STATEMENT

by Prof. Ivan Dimitrov Trendafilov, PhD
of the dissertation for obtaining
the scientific degree "Doctor of Science"
in scientific field 4. Natural sciences, mathematics and informatics
professional field 4.5. Mathematics
specialty Algebra and Number Theory

Title: SOME CLASSES OF NONCOMMUTATIVE RINGS AND ABELIAN GROUPS

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I present my report as a member of the Scientific Jury, determined by the Order No. 78/28.02.2020 of the Director of Institute of Mathematics and Informatics (IMI), Bulgarian Academy of Sciences (BAS).

The statement was prepared in accordance with the requirements of:

- the Law for the Development of Academic Staff in the Republic of Bulgaria (ZRASRB),

- the Rules for the Implementation of the ZRASRB,

- The Rules on the Terms and Conditions for Acquisition of Academic Degrees and Occupation of Academic Positions at BAS and at IMI of BAS.

General information about the candidate

According to the documents submitted for participation in the procedure, Peter Danchev obtained a Master's Degree at the Faculty of Mathematics and Informatics, Plovdiv University in 1996. In the period 2016 – 2018 he is a PhD student at IMI - BAS, section Algebra and Logic, and defends in this section of IMI - BAS in 2018 a thesis on the topic in the specialty Algebra

and Number theory. There in 2019 he became an Assistant Professor.

A brief description of the dissertation

This dissertation, of 230 pages, includes five chapters, a list of 110 references, used author papers and citations in total.

The text starts by providing in Chapter 1 a brief discussion of the fundamental results connected with the dissertation.

In Chapter 2 (one page?) there are some notations used in the next text.

Chapter 3 focuses on the weakly exchange rings. From classical paper of W.K. Nicholson, in 1977, it is well-known that an unitary ring R is clean when each $x \in R$ is a sum of an idempotent and a unit, i.e. $x = e + u$ and exchange when for each $x \in R$ there is an idempotent $e \in xR$, such that $1-e \in (1-x)R$. All clean rings are exchange. A ring R is weakly exchange when each $x \in R$ is a sum or a difference of an idempotent and a unit. In this chapter Danchev prove interesting results connected with the weakly exchange rings. I point the very good results in Theorem 1.64 and Theorem 1.93.

In the last three pages of the considered chapter 3 there are some applications to group rings. But let us note the important result: If G is a locally finite group and R is a ring, then the group ring $R[G]$ is UU - ring if and only R is UU - ring and G is 2-group.

Most of the dissertation is Chapter 4 (over 150 pages), which is about Abelian groups. There are new interpretations of the well-known classes of Krylov transitive groups, weakly transitive groups, IFI-groups, n-balanced projective groups, n-simply presented groups, $\omega_1 - n$-simply presented groups and some other classes of groups.

The first section of this chapter is "Generalizations of transitive and fully transitive Abelian groups". There are a series of results concerning the characterization of both classical classes of transitive and fully transitive groups and their extensions.

The endomorphism ring of an Abelian group is an important classification tool. There are two questions for the endomorphism ring of an Abelian group: when is the ring generated by its commutators, and when is the ring

additively generated by its commutators? In connection with these questions Danchev studies the so-called commutator fully transitive groups and strongly commutator fully transitive groups getting interesting results.

The second section is "Generalizations of simply presented Abelian p-groups". It is important to note the results of Theorem 4.43 and Theorem 4.44 relating to n-simply presented groups

The results established in the dissertation are sound, technically correct, true and perspective.

After the main chapters of the disertation in Chapter 5 the author present six challenging problems concerning noncommutative rings and Abelian groups.

Most of the scientific results of Peter Danchev have been presented at large number of international conferences and seminars.

General description of the presented publications

Peter Danchev presents sixteen scientific articles. Five of the articles are published in international scientific journals without SJR or IF. Two articles have SJR and the remaining nine articles are published in respectable scientific journals with an IF. Five of the articles are own and eleven are joint.

Seven of the articles,[D1] - [D7], concerning associative rings and the remaining nine, [D8] - [D16] are relevant to Abelian groups.

The sixteen articles presented have a total of 31 citations.

Main results

In my opinion the most important scientific results of the candidate are the following:

• A ring R is uniquely weakly nil-clean if and only if R is decomposable as $R \cong R_1 \times R_2$, where either $R_1 = \{0\}$ or $R_1/J(R_1)$ is Boolean with nil $J(R_1)$, and either $R_2 = \{0\}$ or $R_2/J(R_2) \cong \mathbb{Z}_3$ with nil $J(R_2)$.

• If G is a locally finite group and R is a ring, then the group ring $R[G]$ is UU - ring if and only if R is UU - ring and G is 2-group.

• Commutator fully transitive groups are always commutator socle-regular.

• Let G be a group, $n \in \mathbb{N}$ and λ an arbitrary ordinal. Then G is n-simply presented if and only if both $p^\lambda G$ and $G/p^\lambda G$ are n-simply presented.

• For an arbitrary $n \in \mathbb{N}$ a direct summand of an n-simply presented group is again an n-simply presented group, provided that the complement is a countable group.

Recommendations and remarks

I have not found substantial errors, inaccuracies and omissions in the presented materials. I recommend to the candidate to concentrate his efforts in working with PhD students and young researchers.

Conclusion

A substantial part of the publications of the candidate have been refereed in the world-known scientific databases Scopus и Web of Science and have got recognition, which is proved by the numerous citations by foreign authors. The results obtained and the contributions to science justify me to claim that Peter Vassilev Danchev is a built specialist in the field of algebra.

I think that his scientific activity deserves to be highly assessed, and I suggest to the members of the respected jury to vote on a proposal to the Scientific Council of IMI – BAS for awarding the scientific degree "Doctor of Science"to Peter Vassilev Danchev.

15.06.2020 Sign: .....

Sofia (Prof. Ivan Trendafilov, PhD)