Associative
Unital $\mathbb{C}$ -Algebras – Some Local, Global, and Universal Aspects”**

Professional field: 4.5. Mathematics

Doctoral program “Geometry and Topology”

Institute of Mathematics and Informatics at the Bulgarian Academy of Sciences (IMI-BAS)

The review has been prepared by: Assoc. Prof., Dr. Mikhail Shkolnikov (IMI-BAS) as a member of the scientific jury for the defense of this PhD thesis according to Order No 83/14.08.2026 by the Director of IMI-BAS.

1. General characteristics of the dissertation thesis and the presented materials

The dissertation’s page-count is 227 according to NACID’s information card, with the total number of citations being 186 items. The language of the work is English; its main scientific content is contained in the Introduction, Preliminaries Chapter 0, Chapter 1 “Differentiation over $\mathcal{A}$ ”, and Chapter 2 “Integration over $\mathcal{A}$ ”, as well as appendix A “Structure Tensors”.

2. Short CV and personal impressions of the candidate

I before attending the pre-defense I wasn’t quite familiar with the work of thesis’ author, although we met on a few occasions and discussed mathematics. He gave me an impression of a person with broad interests and wide mathematical sensibility. According to other researchers’ opinions Marin Genov prefers the slow working style and favors individual effort. Despite that, the volume of the PhD dissertation and the amount of references therein, as well as keen descriptions of the content referred work, signal of dedication and professionalism, and the adequacy of the time taken to perform the work under review.

From what I see in the shared CV, Marin Genov has been within an official doctorate program for four years, and has been expelled from the program in June 2024 with the right of the PhD defense. A year before that he entered the position of a Mathematician at IMI-BAS in the section “Analysis, Geometry and Topology”, and in 2024 he switched to the section “Algebra and Logic” first in the analogous “Mathematician” position, upgrading it next year to “Assistant” in the same section. He has had some TA experience at Sofia University and “Ludwig-Maximilian” University in Munich with classes being in various kinds of Analysis and Algebra.

In addition to that, his 2014 diploma work at “Ludwig-Maximilian” University under the supervision of Martin Schotenloer is dedicated to L-functions, showing that his competence spans to Number Theory as well. His 2018 Master Diploma work at the Sofia University is entitled “Functions holomorphic over finite-dimensional complex associative commutative unital algebras – some local, formal, global, and universal aspects” under the supervision of Emil Horozov according to the provided CV. The current work under review performed under the supervision of Akad. Vesselin Drensky, bearing a very similar name seems to be a direct continuation and a very substantial extension of the mentioned master diploma work, at least judging by an arXiv 2018 manuscript arXiv:1812.11661v2 “Functions Holomorphic over Finite-Dimensional Commutative Associative Algebras 1: One-Variable Local Theory I”, showing a commendable fidelity to the subject and exceptional perseverance of the author.

3. Content analysis of the scientific and applied achievements of the candidate, contained in the presented PhD thesis and the publications to it, included in the procedure

This thesis is a monumental work. First of all, the Preliminaries Chapter 0 builds a coherent algebraic foundation for the further treatment of the generalization of a holomorphic function to the case of a variable belonging to an open domain inside a commutative associative unital algebra over real or complex numbers. The choice of a coefficient system is dictated by both topological and analytic necessities of the more advanced topics developed in the later two chapters. A lot of the material in Chapter 0 is presented in a wide generality, and even taken on its own it is very impressive, as it might be used as a basis for a self-contained non-conventional course in commutative algebra, covering, nevertheless, many of the essential topics, say, for doing algebraic geometry. The presentation there, as in the next chapters, is meticulous and shows a very well developed mathematical taste and an affection for the detail.

The attitude of the author explicitly stated in the (magnificently written) introduction is admirable and extremely meaningful. I quote him below: *“But despite some such isolated advancements, the sheer number of historical “independent” rediscoveries only attest to the rather sorry state the subject of finite-dimensional commutative $\mathcal{A}$ -analysis is in, which in and of itself should be taken as a negative development, considering its lively origins. Indeed, with some notable exceptions, the area shows a sad tendency of authors picking (oftentimes the same) lowest of low-hanging fruits. We aim to fix that by picking all of the remaining low-hanging fruits. An unfortunate but not unexpected byproduct of the aforementioned development is also the scattered nature of the subject.”*

I believe that this dissertation has achieved its goal. In fact, besides being a masterpiece of scholasticism (in a good sense of the word), it has a significant novelty to it. One such is the introduction of the *spectral approach* in the finite-dimensional commutative analytic theory. A different one is the discovery of generalized Wirtinger operators acting on functions from an algebra to a module over it universal across all commutative algebras. Many classical results in the theory of functions of a single complex variable are pushed to a much larger generality, including such a fundamental Theorem of Cauchy and Goursat giving an contour-integration characterization of complex differentiability. Among the foundational achievements, which I think at first glance may look like a technical loophole facilitating the development of the theory is the introduction of the intermediate-regularity sheaf $\mathcal{F}_p^{\text{str}}$ behaving well under certain categorical constructions.

In addition to the formal theorems, their complete proofs, and precise definitions on which those are based, the work presents an unusual (at least for the reviewer) organic synthesis of algebra, analysis and topology, in the decreasing order of prominence. This, in my view, might be a future progression towards integrating the techniques coherently compiled and developed by the author of the thesis into the mainstream of modern mathematical thought, if he wishes to do so, of course, or by some other researchers working on the adjacent topics. Indeed, the impression I had after reviewing the bibliography that the current state of the subject is that it remains to be relatively obscure, although the corpus of relevant literature has been growing since the second half of XIX century. One exceptional instance of the recognized universal acclaim of the kin mindset is that of Frobenius manifolds developed and popularized by Yuri Manin.

Concerning the two publications enclosed as a part of the procedure’s set of documents, the one in the Proceedings of BAS describes briefly some fundamental results concerning the local aspects of the theory, and the one in Serdica Mathematical Journal is a more substantial writing on the generalization of Cauchy integration formula, extending known results by other authors. Both are subsumed by the presented PhD thesis.

4. Approbation of the results

The older publication

M. Genov, “Holomorphy over finite-dimensional commutative associative $\mathbb{C}$ -algebras: local theory”, C. R. Acad. Bulg. Sci. 77(5), 2024, 646–656.

is a condensed version announcement style version of some results presented and proven in the thesis, more specifically related to the local theory in the morphism setting $\mathcal{A} \rightarrow \mathcal{B}$.

The most recent publication

M. Genov, “The Generalized Homological Cauchy Integral Formula for A-Holomorphic Functions”, Serdica Math. J. 52 (2026), 185–204.

is a more focused and self-contained treatment of the case of a function from an open subset of an algebra to the algebra itself, thus disregarding the more general setup developed in the thesis, where the target is a finitely-generated module over that algebra.

As it is seen from the above, both publications are single-authored. From what I see, neither is cited by other researchers.

From the provided CV, it follows that Marin Genov has given at least half-a-dozen of talk on the topics covered in the thesis. Most of them were at IMI-BAS, but at least two were given at international conferences.

The scientific works meet the minimum national and IMI-BAS requirements for acquiring the PhD degree in Mathematics. No plagiarism has been noticed.

5. Qualities of the “avtoreferat”

The presented “avtoreferat” is extremely substantial and thorough, it describes the content of the theses very accurately. My only concern is that it might be longer than most of the analogous documents, although there doesn’t seem to be a strict and formal page limit for such.

6. Critical notes and recommendations

First of all, the literary awareness of the author is stunning, he seems to have an all-encompassing understanding of the subject and the adjacent fields. Second, as the author notes himself, the study, even at the algebraic level, could have been way more involved, with the introduction, for instance, of a more advanced module theory in the multi-variable case. This, however, is not a flaw but rather a promise for the years of fruitful work. From the geometric perspective, it would be desirable to try seeing what would be the corresponding theories for the concrete cases of more exotic base-algebras (such as higher-dimensional local ones) different from real, complex and dual numbers, as well as interplays of such. The functionality that is stressed in certain parts of the thesis might provide a suitable foundation for the transfer of the results across these geometries.

As a reader, I still feel that more examples might be helpful, although there are some present in the thesis. What I would like to see in a action, in particular, are the analytic continuations. A particular suggestion is as follows: consider the classical lacunary series over $\mathbb{C}$ and $\mathbb{C}[\varepsilon]/\varepsilon^2$ – what are the corresponding domains of analyticity?

Finally, still being naive in analysis and PDEs, it still looks like that the developed techniques might provide a potent framework at least in some of the relevant situations where symmetries are abundant.

7. Conclusion

Having become acquainted with the PhD thesis presented in the procedure and the accompanying scientific papers and on the basis of the analysis of their importance and the scientific and applied contributions contained therein, I confirm that the presented PhD thesis and the scientific publications to it, as well as the quality and originality of the results and achievements presented in them, meet the requirements of the ADAS in the Republic of Bulgaria, the Rules for its Implementation and the corresponding Rules at IMI-BAS for acquisition by the candidate of educational and scientific degree “Doctor” in Mathematics. In particular, the candidate meets the minimal national requirements in the professional field and no plagiarism has been detected in the scientific papers submitted for the competition.

Based on the above, I strongly recommend to award the educational and scientific degree “Doctor” in Mathematics to Marin Genov.