STATEMENT

by **Assoc. Prof. Dr. Valdemar Vasilev Tsanov,**

Section "Analysis, Geometry and Topology" at the
Institute of Mathematics and Informatics at the Bulgarian Academy of Sciences,

on the Dissertation Thesis

entitled

"Functions Holomorphic over Finite-Dimensional Commutative Associative Complex Algebras – Some Local, Global, and Universal Aspects"

by

Marin Vladimirov Genov

submitted for obtaining the educational and scientific degree "Doctor"
in the Area of Higher Education: 4. Natural Sciences, Mathematics and Informatics,
Professional Field 4.5 Mathematics, Doctoral Program "Geometry and Topology"

Scientific Advisor: **Acad. Vesselin Drensky**

I am a member of the Scientific Jury for the defense of this dissertation thesis according to Order No. 83/14.08.2026 of the Director of the Institute of Mathematics and Informatics at the Bulgarian Academy of Sciences (IMI-BAS). The statement is prepared in accordance with the requirements of the Act on Development of the Academic Staff in the Republic of Bulgaria, the Regulations for its implementation, the Rules for the conditions and regulations for acquiring scientific degrees and occupying academic positions in the Bulgarian Academy of Sciences and the relevant regulations of IMI-BAS.

From the required documents and papers submitted by Marin Vladimirov Genov, I have made sure that he meets the requirements of the Act on Development of the Academic Staff in the Republic of Bulgaria, the Regulations for its implementation, the Rules for the conditions and regulations for acquiring scientific degrees and occupying academic positions in the Bulgarian Academy of Sciences and IMI-BAS, as well as the specific requirements for obtaining the educational and scientific degree "Doctor", given in the Rules for the conditions and regulations for acquiring scientific degrees and occupying academic positions in the Institute of Mathematics and Informatics at the Bulgarian Academy of Sciences.

1. General Characteristics of the Dissertation Thesis

The dissertation thesis consists of 227 pages and contains: an introduction, a chapter with preliminaries, two chapters presenting the main results, an appendix with necessary results from tensor algebra, and a bibliography of 186 entries grouped by topic. The research addresses natural and timely questions from contemporary algebra and complex geometry, related to generalizations of classical theorems from complex analysis in the context of finite-dimensional commutative associative complex and real algebras. The focus is on notions of equivariant differentiability and analyticity of (local) maps between algebras with respect to a given morphism, or between an algebra and a finite-dimensional module thereof with respect to the representation. The central results concern generalizations of

the Cauchy–Riemann equations and the Cauchy integral formula, as well as some of their applications. The chapter on preliminaries contains a detailed exposition of the necessary definitions and theory of finite-dimensional commutative associative algebras, enabling the known analytic results to be formulated in a general context and the new results to rest on a well-defined foundation. The diverse aspects touched upon in the introduction and the preliminaries, supported by the extensive bibliography, demonstrate that the candidate has interests and knowledge extending beyond the specific topic of the dissertation thesis, encompassing potential continuations and applications of the results obtained.

The dissertation thesis is theoretical in character and meets the generally accepted requirements for obtaining the educational and scientific degree "Doctor".

2. Main Scientific Contributions

The main results of the dissertation thesis are contained in Chapters 1 and 2, and constitute generalizations of classical theorems from complex analysis to the setting of finite-dimensional commutative associative real and complex algebras. Morphisms $\varphi : \mathcal{A} \to \mathcal{B}$ of such algebras and modules M with representations $\rho : \mathcal{A} \to \mathrm{End}(M)$, respectively real or complex, are considered.

Chapter 1: Differentiation over $\mathcal{A}$. Three notions of $\mathcal{A}$-differentiability are investigated – classical, Fréchet, and strict Fréchet – for maps $f : U \to \mathcal{B}$, or $f : U \to M$, where $U \subset \mathcal{A}$ is an open subset, as well as the notions of analyticity and k-fold regularity. It is proved that k-regularity over $\mathcal{A}$ is equivalent to $\mathcal{A}$-equivariance and k-regularity over $\mathbb{R}$. In the complex case, the classically $\mathcal{A}$-differentiable functions are $\mathcal{A}$-holomorphic, and for these, analyticity is subsequently established. Rules of differential calculus are derived, such as the chain rule and the Leibniz rule. Strict Fréchet $\mathcal{A}$-differentiability plays a key role, and its application in the given context is one of the original contributions of the dissertation thesis. It is shown to be particularly is useful for integration, but has poor compatibility with the chain rule for differentiation, requiring additional hypothesis.

Canonical factorizations of $\mathcal{A}$-holomorphic functions through algebra epimorphisms are constructed. It is shown how the general theory of Artinian algebras allows many questions in the given setting to be reduced to the case of an indecomposable representation of a local algebra. A canonical analytic continuation of $\mathcal{A}$-holomorphic functions $f : U \to M$ to the projective closure of U is constructed, inducing an isomorphism of sheaves.

An analogue of the Cauchy–Riemann equations and their generalization due to Scheffers is derived. This is a key step in the development of the theory. Wirtinger operators are constructed, providing a condition for Fréchet $\mathcal{A}$-differentiability. The universal construction of these operators is one of the interesting contributions of the dissertation thesis, with potential applications as a technical tool in research on related topics.

The 1-form $f(Z)\,dZ$ associated with a given function f is investigated, and it is shown that Fréchet $\mathcal{A}$-differentiability of f is equivalent to closedness of $f(Z)\,dZ$, given Fréchet $\mathbb{R}$-differentiability. Generalizations of the Cauchy and Cauchy–Goursat integral theorems are proved. Important consequences are derived, such as infinite integrability of $\mathcal{A}$-holomorphic functions on a simply connected domain in $\mathcal{A}$.

Chapter 2: Integration over $\mathcal{A}$. Here the base field is that of the complex numbers. The notion of an $\mathcal{A}$-valued, or more generally M-valued, analytic index is introduced for admissible pairs consisting of a point and a 1-cycle in $\mathcal{A}$. Basic properties of this index are studied. The important case of a local algebra $\mathcal{A}$ and the connection between the $\mathcal{A}$-valued index and the classical index of the spectral projection of the 1-cycle into the local field of the algebra are examined. Invariance of the index under (spectral) homotopy of 1-cycles is proved, and a structural interaction with the multiplicative group $\mathcal{A}^\times$ of $\mathcal{A}$ is developed. A generalization of the Cauchy integral formula is proved in the most general

homological variant possible for the given context. Many special cases (for example, simply connected domains) and corollaries generalizing standard consequences from the classical one-dimensional case are considered. A generalization of Morera's theorem is proved, as well as the equivalence between $\mathcal{A}$-holomorphicity and $\mathcal{A}$-analyticity of functions. A canonical $\mathcal{A}$-analytic continuation of an $\mathcal{A}$-holomorphic function to the spectral closure of its domain of definition is constructed, and domains of $\mathcal{A}$-holomorphicity are characterized. A Cauchy transform is constructed. A generalization of the Cauchy–Pompeiu integral formula is proved, and its applications in the theory of Bochner–Martinelli are given.

Among the contributions of the dissertation thesis one should also count the systematic formalization of the fundamental notions within a single well-defined general context, encompassing the category of finite-dimensional commutative associative complex algebras and the category of their finite-dimensional modules.

3. Approbation of the Results

The results of the dissertation thesis have been published in the following two articles, [1] with impact factor and [2] with SJR:

[1] M. Genov, "Holomorphy over finite-dimensional commutative associative $\mathbb{C}$-algebras: local theory", C. R. Acad. Bulg. Sci. **77**.5 (2024), pp. 646–656.

[2] M. Genov, "The Generalized Homological Cauchy Integral Formula for $\mathcal{A}$-Holomorphic Functions" (2026), to appear in Serdica Mathematical Journal.

4. Qualities of the Dissertation Summary

The dissertation summary consists of 41 pages, follows the structure of the dissertation thesis in its content, and also contains an extensive bibliography. The introduction describes motivating known results and related topics in a broader context. The main results are formulated in technical detail, accompanied by the relevant definitions and statements of a significant portion of the propositions constituting the proofs. The key observations are well highlighted. The dissertation summary gives a clear picture of the results of the dissertation thesis and the main thread of its content.

5. Overall Assessment and Recommendations

The dissertation thesis is well structured, contains original and important results, a detailed exposition of the necessary theory, and a general formalization of already known results on the topic. A large part of the results are generalizations of theorems fundamental to complex analysis, such as the Cauchy integral formula, which have been the subject of previous studies, including some results in the given context. The generalizations offered here have a complete character and formulations convenient for application. For these reasons, the present work could serve well in further development of the theory.

As a recommendation, I would note that it would be interesting to emphasize and explore the presence of new phenomena in the generalized theory. For instance, the role of strict Fréchet $\mathcal{A}$-differentiability is particularly interesting, as it exhibits a phenomenon distinguishing the generalized theory from the classical one.

Some of the results of the dissertation thesis have been published in established journals and presented at seminars and international conferences. I have personally attended two seminar talks by Marin Genov, from which I came away with a very strong impression.

I assess the dissertation thesis as excellent.

6. Conclusion

The above analysis shows that the submitted dissertation thesis fully meets the requirements of the Act on Development of the Academic Staff in the Republic of Bulgaria, the Regulations for its implementation, the Rules for the conditions and regulations for acquiring scientific degrees and occupying academic positions in the Bulgarian Academy of Sciences and IMI-BAS, as well as the specific requirements for obtaining the educational and scientific degree "Doctor", given in the Rules for the conditions and regulations for acquiring scientific degrees and occupying academic positions in the Institute of Mathematics and Informatics at the Bulgarian Academy of Sciences.

All of the above gives me a good reason to confidently give my positive evaluation of the dissertation thesis **"Functions Holomorphic over Finite-Dimensional Commutative Associative Complex Algebras – Some Local, Global, and Universal Aspects"** and strongly recommend the Scientific Jury to award **Marin Vladimirov Genov** the educational and scientific degree **"Doctor"** in the Area of Higher Education 4. Natural Sciences, Mathematics and Informatics, Professional Field 4.5 Mathematics, Doctoral Program "Geometry and Topology".

20.09.2026
Sofia

Member of the Scientific Jury:

/Assoc. Prof., Dr. V. Tsanov/