

STATEMENT REPORT

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on the PhD Thesis of Svetozar Zlatkov Stankov
entitled "Symmetry and Metric Geometry in Banach Spaces"
submitted for obtaining the educational and scientific degree "Doctor"
in the Area of Higher Education 4. Natural Sciences, Mathematics and Informatics,
Professional field 4.5 Mathematics, Doctoral program "Mathematical Analysis"

I am a member of the Scientific Jury for the defense of this thesis, in accordance with the Scientific Council meeting of the Institute of Mathematics and Informatics, (Protocol No. 2 of 14 February 2025) and by the order of the Director of the Institute of Mathematics and Informatics at the Bulgarian Academy of Sciences No. 17 of 17 February 2025. The report is prepared in accordance with the requirements of the Act on Development of the Academic Staff in the Republic of Bulgaria (ZRASRB), the Regulations for its implementation, the Rules for the conditions and regulations for acquiring scientific degrees and occupying academic positions in the Bulgarian Academy of Sciences and the relevant regulations of IMI-BAS.

1. General Description of the Thesis.

The submitted thesis is written in English, comprises 70 pages, and is structured into 6 chapters, followed by a list of 54 bibliographic references. The thesis includes figures and program code. The bibliography demonstrates that the doctoral candidate has a solid command of the field.

2. Main Scientific and Applied Achievements.

Chapter 1 provides a detailed overview of the development of the structural theory of Banach spaces. It describes the Tsirelson space T , which has had significant impact in the theory of B -spaces. This space was introduced by Figiel and Johnson; the original Tsirelson space is the dual space T^* . The construction of Tsafiri is discussed, along with the notions of type, cotype, and distortability. Notable authors mentioned include Odell, Schlumprecht, Gowers, Maurey, and Argyros. Several authors, including the doctoral supervisor Prof. Dr. D. Kutsarova, posed the question of a Banach space in which all subsymmetric basis sequences are equivalent to a single basis that is not symmetric. The first example was given in [7] in 2024. The dissertation presents additional examples of spaces with a unique subsymmetric basis sequence up to equivalence.

Theorem 29 (Chapter 2) asserts that every subsymmetric sequence in the dual space $Ti^*(p, \gamma)$ for $p \in (1, \infty)$ and sufficiently small $\gamma > 0$ is equivalent to the canonical subsymmetric basis, which is not symmetric. The proof establishes several interesting facts, such as that $Ti^*(p, \gamma)$ does not contain an isomorphic copy of ℓ_q , where q is the conjugate exponent of p . The reflexivity of $Ti(p, \gamma)$ guarantees that any subsymmetric basis in its quotient spaces is equivalent to the canonical basis.

Chapter 3 (Theorem 33) shows that the symmetrization $S(S^*)$ of the dual of Schlumprecht's space (in contrast to the symmetrization $S(T^*)$ of the original Tsirelson space) contains a subspace isomorphic to ℓ_1 .

Chapter 4 (Theorem 34) examines the “yardstick” construction in $Ti(p, \gamma)$, proving that c_0 is finitely representable in a non-overlapping manner with respect to the canonical basis for $p \in (1, \infty)$ and $0 < \gamma < 3^{\frac{1}{p}-1}$.

Chapter 5 discusses bilipschitz embeddings and reviews results on embedding the diamond graphs D_n into non-superreflexive spaces, specifically L_1 . Bilipschitz embeddings into $L_1[0, 1]$ are important in computer science. The thesis also examines embeddings of Laakso graphs into non-superreflexive spaces and into $L_1[0, 1]$. Theorems 47 and 49 address embeddings of Laakso graphs into non-superreflexive spaces and into $L_1[0, 1]$, respectively, with Theorem 49 providing the stronger result for $L_1[0, 1]$. Using a computer program, the author obtained lower bound estimates for the distortion of embeddings of Laakso graphs $\mathcal{L}_2$ (Theorem 52) and the diamond graph D_2 (Theorem 54).

Chapter 6 contains a summary of the main contributions and details on the defense of the results.

The overall impression is of a very thorough and successful piece of work on the topic.

3. Approbation of Results.

The results presented in the thesis have been published in three articles in impact-factor journals and presented at international mathematical conferences.

4. Comments.

I have no serious criticisms. Minor remarks include the use of foreign words in the Bulgarian summary and some minor typographical errors. For example, the subscript X is missing in two inequalities in the definitions of bilipschitz embedding (and distortion) in the summaries, an omission that does not appear in the dissertation itself.

5. Qualities of the Autoreferat.

The 21-page autoreferat of the thesis accurately reflects the content of the thesis. It provides detailed justification of the relevance and motivation for the chosen topic, clearly formulates the research tasks and objectives, and correctly presents the main scientific contributions.

6. Conclusion.

I consider that the submitted thesis fully meets the requirements of the Act on Development of the Academic Staff in the Republic of Bulgaria, the Regulations for its implementation, the Rules for the conditions and regulations for acquiring scientific degrees and occupying academic positions in the Bulgarian Academy of Sciences and IMI-BAS, as well as the specific requirements for obtaining the educational and scientific degree “Doctor”, given in the Rules for the conditions and regulations for acquiring scientific degrees and occupying academic positions in the Institute of Mathematics and Informatics at the Bulgarian Academy of Sciences.

I give an unequivocally positive evaluation of the thesis “Symmetry and Metric Geometry in Banach Spaces” and strongly recommend that the Scientific Committee grant Svetozar Zlatkov Stankov the degree of Doctor in the field of 4. Natural Sciences, Mathematics and Informatics, professional field 4.5 Mathematics, doctoral program “Mathematical Analysis.”

May 14, 2025
Sofia

Member of the Scientific Jury: Prof. Dr. L. Nikolova