REVIEW

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Of a dissertation for obtaining the scientific degree "Doctor of Sciences"
in the field of higher education: 4. Natural sciences, mathematics and informatics.
professional field: 4.5. Mathematics.

Author: Peter Vassilev Danchev
Topic: "SOME CLASSES OF NON-COMMUTATIVE RINGS AND ABELIAN GROUPS".

1. General description of the dissertation

By order № 75 of 28.02.2020 of the Director of the Institute of Mathematics and Informatics - BAS and based on a decision of the SC of IMI (protocol №3 of 21.02.2020) I was appointed a reviewer in the scientific jury in connection with the defense procedure of the dissertation on the topic "SOME CLASSES OF NON-COMMUTATIVE RINGS AND ABELIAN GROUPS" for obtaining the scientific degree 'Doctor of Sciences' in the field of higher education 4. Natural sciences, mathematics and informatics, professional field 4.5. Mathematics. The author of the dissertation is Petar Vassilev Danchev.

The dissertation consists of an abstract, introduction, notation, two chapters, a list of unsolved problems, a bibliography and a list of citations. The total volume of the dissertation is 230 pages. Also presented is an abstract, an author's report on the contributions of the dissertation, a list of publications, a CV and a report on the fulfillment of the minimum requirements by a candidate for the degree of "Doctor of Science".

2. Brief biographical data about the author

Peter Vassilev Danchev graduated from high school in 1989 at MH "Acad. K. Popov", Plovdiv. He graduated from higher education in 1995 in - FMI of PU "P. Hilendarski ", Plovdiv with a specialization in" Algebra and Number Theory ".

His teaching activity is carried out in PU "P. Hilendarski ", Plovdiv - 1993-1994, " Acad. K. Popov ", Plovdiv - 1995, TsUNT, Plovdiv - 1995-1996, 127 SOU" I. N. Denkoglu ", Sofia - 2002-2016, NUTI, Sofia - 2010-2011, Technical University, Sofia - 2017-2018, IMI of BAS, Sofia - 2018-2019. He has a doctor's degree from 2018. He speaks Russian and English.

3. Minimum requirements

This review has been prepared in accordance with Law for Development of the Academic Staff in the Republic of Bulgaria (LDASRB) and its rules. The presented materials and dissertation results fully comply with the specific requirements of IMI, adopted in connection with the Rules of IMI for application of the LDASRB.

Submitted by the applicant are all documents and materials required by the procedure, which allow for a complete and accurate assessment in accordance with the requirements of LDASRB, the Regulations for its implementation and the Regulations of IMI-BAS.

The candidate has submitted a "doctor's" diploma and a completed table with publications that have not participated in previous competitions. The total number of points is 712, which exceeds the minimum required 350 points. They are distributed as follows: A Indicator 1 - 50 points, B Indicator 2 - 100 points, D Sum of indicators 5-9 - 410 points, E Sum of indicators 10-12 - 152 points.

Based on the above, it can be seen that the candidate fully meets the set of criteria and indicators for obtaining the scientific degree "Doctor of Sciences".

3. Characteristics and evaluation of the dissertation

The dissertation is structured by an abstract, introduction, notations, two chapters, a list of unsolved problems, a bibliography of 110 titles, a list of 16 articles by the author used in the dissertation and a list of 31 citations. The total volume of the dissertation is 230 pages.

The introductory part is formed in two paragraphs. The first describes in detail the relevance of the issues, goals and objectives of the dissertation. The second summarizes the main indications.

The first chapter covers 50 pages and is devoted to non-commutative unitary rings and group rings.

In § 1 are considered the so-called weakly clean rings. The ring R is called weakly clean, if every element $r \in R$ can be written as $r = u + e$ or $r = u - e$, where $u \in U(R)$ and $e \in Id(R)$. In this regard, the ring R is called weakly exchange, if for any element $x \in R$ there exists $e \in Id(R)$ such that $e \in xR$ and $1 - e \in (1 - x)R$ or $1 - e \in (1 + x)R$. The study of this variety of rings is not fully completed, which continues to this day. In Theorem 2.1 Danchev proves that if the factor-ring $R / J(R)$ is weakly clean (respectively, weakly exchange) and all idempotent are lifted by module Jacobson's $J(R)$, then R $R / J(R)$ is weakly clean (respectively, weakly exchange). In particular, if $2 \in J(R)$, then the inverse implication also holds.

The next type of rings that are considered are the so-called invo-clean. A ring R is called (strongly) invo-clean, if for any $r \in R$ there exist $u \in U(R); u^2 = 1$ and $e \in Id(R)$ with the property $r = u + e$ (in addition to $ue = eu$)). In Theorem 2.2 Danchev proves that the ring R is invo-clean if and only if $R \cong R_1 \times R_2$, where R_1 is an invo-clean ring with a characteristic of at most 8 (which ring is also nil-clean), and R_2 is either $\{0\}$ or a commutative semi-primitive (and therefore reduced) invo-clean ring with characteristic 3 such that each of its elements is the sum of two idempotents (respectively of two involutions). Additionally, R_2 can be embedded isomorphically as a subring in the direct product of copies of the field $\mathbb{Z}_3$.

By naturally expanding this type of ring, the following new concept is reached: The ring R is called (strongly) n - periodically clean for some positive integer n, if for any $r \in R$ there exist $u \in U(R); un = 1, e \in Id(R)$ with the property $r = u + e$ and n is the minimum number in this record. In Theorem 2.3 Danchev proves that if R is an abelian ring, then the ring R is clean, having a multiplicative group with finite exponent, if and only if R is n - periodically clean for some positive integer n.

Another main result is the following statement, which reads as follows: Theorem 2.4. Let n be an odd natural number. The ring R is strongly n - periodically clean if and only if it is a clean ring in which the orders of all inversible elements are odd and bounded by n so that there is (at least one) reversible element of the order n.

In § 2 are discussed the conditions for this group rings $R[G]$ to be UU ring. A ring R is called UU ring, if $U(R)=1+Nil(R)$. In Theorem 4.2.1 Danchev proves that if G is a locally finite group, then the group ring $R[G]$ is UU ring then and only when R is UU ring and G is 2-group.

The second chapter of the dissertation covers 160 pages and is dedicated to Abelian groups.

The ring of endomorphisms of broad classes of Abelian groups was studied in accordance with the group structure, and some of its ring properties were obtained, which are significant amplifications of some well-known classical results.

A new reduction homological theorem is proved using additional axiomatics from set theory (ie, outside the usual ZFC theory) for the class of $p^{\omega+1}$ - projective Abelian groups, the result improving Nunke's classical homological theorem (Topics in Abelian Groups, 1962) for this class of groups.

The basic homological theorem of Nunke (Math. Z., 1967) for direct sums of enumerable Abelian p-groups to a new, much wider class of Abelian groups, called n-totally projective Abelian p-groups, which class entirely contains the well-known and important class of totally projective Abelian p-groups.

Some of the main results in this chapter of the dissertation are these:

1. Let $G=A\oplus B$ be a group with subgroups *A,B*. Then

1.1 G is socle-regular $\Leftrightarrow A$ is socle-regular, assuming that B is separable.

1.2 A is socle-regular if G is socle-regular, i.e., the direct additive of a socle-regular group is again a socle-regular group.

1.3 Krylov-transitive groups are always socle-regular, while the inverse implication is incorrect.

1.4 There is a weakly transitive group that is not socle-regular.

1.5 Each totally projective group with length $\le\omega^2$ is highly projective fully transitive.

1.6 If G is a group so that its first Ulm subgroup $p^{\omega}G$ is elementary, then G is completely transitive $\Leftrightarrow$ the Cartesian square $G\oplus G$ is highly projective completely transitive $\Leftrightarrow$ the Cartesian square $G\oplus G$ is highly switching fully transitive.

1.7 Each totally projective group with length $<\omega^2$ is a commutator socket-regular.

1.8 Direct additive of a commutator socket-regular group is not necessarily again a commutator socket-regular group, the same being true for the case of a commutator - completely transitive group.

1.9 Projectively the socle-regular and commutator socle-regular groups are independent of the transitive and fully transitive groups.

1.10 Commutator fully transitive groups are always commutator base-regular.

1.11 The direct sum of a fully transitive IFI group without periodic elements is again a fully transitive IFI group without periodic elements.

1.12 If G is IFI- group, then the Cartesian square $G\oplus G$ is also IFI-group.

1.13 Any strongly servant-indecomposable group for which inequality is valid $|G/pG| \leq p$ for every prime number p, is an IFI group.

2. Let's assume that G is a group for which the factor group $G/p^{\omega+1}G$ is $p^{\omega+1}$ - projective. If the subgroup $p^{\omega+1}G$ is enumerable, then G is a direct sum of $p^{\omega+1}$ - projective group and enumerable group. In addition, there is a group G, for which the factor group $G/p^{\omega+2}G$ is $p^{\omega+2}$ - projective and the subgroup $p^{\omega+2}G$ is enumerable, but G is not a direct sum of $p^{\omega+2}$ - projective group and enumerable group. In particular, if $0 < n < \omega$, it class from $\omega + n$ - totally $p^{\omega+n}$ - project groups is not closed (invariant) on finite or infinite direct sums.

3. We assume that G is a group, n is an arbitrary natural number and is an arbitrary ordinal. Then G is n - simply presented $\Leftrightarrow$ the groups $p^{\lambda}G$ and $G/p^{\lambda}G$ are n - simply presented.

4. For any $n \in \square$, the direct sum of a *n*-simply represented group is again a *n*-simply represented group assuming that the additional direct summand is an enumerable group.

5. Let $n \in \square$. Then the following two statements are true:

(1) Nicely $\omega_1 - n$ - simply the represented length groups $< \omega^2$ are n - simply represented.

(2) Let G is a group whose factor group $G/p^{\lambda}G$ is n - simply represented for some ordinal λ. Then the group G is good $\omega_1 - n$ - simply represented $\Leftrightarrow$ the subgroup $p^{\lambda}G$ is nicely $\omega_1 - n$ - simply represented.

4. Evaluation of the publications on the dissertation

There are 16 published publications on the topic of the dissertation (5 own and 11 joint). All are in English in foreign magazines. Of these, 9 have an impact factor and 2 have an impact rank. The total value of the impact factor (JCR according to Thomson-Reuters) is 5,855. The high scientific value of the publications is evidenced by the fact that they have a total of 31 citations. The scientific potential of the candidate is evidenced by the fact that the total number of his publications is 354, of which 79 with impact factor.

5. Autoreview

The autoreview is 40 pages long and contains the main results obtained in the dissertation. It fully reflects the content of the dissertation and the main contributions of the dissertation. The autoreview gives an idea of the studied problems and the obtained results. The author's autoreview gives the goals and tasks of the dissertation, highlights the main results and prospects of research on this topic.

6. Critical remarks and recommendations

It makes a good impression that there are additional explanations and examples between the results. My only remarks are on the technical layout of the electronic version of the dissertation. Chapter II are notations that are collected on 1 page and should be the second paragraph of Chapter I - Introduction. Chapter V - Open Issues is also on page 1 and should be included as a paragraph in the relevant chapters. Thus, the dissertation should have an Introduction and two chapters in which the results are presented. My other remark is that any citation of a literature or theorem can be made as an active link so that the reader can easily go

to the title in question. In this regard, it is good to have a field with content with links to the relevant chapters and paragraphs.

7. Recommendations for future use of dissertation contributions and results

I am convinced that the results obtained will be used in future research in this area. In addition, Petar Danchev pointed out what open problems can be solved using the results of the dissertation.

CONCLUSION

The dissertation **contains scientific, scientific-applied and applied results, which represent an original contribution to science and meets** the requirements of the Law for Development of the Academic Staff in the Republic of Bulgaria (LDASRB) and its rules. The presented materials and dissertation results fully comply with the specific requirements of IMI, adopted in connection with the Rules of IMI for application of the LDASRB.

The dissertation shows that Peter Vassilev Danchev has in-depth theoretical knowledge and professional skills on the topic of labor, demonstrating qualities and skills for independent research.

Due to the above, I give my positive assessment of the research presented by the above-reviewed dissertation, autoreview, results and contributions, and **I recommend** the esteemed scientific jury to award the degree of **'Doctor of Sciences'** to Peter Vassilev Danchev in higher education area 4. Natural sciences, mathematics and informatics, professional field 4.5. Mathematics.

1.06. 2020 **Reviewer:**
(Prof. D.Sc. Ivo Michailov Michailov)