REPORT

by Assoc. Prof. Alexander Vladimirov Petkov, Ph.D.,

Sofia University "St. Kliment Ohridski",

Faculty of Mathematics and Informatics,

Professional field: 4.5. Mathematics (Differential geometry),

as a member of the Scientific Jury on this procedure,

according to Order № 15/17.02.2025 of the Director of IMI – BAS

on

the competition for academic position "Associate Professor",

Area of Higher Education: 4. Natural Sciences, Mathematics and Informatics,

Professional field: 4.5. Mathematics (Geometry and Topology – Tropical geometry),

announced in SG, issue 106/17.12.2024

for the needs of IMI - BAS

1. ***Information about the competition***

Documents for participation in the competition were submitted only by Mikhail Shkolnikov, Ph.D. The Scientific Jury was formed by Order № 15/17.02.2025 of the Director of IMI - BAS, and the first meeting of the Jury was held on February 19th 2025.

The documents presented by the applicant satisfy the minimal national requirements of the Act on Development of the Academic Staff in the Republic of Bulgaria, the Regulations for its application, and the additional requirements of IMI – BAS for Area of Higher Education 4. Natural Sciences, Mathematics and Informatics, Professional field 4.5. Mathematics.

2. ***Information about the applicant***

The applicant Dr Shkolnikov has completed his bachelor's degree in the specialty "Mathematics" at St. Petersburg State University in 2012, and his

master's degree in the specialty “Mathematics” at the University of Geneva in 2013. In 2017, the applicant successfully defended his doctoral dissertation (Ph.D.) in Mathematics again at the University of Geneva.

In the period 2017 - 2019, the Dr Shkolnikov was a postdoctoral fellow at the Institute of Science and Technology in Klosterneuburg, Austria, and in the period 2019 - 2020 - a postdoctoral fellow at the University of Geneva.

From 2023 to the present, Dr Shkolnikov is a researcher at the IMI – BAS.

The applicant speaks Russian (native), English, French and Bulgarian languages.

3. Description of the scientific works and the scientific contributions

Dr Shkolnikov's scientific interests and achievements are mainly in the realm of an avant-garde field of Mathematics called *Tropical geometry*, which unites ideas from real algebraic geometry, mathematical physics, and computer science, and which is expected to have numerous and profound applications in the natural and computer sciences.

The applicant has submitted for the competition 13 scientific articles, which are published in prestigious scientific journals, indexed by Web of Science and/or Scopus.

Dr Shkolnikov has comprehensively described his scientific achievement in the submitted abstracts of the papers, as well as in the Reference for original scientific contributions.

The results of the scientific publications can be grouped as follows:

a) Non-abelian tropicalization

Generally speaking, a non-commutative version of phase tropicalization has been studied in this direction, as well as the properties of hyperbolic amoebas and their comparison with commutative amoebas in the n-dimensional space $\mathbf{R}^n$.

The articles numbered by [11] and [13] from the publications list are devoted to this area.

b) Tropical sandpile

The articles numbered by [3], [4], [5], [9] and [12] are related to this group. In this direction, the article with number [4] is noteworthy, in which a new tropical model with self-organized criticality (SOC) behavior is presented, which, unlike all known SOC models, is continuous and represents a certain scaling limit of the sandpile model, which is the first and archetypical SOC model.

I will also mention the paper numbered by [9], in which the theory of smoothing is developed, in order to show the existence of so-called solitons in a sandpile model, studied by S. Caracciolo, G. Paoletti, and A. Sportiello. In this way, a step towards is taken, understanding the phenomena of the identity in the sandpile group for planar domains, where solitons appear according to experiments. It is proven that the sandpile states, defined by the smoothing procedure, remain unchanged when the wave operator is applied, and can interact, forming triads and nodes.

c) Tropical optics

The candidate has dedicated the paper numbered by [7] to this direction, in which he demonstrates a general method for obtaining certain summation formulas over a part of **SL(2,Z)**, as well as another paper, which is not included in the list for the competition.

d) Extended sandpile

The papers [8] and [10] are related to this area. In [8], the evolution of the sandpile identity under the action of harmonic fields of different orders is analyzed. In [10], a tiling problem between bounded open convex polyforms in $\mathbf{R}^2$ with colored directed edges is introduced and studied. Several examples of infinite series of such tilings converging to $\mathbf{R}^2$ are provided, and thus the limit of the sandpile group on the plane is defined.

e) Quantum topology

The papers numbered by [1], [2] and [6] are devoted to this realm. A notable achievement was made in [1], where a solution to a part of Problem 1.60 from

Kirby's list of unsolved problems in topology was presented, thus answering positively a question posed in 1987 by J. Przytycki.

The applicant has submitted a list of 20 citations in refereed and indexed by Web of Science and/or Scopus volumes (without self-citations).

4. Personal impressions about the applicant

I have known Dr Shkolnikov since he took up a position at the IMI – BAS. I have had the opportunity to attend several conferences in Bulgaria dedicated to Algebraic and Differential Geometry, which he also attended. He gives the impression of an extremely erudite, earnest and diligent mathematician.

5. Conclusion

Based on the documents presented by Dr Mikhail Shkolnikov at the competition and due to the analysis of the significance of the scientific results, **I confirm**, that the scientific achievements correspond to the requirements of the Act on Development of the Academic Staff in the Republic of Bulgaria, the Regulations for its application, and the additional requirements of IMI - BAS, for the occupation by the applicant of the academic position "Associate Professor" in the scientific field of the present competition. The applicant not only satisfies, but exceeds the minimal national requirements and the specific requirements of IMI – BAS in the professional field and no plagiarism has been established in any form in the scientific papers submitted at the competition.

Based on the above, I give my **positive estimate** of the candidacy of Dr Mikhail Shkolnikov and **strongly recommend** to the Scientific Jury to propose to the Scientific Council of IMI - BAS **to elect Dr Mikhail Shkolnikov to occupy the academic position "Associate professor" in the professional field 4.5. Mathematics (Geometry and Topology – Tropical geometry).**

March 18th 2025

This report is prepared by:…….

/Assoc. Prof. Alexander Petkov, Ph.D./