O P I N I O N

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member of the Scientific Jury according to Order No. 83/14.08.2026

of the Director of the Institute of Mathematics and Informaticsна – BAS

R E L A T I N G T O

a dissertation for the acquisition of the educational and scientific degree ''**Doctor"**

Author of the dissertation– **Marin Vladimirov Genov**

Topic of the dissertation – ''**Functions Holomorphic over Finite-Dimensional Complex Commutative Associative Algebras – Some Local, Global and Universal Aspects''**

Field of higher education – **4. Natural sciences, Mathematics and Informatics**

Professional field – **4.5 Mathematics**

Doctoral program – **Geometry and Topology**

Scientific supervisor**– Acad. Vesselin Drensky**

General characteristics of the dissertation work

The dissertation is 227 pages long, presented in English and consists of Declaration of Originality, Acknowledgements, Abstract, Notations and Conventions, Introduction and Overview, 3 Chapters, Appendix on Structure Tensors and Bibliography. List of author's publications is given and the reports on the dissertation as well. The bibliography list includes 135 items of authors abroad.

The publications on the topic of the dissertation are two. They are in journals published by BAS with a corresponding index IF=0.3 for [Gen24] and SJR=0.173 for [Gen26]. The reported results on the topic of the dissertation are included in 6 talks - on 3 institutional seminars and on 3 international scientific events in the country.

The Abstract is 46 pages long. It is formatted according to the requirements and essentially reflects the goals set, the results obtained and the contributions in the dissertation.

Remark – In some places there is a difference in the numeration of propositions in the Abstract and the Dissertation. In my Opinion the numeration according to the dissertation is followed.

The main goal and motivation of the dissertation is to give a substantial base for developing the geometry over finite dimensional commutative associative algebras in the spirit of Riemann manifolds theory and more generally of the complex analytic geometry. Using the basic language of commutative algebra, algebraic topology and differential geometry a comprehensive study is provided on differentiation and integration over finite dimensional

representations of the considered types of algebras via the interaction between analysis and basic algebraic category-theoretical constructions **.**

Knowledge of the problem - The dissertation undoubtfully shows a detailed exposition of the considered problems. The complex scientific potential of the author is seen as well in the cited publications, which are from 16 different subfields in algebra, geometry and topology.

A brief description of the contents

A starting point of the dissertation are the finite dimensional representations ρ of the considered finite dimensional commutative associative algebras and as a continuation of these representations the investigation of morphisms of these algebras.

Let ***A*** be a **K**-algebra of the considered type, ***K*** ϵ { ***R, C***} and $_A$**M** be a finitely generated ***A***-module. The exposition is a generalization of the theory of the vector valued holomorphic functions in one variable over the set of functions with a ***A***-linear differential where the role of ***C*** is played by ***A*** and that of the corresponding complex vector space – by the finitely generated ***A***-module.

Chapter 0 includes preliminaries and definitions structured in 3 sections. Section 0.1 is the algebraic preparation with 13 subsections. Section 0.2 gives facts for the group of units of the algebra ***A***. Section 0.3 is devoted to norm algebras and matrix analysis.

Chapter 1 includes results connected with the purely algebraic base of the investigations made in the dissertation.

Chapter 2 grounds the considered topological apparatus.

Section 1.1 is devoted to differentiation over the presentation ρ.

In Section 1.2 different automatic ρ-differentiable continuations of ρ-differentiable functions are proved as well as full canonical factorizations /for the considered representations ρ/.

In Section 1.3 the homogeneous generalized Cauchy-Riemann-Scheffers equations are stated /included in publication [Gen24] of the author/ for the corresponding ρ-differentiable functions. Generalized Wirtinger operators are derived universal for the considered algebras.

In Section 1.4 different versions of the Generalized Integral Cauchy Theorem are stated.

Section 2.1 is devoted to the complex case exposed via spectral theory.

In Section 2.2 ρ-analogue of the analytic index is introduced and its full characteristics is given via spectral homological approach /included in [Gen 26]/.

In Section 2.3 different versions of the Cauchy Integral Formula over the corresponding algebra ***A*** are proved /included in [Gen26]/ starting with the local ”point ’ formulation, moving through the homotopic ‘point” and local formulations up to the most general homological version.

In Section 2.4 Global ‘’Spectral’’ and Generalized Cauchy-Pompeiu formulas are proved for sufficiently differentiable mappings and orientable compact manifolds in a general position.

From the exposed above concerning the 2 chapters as mostly sufficient the following could be outlined.

Contributions of the dissertation

- Basic properties of the holomorphic functions are introduced and investigated when the field of the complex numbers ***C*** is substituted by a finite dimensional commutative associative algebra ***A*** /over ***R*** or ***C***/ and the corresponding vector space - by a finitely generated ***A***-module.
- A series of basic factorizations and automatic continuations of ρ-differentiable functions are derived and their full standard factorization is proved in the general case /Theorem 1.2.13/.
- A stronger form of the first basic theorem in [GM96] is proved, cited in subfield Finite-Dimensional Commutative Real ***A***-Analysis, for one ***A***-variable /Proposition 1.1.16/.
- A Generalized Cauchy-Gursa Theorem for ρ-differentiable functions is proved as well as the infinite integrability of ρ-differentiable functions over a wide range of domains /Proposition 1.4.11/.
- A full characteristics is given for the analogue of the analytic index for a curve in the complex plain /mainly Lemma 2.2.2, Lemma 2.2.3, Corollary 2.2.6 and Lemma 2.2.12/.
- Graded in importance Generalized Integral Cauchy formulas are derived / Proposition 2.3.1, Proposition 2.3.4 and Theorem 2.3.6/.
- A Spectrally Null-Homological Cauchy Differentiation Formula over ***A*** is formulated and proved /Proposition 2.3.8/.
- A Cauchy Transformation over ***A*** is formulated and proved /Proposition 2.3.16/.
- A Generalized Cauchy-Pompeiu Formular over ***A*** is proved concerning compact manifolds in a general position about the radical /Theorem 2.4.7/.

Assessment of the publications on the dissertation work

In Paper [Gen24] it is shown that to a morphism of finite dimensional commutative associative unital Banach ***C***-algebras one could associate topological spaces of differentiable functions with a linear differential. For such spaces the author gives an overview of some new results concerning the fundamentals of the corresponding local theory and strengthens various already existing results.

After the algebraic and analytic preliminaries and notations the author comes to Generalized Cauchy-Riemann-Scheffers equations /Proposition 5.1/ and Generalized Wirtinger operators. The generalized index and Cauchy Integral Formula are considered as well /Proposition 7.4/. The Generalized Cauchy Transform is the topic of Proposition 8.4 and Proposition 8.6 is a generalized Morera version. By using spectral homology the Generalized Homological Cauchy Integral Formula is proved /Theorem 9.2/.

Paper [Gen26] – Using the mathematical apparatus from [Gen24] significant results are proved. We'll mention the Centered Integral Cauchy Formula over ***A*** with index /Proposition 5,1/, the Local Cauchy Integral Formula over ***A*** /Proposition 5.2/, the Automatic

Analytic Continuation to spectral cylinders /Theorem 5.3/ as well as the Spectrally Null-Homological Cauchy Integral Formula over ***A*** /Theorem 5.5/.

I appreciate highly the quality of these 2 publications.

Opinion, recommendations and remarks

The dissertation is developed thoroughly and precisely. The results obtained include scientific contributions mainly and convincingly meet the goal and the aims of the dissertation.

I have no critical remarks on the substance of the presented dissertation work. My recommendation to the doctoral student is to improve the style in some paragraphs, to clear the connections in the many considered mathematical subfields, to present his work to more various mathematical audience and future joint work and citations.

C O N C L U S I O N

I **positively** assess the work done and the results obtained in the dissertation. The dissertation work meets all the requirements of the Law on the development of the academic staff in the Republic of Bulgaria**,** the Regulations for its implementation, as well as the Regulations on the terms and procedure for acquiring scientific degrees and occupying academic positions at the Institute of Mathematics and Informatics – BAS.

I **strongly propose** to the esteemed Scientific Jury to award Marin Vladimirov Genov the educational and scientific degree ‘’doctor’’ in the field of higher education 4. Natural Sciences, Mathematics and Informatics, professional field 4.5 Mathematics (Geometry and Topology).

Rousse, Compiled by;

16.09.2026 / Assoc. Prof. Dr. Ts. Rashkova/