Bulgarian Academy of Sciences
Institute of Mathematics and Informatics

DEVELOPMENT OF DIGITAL COMPETENCE IN MATHEMATICS EDUCATION

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DISSERTATION ABSTRACT

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Professional field 1.3. Pedagogy of Education in...
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INTRODUCTION

The goals and vision for digital transformation of the Republic of Bulgaria for the period up to 2030 are set out in a national strategic document (Digital Transformation of Bulgaria for the period 2020-2030).. One of the areas of impact described in it concerns education at all levels, with a focus on improving technological knowledge and digital skills. In 2013, a European framework for digital (digital and digital we consider them a different translation of digital) competence in Europe DigCom 1.0 was published (Ferrari, 2013), in which five areas of digital competence are separated: information, communication, content creation, safety, problem solving. Research in this area has been accelerated and several more reference frameworks have been developed, aimed at improving citizens' digital competences DigCom 2.0, DigCom 2.1 and DigCom 2.2 are (Vuorikar et al, 2016), (Carretero et al, 2017) (Vuorikar et al, 2022). The five areas of competence (information literacy and data literacy; communication and cooperation; digital content creation; security; problem solving) are divided into eight levels, depending on the level of knowledge, complexity of the task and autonomy of its implementation. The DigCompConsumers Digital Competence Framework aims to increase consumer confidence in e-buying and sales and to create the conditions for their active and confident activity in the digital market (Brečko & Ferrari, 2016). The Digital Competence Framework (Punie, 2017) is aimed at educators of all levels and forms of education.

The COVID-19 pandemic has caused significant changes in all areas of people's lives. Account shall be taken of the need for well-trained teachers, trainers, educators and administrative staff, for appropriate learning content, for providing security (Council conclusions on digital education in European knowledge societies. 2020/C 415/10). Emphasis is also placed on the importance of pedagogical concepts, of tools and methods of learning and teaching, on educational research contributing to the development of innovative concepts in education.

The Ministry of Science and Education in Bulgaria has carried out a number of activities to ensure digital transformation in the Bulgarian educational system. They are aimed both at changing the environment and at developing the digital competence of teachers and learners.

Digital transformation is one of the main priorities at European level. The main objectives of the policy agenda for the digital decade at European level are: a population with digital skills and highly qualified digital specialists; secure and sustainable digital infrastructures; digital transformation of business; digitalisation of public services. Since 2014, digital progress in EU Member States has been taken into account through the Digital Economy Index and Society. According to Bulgaria's progress report, in 2022 Bulgaria ranks second to last (Digital Economy and Society Index (DESI), 2022 - Bulgaria). Bulgaria's lag behind the average European levels in the implementation of digital technologies and human capital is significant, despite the efforts made by the Ministry of Education and Science.

At European and national level, digital transformation, and in particular the development of digital competence, have been prioritised. It is important to carry out specific research, measures and implementation actions in order to have results.

The aim of the study is to develop a toolkit for the development of digital competence of students in their education in mathematics.

The object of the study is the students in their education in mathematics.

The subject of the study is the development of digital competence of students in their education in mathematics.

Hypothesis: The use of the developed toolkit provides conditions for the development of digital competence of students through mathematics education.

To realize the hypothesis, we set ourselves **the following tasks**:

- Analysis of strategic documents, scientific research and training documentation.
- Analysis of good practices and technologies relevant to the subject of the study.
- Development of a toolkit for the development of digital competence in mathematics education.
- Performing a pedagogical experiment.

The following **methods were used**:

- Study of strategic documents, scientific literature
- Analysis of training documentation

- Observation, polling
- Pedagogical experiment
- Statistical methods.

Chapter 1 analyzes the results of studies of theories and practices related to digital competence.

A study of international, European and national strategic documents presenting a definition and structure of digital competence is done. In the study, we use the European Parliament's and the Council's understanding of competences as a combination of knowledge, skill and attitudes, and digital competence as "confident, critical and responsible use and engagement with digital technologies for learning, in the workplace and for participation in society". We view digital competence and digital competence as synonymous. In the study we build on the five areas of competence (information literacy and literacy for data use; communication and cooperation; creation of digital content; security; problem solving).

In the first chapter are also set out basic examples of the development of digital competence in mathematics education, with an emphasis on:

- specialized software for mathematics education
- Synchronous online learning platforms
- Games as a Tool for Teaching Mathematics
- Video Tutorial
- competitive mathematics and digital competence
- Augmented Reality and Virtual Reality in Mathematics Education
- learning the peculiarities of a digital tool
- multidisciplinary lessons and digital competence.

Illustrations are made with digital resources from the Virtual School Cabinet in Mathematics of the Institute of Mathematics and Informatics of the Bulgarian Academy of Sciences, with tasks from the online competition "VIVA Mathematics with Computer", etc. The importance of dynamic constructions for the study and formulation of hypotheses, for providing a link between mathematics and fine arts, for organizing mathematical games, to provide for inspection and self-checking. An example is also given of the use of specialized software to work with Olympic tasks at school. Depending on the organization of the file, the corresponding dynamic composition can be used to pose a task, to guide the formulation of hypotheses, to explain the solution of a problem (on Fig. 1 there is a dynamic construction of the problem of 23 BMO in 2006. A straight line intersects its sides AB and AC and the continuation of BC at points D, F and E respectively, C being between B and E. The chords AA', BB' and CC' of k are parallel to DE. To prove that you do A'E, B'F and C'D go through one point").

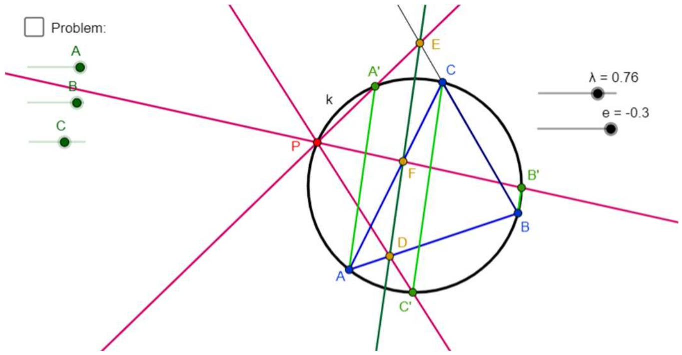

Figure 1. Dynamic construction of a task on BMO

https://cabinet.bg/content/bg/html/d18401.html

Conclusions from Chapter 1.

There are international, European and national strategic documents that are relevant to the digital transformation of education. In Bulgaria, many activities have been implemented to increase digitalization, including in education. However, Bulgaria is in the last places in various European rankings related to digitalization.

A competency approach is used when creating current curricula and programs in Bulgaria. The development of digital competence is also embedded in the curricula and programs in mathematics. Activities have been carried out to prepare teachers in mathematics for the use of digital educational resources, as well as for the use of platforms for synchronous distance learning. Currently, the so-called "Digital Backpack" is being developed and used, with access to it for all teachers and students in Bulgaria.

The need to conduct synchronous distance learning, as a result of the COVID 19 pandemic, has led to the development of videoconferencing platforms, as well as their widespread use. The education system in Bulgaria is largely prepared for the implementation of synchronous online learning, i.e. in the area of competence *communication and cooperation* there is a great development.

Digital educational resources are available for mathematics education, both free and paid. Some of them have translated materials. A disadvantage of some of them is a discrepancy with the Bulgarian curriculum when it comes to formal education.

National and European projects provide support to teachers and students with an interest in digital resources in mathematics and STEAM.

Digital educational aids for school mathematics education are growing both in quantity and quality. A decade ago, textbook publishers associated digitalization with the provision of textbooks not only on paper, but also in PDF format. Now textbooks in mathematics are accompanied by virtual resources for research and verification on a number of topics. There are also specialized sites, such as the Virtual School Classroom in Mathematics, developed and maintained by the Institute of Mathematics and Informatics of the Bulgarian Academy of Sciences. Interest in educational videos in mathematics has increased since COVID 19.

Several online competitions, such as "VIVA Mathematics with Computer", "Theme of the Month", "COMPMATH – National Olympiad in Computational Mathematics" are of great importance for the promotion of digital tools to support mathematics education, as well as for the development of digital competence of the participants. Some classic math competitions are held online. Although their format has not changed, its participation in them contributes to the development of the digital competence of the participants.

In mathematics education there is an opportunity to develop digital competence in all five areas of competence (information literacy and literacy for data use; communication and cooperation; creation of digital content; security; problem solving). Most of this development can also take place in other activities both in school and outside it; both in the learning process and in everyday life. For some of them, it is most natural for the development to take place through mathematical education, for example, the formation of competence to use specialized software to create computer models.

In the Bulgarian school there are traditions of using a computer in the study of mathematics. Currently, there is research related to the use of some systems such as GeoGebra in studying mathematics. There are resources developed. Some of them are used, but not enough in the Bulgarian school. Activities are needed to support teachers more extensively and systematically in deploying these resources.

We believe that it will be useful to create educational resources for the training of mathematics with augmented reality and virtual reality, educational games and videos. The establishment of STEM centers will ensure their technical use in school.

In Chapter 2. the StruniMa educational system *is presented* as a tool for the development of digital competence in mathematics education.

StruniMa is an educational system in the form of a multiplayer video game aimed at learning topics such as "Symmetry on a board", "Board coverings", "Graphs and chains", "Nodes and connections", etc. Its aim is to support the simultaneous development of digital and mathematical competence.

The StruniMa educational system is made using the Unity3D game engine. Thus, there is an opportunity both to generate many specific games and to provide online communication, observation, training, feedback, evaluation, etc. For example, two preliminary applets were made – "Game with axial symmetry" and "Game with central symmetry". They require a separate installation and can be found at the following address:

https://cabinet.bg/index.php?pageid=video&contenttype=virtual_reality

The StruniMa educational system has three main components – "Shkola", "Learn" and "Duels", which correspond to components in the learning process – collaboration with a teacher, independent work activities, knowledge verification activities and competitions (Figure 2).

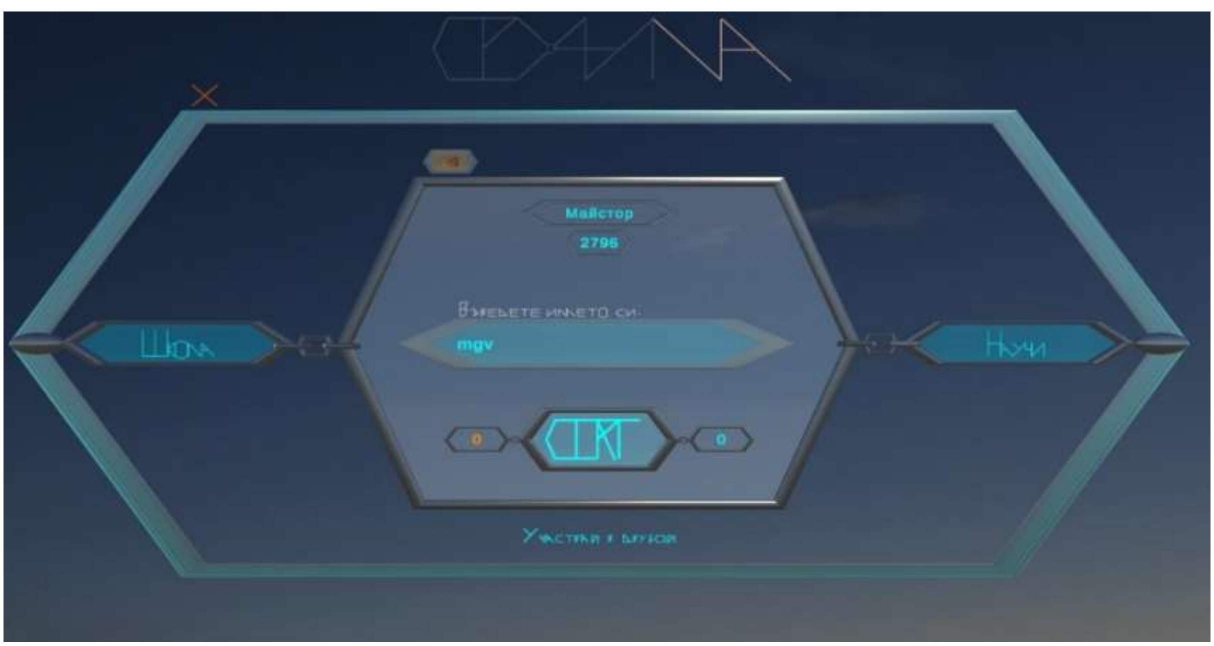

Figure 2. Main components and schematic - https://strunima.free.bg/MainComponents.html

StruniMa is available for Windows and Android, and it can also be exported to MacOS and Linux. For Windows, downloading a new version is done through an updater.

Fig. 3 presents the Component "Shkola" for synchronous distance learning through the classroom in *StruniMa.*

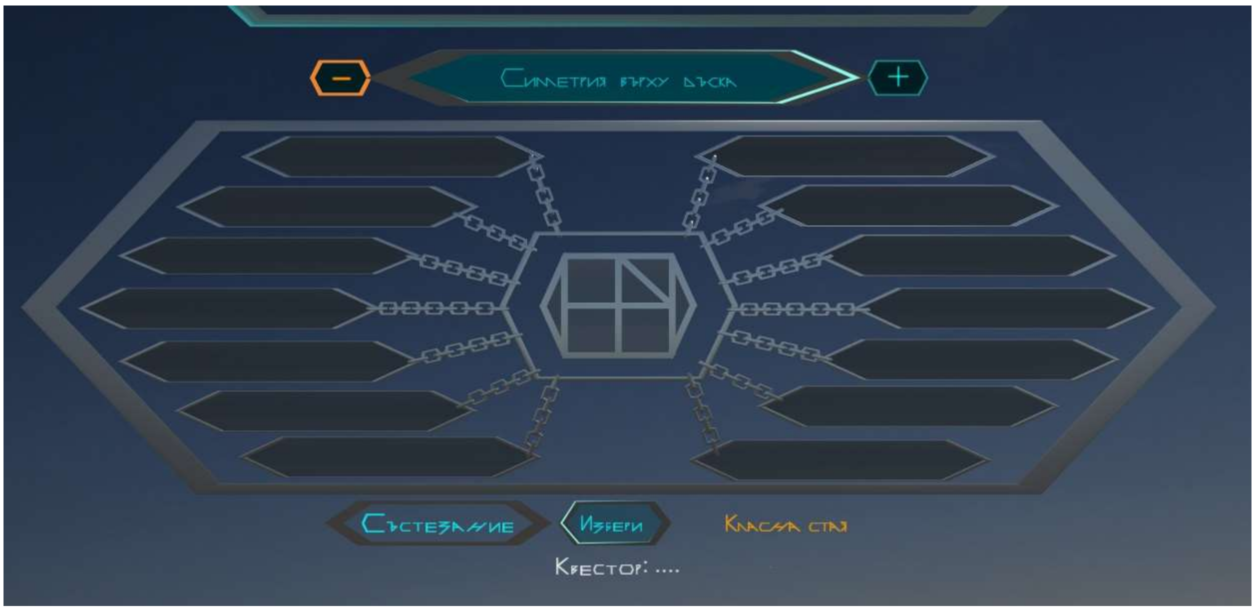

Figure 3. The „Shkola“ component - https://strunima.free.bg/Shkola.html

Fig . 4 presents the "Duel" component for games with standings, groups and points. The matches are a competitive component that participates in determining the total sum of points of the player in *StruniMa* (along with the result of the competition outside the network in the section for independent work).

Figure 4. Match-making component “Duels” - https://strunima.free.bg/Dvuboi.html

Some of the main components of *StruniMa* are described.

The competition level is a building block in the competition part of *StruniMa* and determines the level of complexity of the task according to predefined parameters – a primitive with certain dimensions, time, etc. (Figure 5).

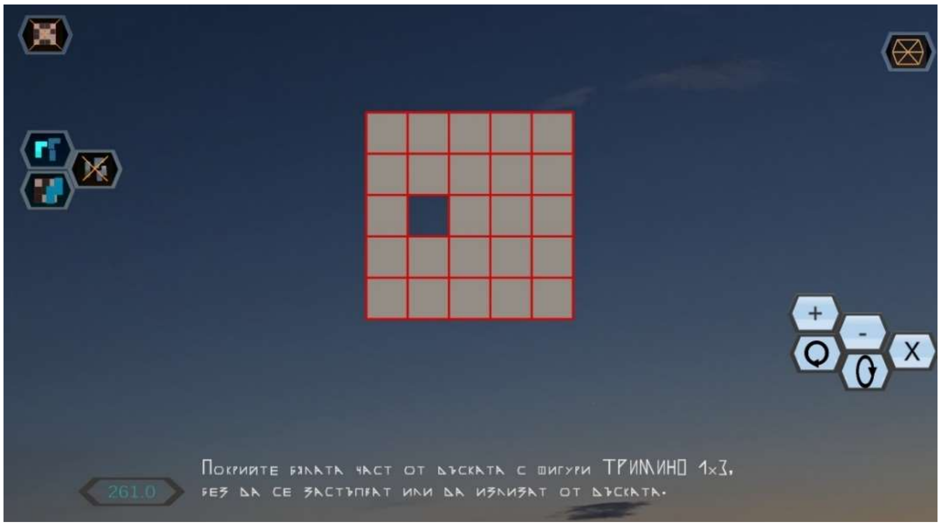

Figure 5. Competitive level from Board Coatings - https://strunima.free.bg/CompLevel.html

The training topic is a sequence of steps through which the player gets acquainted with both the controls (buttons, drop-down menus, etc.) and theoretical knowledge and methods related to the sub-topic. They are available in the section for independent work and are accessed from the "Learn" button in the research part of each main topic (Fig. 6).

Figure 6. Buttons for login to the training part and the competition part of the topics

Training topics can also be loaded by the Quaestor in the Classroom of the “Shkola”. The Quaestor can observe each participant and can give guidance through the writing board.

The writing board in *StruniMa* includes both manual typing (with the mouse on a computer or with touch from a touchscreen device) and mechanical writing of text with the keyboard (Fig. 7). Both are synchronized in the network part of the game:

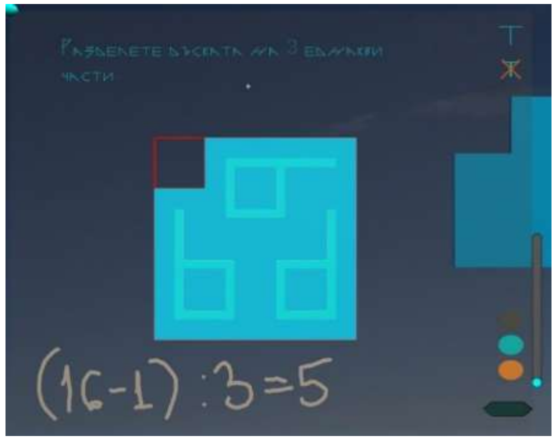

Figure 7. Use of the writing board to represent a condition and solution of a coating problem - https://strunima.free.bg/DrawingBoard.html

The combination of the two allows for a complete spelling of a condition and a solution to a problem or study. The board also allows you to solve problems on topics beyond those set in the game.

Each topic has developed sub-themes, for each of which there are training steps, competitive levels and support functionality in the research parts of the topics (Fig. 8).

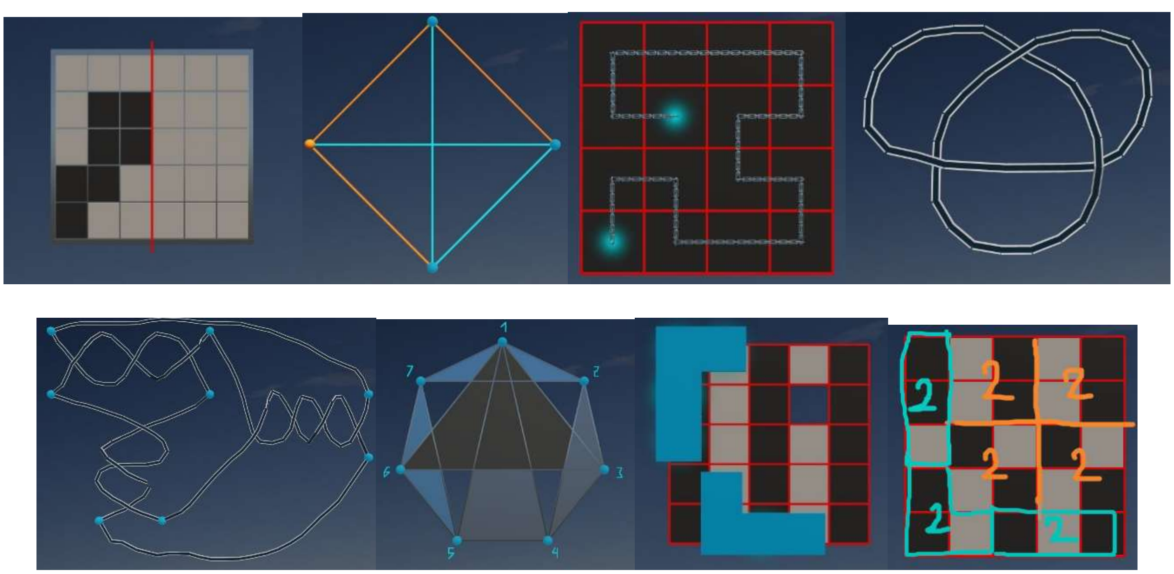

Figure 8. Sub-themes in StruniMa - https://strunima.free.bg/Applets.html

In **Chapter 3.** A pedagogical experiment is presented. The aim of the pedagogical experiment is to establish the possibility of providing conditions for the development of digital competence of students in the use of *StruniMa*. The possibilities for the development of digital competence of students in propaedeutics, introduction, studying, application, updating of types of symmetries are checked, so students from 1st to 9th grade are included in the experiment. It is used both individually training and group training. Group training is conducted with whole school classes, subgroups of school classes, as well as one group for extracurricular learning. Several topics have been studied with the extracurricular learning group. Some of the trainings are in face-to-face form and others – in remote form. In the distance form is applied both synchronous learning in an electronic environment and asynchronous learning. For synchronous learning in an electronic environment, in some cases, the created opportunity in *StruniMa is used*, and in others - the software platform for video conferencing ZOOM.

Individual face-to-face trainings were held in 2020-2023. Participants are predominantly interested in mathematics, information technology or games. Some of these trainings are realized within stands at the Symmetry Festival, UMB Spring Conference, etc. Individual distance trainings were conducted in 2020-2023 through the software platform for videoconferencing ZOOM and synchronous distance learning at *StruniMa* (more than 10 students). The group trainings were held in Shumen, Kaspichan and Sofia, in four types of groups, with a total of over 70 students. Expert assessment of 21 mathematics teachers was used.

The opportunities for the development of digital competence in terms of the first main area (Information literacy and literacy for data processing and content) when downloading the software are observed. For example, a version for the specific operating system, working with links in the website and working with the documentation of the softwares. On the development of digital competence in terms of a second major area (Communication and cooperation and participation in society) is facilitated through the work on common objects in the network part of *StruniMa*. Activities related to the third main area (Creation of digital tools and ethical principles) have been observed in the offer of new types of levels or functionalities by students during the work with the software. Assists in the development of digital competence in terms of the 4th main area (Safety and Security) by familiarizing or applying knowledge and skills for peculiarities of registration and entrance; an understanding of the importance and ways to properly store the password when registering and login. The development of competences from the 5th main area (Problem Solving) is realized through basic activities such as downloading and unzipping software; independent navigation on UI elements of the software (without prior knowledge of them); the identification of problems in the developed software – for example, the understanding that a situation is not expected behavior (many of them are observed during the work with students), the creation of computer models for solving problems.

Two standalone games have been developed, in different variants for different operating systems and environments (Fig. 9 and Fig. 10), as well as the theme in *StruniMa* "Symmetry on a board" (Fig. 11).

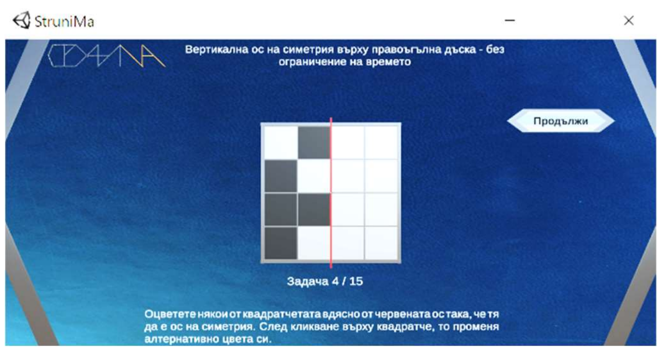

Figure 9. Playing with axial symmetry

http://cabinet.bg/content/download/NoTimeLimit.zip,

https://cabinet.bg/content/download/SepTime.zip

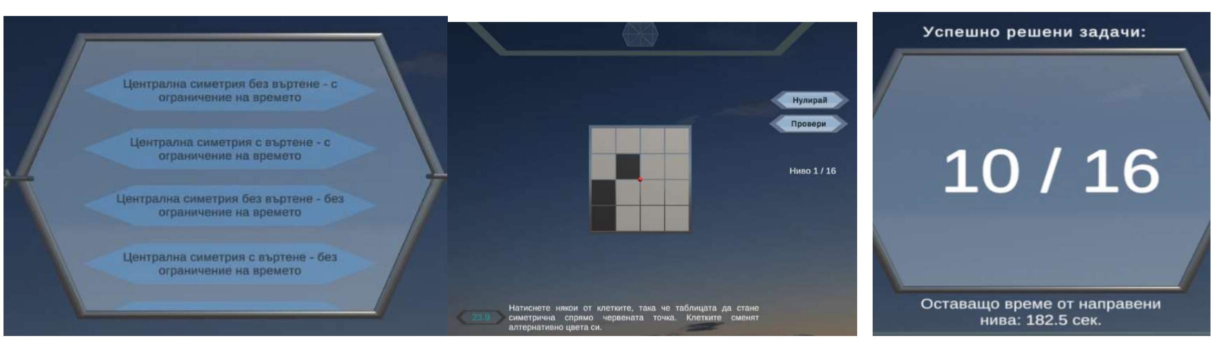

Figure 10. Variants of the game "Central symmetry on a board

https://cabinet.bg/content/download/CentralSymmetryDemo.zip

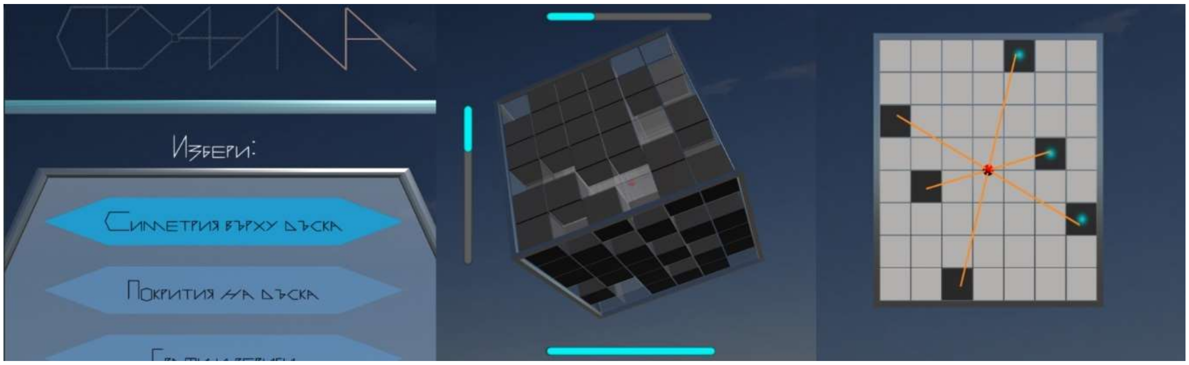

Figure 11. Symmetry table theme - [http://strunima.free.bg/download.html,](http://strunima.free.bg/download.html) [https://strunima.free.bg/SymmetryOnTable.html,](https://strunima.free.bg/SymmetryOnTable.html) https://strunima.free.bg/SymmetryOnTable3D.html

We present a description of one of the games used in the experiment. The object of a game with central symmetry is to color some of the cells on a rectangular board so that the board is symmetrical relative to its center. It is offered in four versions, depending on the availability of animation, as well as a limitation of the operating time:

- "Central symmetry without rotation – with time limit";
- "Central symmetry with rotation – with time limit";
- "Central symmetry without rotation – no time limit";
- "Central symmetry with rotation – no time limit" (Fig. 12).

Each of the variants contains 16 examples that have the role of levels. Each example is generated in a random way from boards of different size and different number of colored squares.

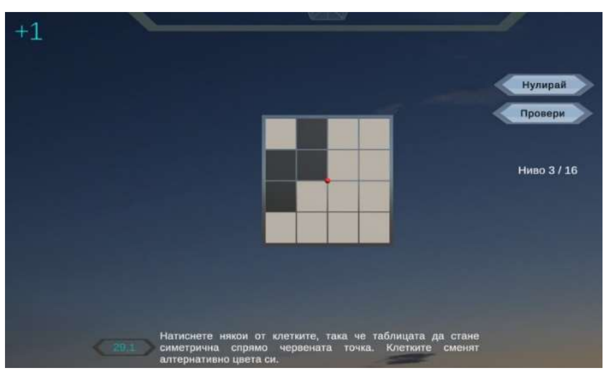

Figure 12. Example from variant "Central symmetry without rotation – with time constraint"

Several variants of organization were experimented: with or without installation; with updating knowledge through *StruniMa* or through educational resources with Geogebra; with providing a computer for each student or using 1 computer and projector, etc.

Experiment 1 was carried out in three stages at Panayot Volov Secondary School in Kaspichan (Fig. 13). The first stage was held in 2 school hours on the same day, the second stage - one month after the first, and the third - seven months after the second stage (in the next school year). The main content is related to propaedeutics and the introduction of axial symmetry and central symmetry by using a video game. 16 students from 5th grade took part in the classes.

Figure 13. In SU "Panayot Volov" in Kaspichan

Main activities of students in the first lesson of the first stage:

- Download the game from a specified link (each of the students writes the link manually)
- Open the archive and unzip content onto the desktop
- Release of "Central Symmetry Game"
- Record the IP address of the server that will collect their results
- Repeatedly passing through sections 1 and 3 of the game, which include a board with central symmetry without constant rotation of a board.

The activity was included in the STEM Discovery Campaign in 2022 and awarded (Fig. 14).

Figure 14. In the STEM Discovery Campaign in 2022, the relevant blog and the award won https://sdw-blog.eun.org/2022/06/30/a-lesson-in-central-symmetry/

And during the next two stages, all students retained an interest in the game and a competition was successfully held, including several levels with axial and central symmetry. Examples with central symmetry on a three-dimensional board are also included (Fig. 15).

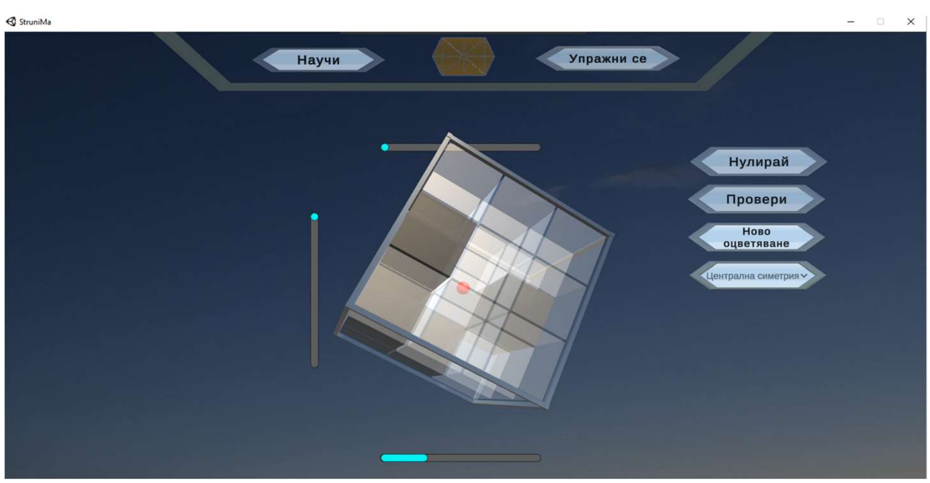

Figure 15. Three-dimensional board with central symmetry

Experiments were conducted with students from PPMG "Nancho Popovich" in the city of Shumen (Experiment 2 and Experiment 3). The main activities of the students are:

- Download a zip archive from a predefined archive
- installation of *StruniMa*
- Sign up
- updating knowledge about axial and central symmetry
- self-exploration in the research part of the game
- passing through the sections of tasks in the exercise part of the game – 4 levels with difficulty I and 4 levels with difficulty II.

The following criteria and indicators were used:

Time

- time to install and run the tool
- time to create a username and password
- Tool Launch Time on Reuse
- time to deal with the set mathematical problems (according to the type of game), including orientation in the software.

Autonmy for registration and work with the product

- without support
- with an answer to 1 question
- with an answer to 2 questions
- with effective help.

In addition, we take into account:

Motivation

- Retention of interest until the end of the experiment
- Impact of the competition environment
- Level of interest in multiple use
- use of the instrument after the experiment has been carried out;

Performance in terms of mathematics

- number of attempts until a preset level is achieved
- Maximum score
- Success after reuse
- results of a competition.

During the experiment, technical problems related to the digital tool were observed – when installing, shutting down the Internet, software used outside the digital tool; level of performance of the digital tool relative to the hardware used.

Part of the checks are carried out for the first and second performance of the activity under consideration by the same group, and others – for the results of an experimental and control group. When using a control group and an experimental group, alignment is carried out according to information technology or computer modeling estimates.

In 2023, an experiment was conducted with students from two 8th grade classes at the Nancho Popovich High School of Mathematics in Shumen (Experiment 2) in two stages.

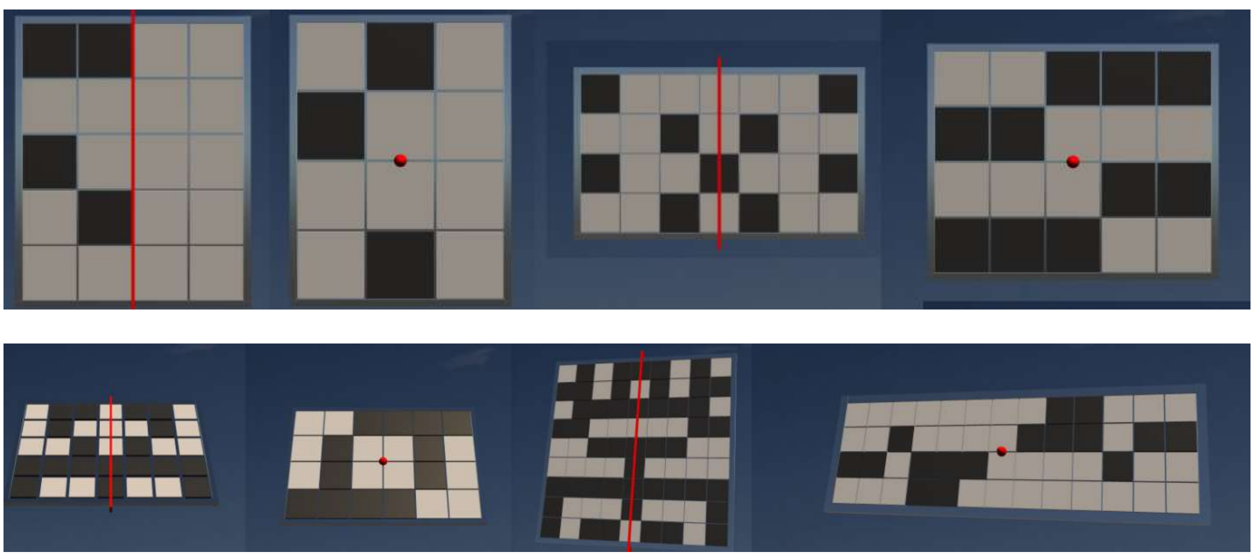

Figure 16. Group I and Group II levels used in experiment 2 and experiment 3

Both classes had preliminary knowledge of two types of symmetry in the plane through lessons "Axial symmetry", "Properties of Axial Symmetry" and "Central Symmetry" from the compulsory preparation in mathematics.

In the competition, held at the end of the hour, 8 levels with difficulty I and II were included (Fig. 16). The average score of the participants' difficulty I levels was 683 points, and the average score of those with difficulty II of the participants was 482 points, with each successfully decided level yielding between 100 and 200 points.

Experiment 3 was conducted with 9th grade pupils from the same school as well as with 5th and 6th grade students.

The results show a significant development of digital competence in the ability for some features in the registration of a profile:

- User Name Character Restrictions
- Restrictions on password symbols
- Valid components of email – prefix, '@' symbol and domain, which must be composed of two parts separated by a period such as "strunima.bg".

The average registration and entrance time measured for the students of the 2nd class of the experimental group (9 g and 9 e grade) was 2 minutes and 56 seconds.

The average registration and entrance time measured for the 9 students in a class of the control group is 4 minutes and 41 seconds.

The information technology grades of grades 9 differ negligibly. We tested a hypothesis:

Null hypothesis H01: The registration and input time of the control group and the experimental group is statistically indistinguishable.

Alternative hypothesis H1: The registration and input time of the control group and the experimental group is statistically distinguishable.

Using Student's T-method for comparison of mean between two independent samples we obtain that temp=2.12, which is more than , which means that the alternative hypothesis H$t_{\alpha} = 2{,}06$1 is fulfilled.

Comparing the average number of points of 9d and 9d PPMG-Shumen of Group I and Group II levels from Experiment 2 and Experiment 3 can be noticed a slight improvement – from 683 and 482 (respectively for levels with difficulty I and II, Fig. 16) rises to 693 and 495. Here we can draw a conclusion, which was also noted in experiment 1 – the knowledge and the acquired understanding of the types of symmetry on a board was preserved after a period of more than 6 months.

The average registration and entrance time measured for the 6b grade students of the control group was 2 minutes and 27 seconds. The average registration and entrance time measured for the 5b grade students of the control group was 7 minutes and 54 seconds.

In terms of the degree of autonomy at registration and entrance, the number of additional questions asked by students from experimental and control groups did not differ significantly. However, students in the control groups had a higher number of unsuccessful login attempts (after registration).

With regard to the criteria related to the motivation of the groups to maintain interest until the end of the experiment, the impact of the competitive setting and the level of interest in repeated use, there is no difference between the control and experimental groups. All students remained interested until the end of the class. The competitive environment was motivating for almost all students. In few cases, there is an independent use of the digital tool without the presence of a teacher. – some of the factors for this are the difficult installation and the level of development of the tool's website.

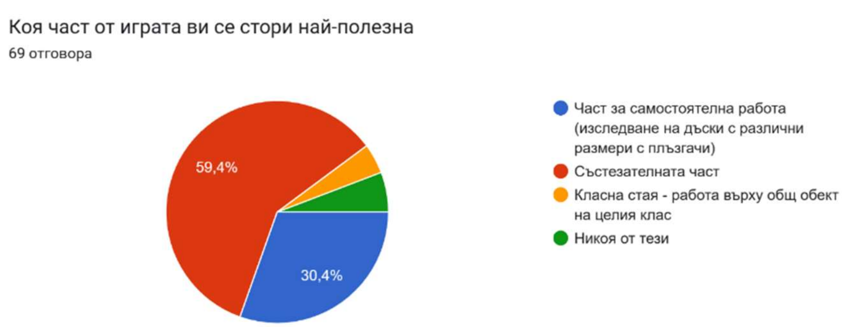

Figure 17. Survey with students from Experiment 3

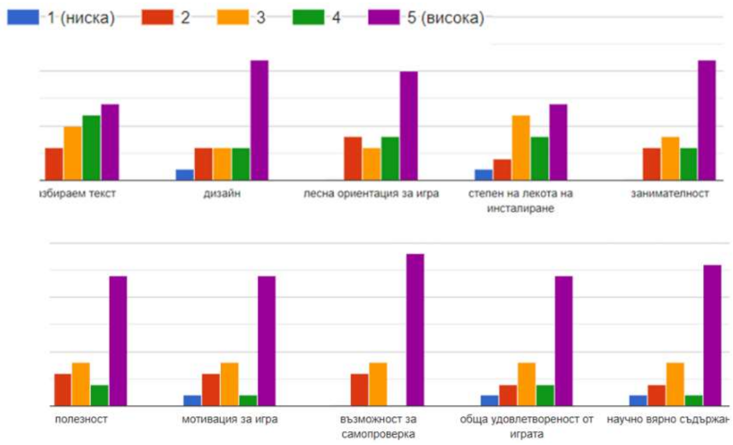

Figure 18. Teacher Poll for "Central Symmetry Game"

From the results of experiment 2 and experiment 3 several more general conclusions can be drawn.

1. After the training, students successfully manage to download and unzip files. Students successfully manage to navigate between menus in the digital tool, as well as to recognize buttons and drop-down menus for certain functions. Students are able to create user input data and understand the limitations of selecting a name and password. This follows from successfully passing the I group of levels of all students.

2. Students have gained a sense of the basic properties of the axial and central symmetry in the plane in mathematics classes. The difficulty of students in rotating the board (group II of levels) is due to the need to consider combinatorial properties of a board divided into cells (for example, a relationship between the numbers of the pillars and the rows of symmetrical cells), as well as developed spatial thinking.

3. Each student is actively involved in the class. This is expected given the non-standard situation.

4. Installing software takes up much of the time in the hour. This indicates a need for good ditigal competence of students in this respect, or for a program to automatically download or update the software, or for the software to be available as a web application.

5. The presence of already installed software allows the organization of a competition among all students, including, for example, three tasks from axial symmetry and three tasks from central symmetry. This activity plays the role of testing knowledge at the end of the hour.

6. Developed digital competence (both teacher and pupils) is key to reducing the time for technical activities related to the students' work environment. This leaves more time for activities related to the specific topic of the hour.

7. With the number of students between 10 and 15 in an attendance environment, i.e. when dividing the class into two parts, there is no load on the Internet connection. Having a larger number of students in a face-to-face environment poses risks for successful classes using digital devices and software. Many of the activities require network access, which in some cases can be burdensome for the classroom's Internet connection. Some computer labs use a Windows server machine, which basically means that all students use the same computer. In these hours, the number of students depends on the characteristics of the machine. Activities that do not require entry/registration must also be carried out. A large number of students have difficulty preparing their work environment, so the time for technical activities may be too long.

The topic "Symmetry on a board" by *StruniMa* has been provided for testing students in multiple cases, both in person and remotely. To solve levels with central symmetry, different strategies have been observed that are based on different properties of central symmetry on a board – for example, the corresponding symmetrical columns are the same, but turned to each other. Also, the composite figures of the left half can be transferred to the right half (Fig. 19).

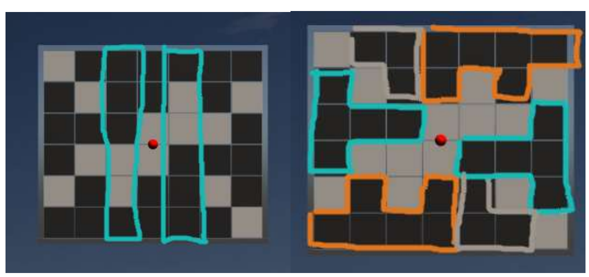

Figure 19. Divide the left half into shapes or areas that move to the right half of the board

Experiment 4 is related to continuous work with an extracurricular learning group. Many of the components in the *StruniMa system* were created as a result of many years of pedagogical activity with students from 1st to 8th grade. Many of the lessons and functionalities described in the second chapter are developed on the basis of activities in a learning environment (and the activities themselves are then used repeatedly in a learning environment). Some of the functionalities and interactive elements in *StruniMa* are based on solving combinatorial math problems with students, such as problems with rectangular boards and triangular meshes (Fig.

20). Such examples are the three-dimensional writing board on objects and the main interactive objects in the topics.

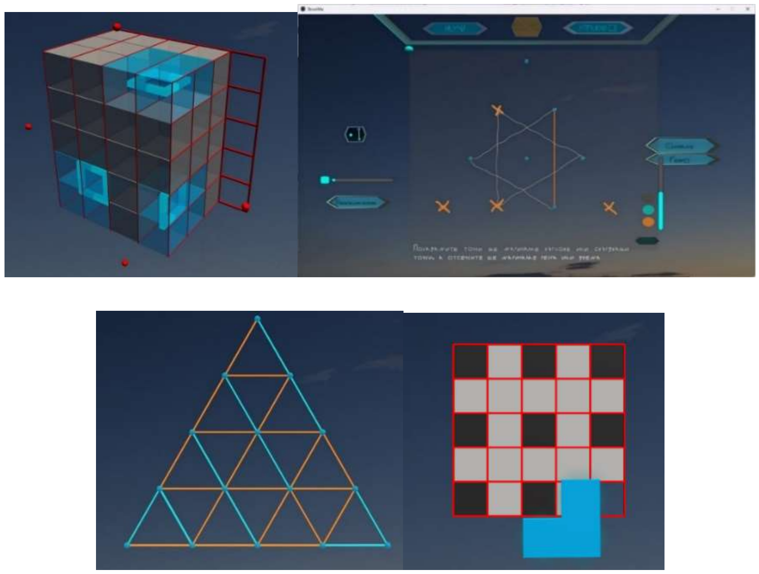

Figure 20. Examples of using StruniMa in extracurricular activities

Students develop their digital competence during and outside the math learning process. For this reason, mathematics education must be properly adapted to the use of software. We can draw several basic conclusions as a result of monitoring the learning process when using *StruniMa*.

- Any small problem in the software or insufficient preparation of a lesson that uses software may take a tangible amount of time from it, which may lead to incompleteness in the material shown or failure to realize the objectives of the lesson. This time is reduced when students have developed digital competence, as the problems encountered are solved faster.
- The gamification of knowledge and the use of software in school hours can make the transmitted knowledge accessible to all students in the lesson. Teacher feedback is facilitated. Students can successfully get acquainted with areas such as combinatorics, in the presence of easy access to the generation of arbitrary constructions with graphs, tables, nodes, etc., objects considered both in the plane and in the three-dimensional space.
- The use of software in school hours and extracurricular activities makes it possible to implement a new kind of tasks that require both rich mathematical knowledge and developed digital competence.

- Developing the ability to describe tasks can be helped with the development of students' digital competence. The ability to easily save and open saved solutions, as well as the ability to mechanically and manually write text in the solutions are helped by the virtual board.
- The introduction of a competitive element in the school hours (through the use of software) contributes to the motivation of students to perceive the target mathematical knowledge. The software used must be updated and developed, and monotony can lead to a decrease in motivation.

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Conclusion

There are international, European and national strategic documents that are relevant to the digital transformation of education. In Bulgaria, many activities have been implemented to increase digitalization, including in education. However, Bulgaria is in the last places in various European rankings related to digitalization.

The development of digital competence is also embedded in the curricula and programs in mathematics in Bulgarian school education. Activities have been carried out to prepare teachers in mathematics for the use of digital educational resources, as well as for the use of platforms for synchronous distance learning. The need to conduct synchronous distance learning as a result of the COVID 19 pandemic has led to the development of videoconferencing platforms as well as their widespread use. The educational system in Bulgaria is largely prepared for the implementation of synchronous distance learning, i.e. in the area of competence communication and cooperation there is a great development. Digital educational resources are available for mathematics education, both free and paid. Some of them have translated materials. A disadvantage of some of them is a discrepancy with the Bulgarian curriculum when it comes to formal education. National and European projects provide support to teachers and students with an interest in digital resources in mathematics and STEAM.

In mathematics education there is an opportunity to develop digital competence in all five areas of competence (information literacy and literacy for data use; communication and cooperation; creation of digital content; security; problem solving). Most of this development can also take place in other activities both in school and outside it; both in the learning process and in everyday life. For some of them, it is most natural for the development to take place through mathematical education, for example, the formation of competence to use specialized software for mathematical models.

In the Bulgarian school there are traditions of using a computer in the study of mathematics. Currently, there is research related to the use of some systems such as GeoGebra in studying mathematics. There are developed resources, but they are not widely used in the Bulgarian school. Activities are needed to support teachers more extensively and systematically in deploying these resources. It is appropriate to create educational resources for the education of mathematics with augmented reality and virtual reality, educational games and videos.

The training system *StruniMa* provides the opportunity to generate many specific games, for online communication, observation, training, feedback and evaluation. The generated specific games provided with free access contribute to the development of competence for the use of augmented reality and virtual reality. The developed methodological guidelines for the use of the educational system are in support of teachers, parents and students.

Through the pedagogical experiment, the capabilities of the developed toolkit for the development of digital competence of students through mathematics education have been proven.

Offerings

An analysis of international, European and national strategic documents, research and educational literature related to the digital transformation of education is made. On the basis of several models of digital competence – citizens, consumers, education, specification has been made regarding school education in mathematics. Analyzed are approaches, methods, technologies and tools related to the development of digital competence in mathematical education, such as research approach, project-based approach, AR, VR, video training, online competitions, specialized software for creating computer models of mathematical objects, etc.

The educational system "*StruniMa*" in the form of a network video game has been developed. It can generate many specific games, provide online communication, observation, training, feedback and evaluation. Several specific games have been generated and made available with free access, respectively for computer use, augmented reality and virtual reality. Methodological guidelines for the use of the educational system have been developed. There are opportunities to use the functionalities of StruniMa in training on some topics such as "Symmetry on a board", "Board coverings", "Columns and chains", "Nodes and connections".

A toolkit for conducting a pedagogical experiment has been developed, which has proven the possibility of providing conditions for the development of digital competence in school mathematics education using the StruniMa educational system.

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Reporting to scientific forums on the topic of the dissertation

- Digital Resources for the Study of Axial Symmetry, prof. Tony Chehlarova, Mladen Valkov. OMI Section Report Session,
- Digital applet with L-trimino board cover. Mladen Valkov, National Seminar on Mathematical Education 2020, http://www.math.bas.bg/omi/nso/?cat=21
- Apple Mathematics, Darinka Valkova, Mladen Valkov, "Dynamic Mathematics in Education", 15 February 2020, IMI-BAS
- Digital technologies in working with young talents and special target groups - Fiftieth Jubilee Spring Conference, Burgas, 1-5 September 2021, akad. Petar Kenderov, prof. Toni Chehlarova, Georgi Gachev, Mladen Valkov
- Struma Game School, Mladen Valkov, National Seminar on Mathematical Education 2021.
- Examples of three-dimensional models for mathematics education. Petar Kenderov, Tony Chehlarova, Mladen Valkov, National Seminar on Mathematical Education 2021.
- *StruniMa* - combinatorial problems in three-dimensional boards, Mladen Valkov, National Seminar on Mathematical Education 2022.
- Digital applet "Coloring without a monochrome clique", Mladen Valkov, "10 years JUMPMath"
- Game with vertical axis of symmetry in a rectangular board - Symmetry Festival, Sofia, Bulgaria, 9-12 July 2021, Toni Chehlarova, Mladen Valkov and participation with a workshop in the "Symmetry Festival" - 9-12 July 2021, Sofia (T. Chehlarova, M. Valkov)
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