R E P O R T

by Prof. Mikhail Krastanov

Institute of Mathematics and Informatics, Bulgarian Academy of Sciences

of the PhD Thesis

entitled

“ Symmetry and metric geometry in Banach spaces”

by

Svetozar Zlatkov Stankov

submitted for obtaining the educational and scientific degree “Doctor” in the Area of Higher Education: 4. Natural Sciences, Mathematics and Informatics,

Professional Field 4.5 Mathematics, Doctoral Program “Mathematical Analysis”

I am a member of the Scientific Jury for the defense of this PhD thesis, appointed by Order № 17 dated 17.02.2025 of the Director of the Institute of Mathematics and Informatics at the Bulgarian Academy of Sciences (IMI-BAS). This report has been prepared in accordance with the requirements of the Act on the Development of the Academic Staff in the Republic of Bulgaria, its implementing regulations, the Rules for acquiring scientific degrees and holding academic positions at the Bulgarian Academy of Sciences, and the specific regulations of IMI-BAS.

Based on the documents and materials submitted by Svetozar Zlatkov Stankov, I have verified that he meets the requirements set out in the Act on the Development of the Academic Staff, its implementing regulations, the BAS Rules, and the IMI-BAS regulations. He also satisfies the specific criteria for obtaining the educational and scientific degree "Doctor" as defined in the IMI-BAS Rules.

1. General characteristics of the PhD thesis

The dissertation comprises 70 pages and includes an introduction, four chapters, a conclusion, a bibliography with 54 references, and a table of contents.

The research addresses current problems in the structural theory of Banach spaces, as well as aspects of metric geometry, particularly concerning the existence of bilipschitz embeddings into classical Banach spaces.

The PhD thesis is theoretical in nature and meets the generally accepted standards for awarding the educational and scientific degree of "Doctor."

The bibliography includes a substantial number of references, demonstrating the doctoral student's comprehensive knowledge of the research field.

2. Major scientific and scientific-applied achievements

Chapter One serves as a concise introduction to the main results of the PhD thesis. It provides an overview of existing achievements in the field and offers a well-founded motivation for the problems under consideration. The content of this chapter clearly demonstrates that the doctoral student, Svetozar Stankov, possesses in-depth knowledge of contemporary topics in functional analysis.

Chapter Two is devoted to examples of Banach spaces that possess a subsymmetric sequence but not symmetric basis sequence, and in which the subsymmetric basis sequence is unique up to equivalence. The chapter examines Tzafriri spaces, introduced in [48] of the PhD thesis's bibliography, along with their modifications, known as Tirilman spaces Ti(p,γ), $1<p<\infty$, $0<\gamma<1$ (introduced by Casazza and named in honor of Tzafriri's Romanian name). Following the exposition in the book by Casazza and Shura, the main properties of these spaces are carefully presented. The central result of this chapter establishes the following:for each $1<p<\infty$ and for each sufficiently small $\gamma>0$, each subsymmetric basis sequence in the dual space of Ti(p,γ) is equivalent to the canonical basis, which is subsymmetric but not symmetric. It is worth noting that Lemma 25 and Lemma 27, which support this result, are of independent mathematical interest.

Chapter Three demonstrates that the symmetrization $S(S^*)$ of the dual of the Schlumprecht space contains a subspace isomorphic to ℓ_1. The proof employs the so-called "yardstick" construction within the Schlumprecht space.

Chapter Four extends a result from paper [26] by M. Junge, D. Kutsarova, and E. Odell, which establishes that the space c_0 is finitely representable in Ti(2,1/2) in a non-intersecting manner with respect to the canonical basis. By employing a similar "yardstick" construction and introducing the necessary refinements to the original proof, the author demonstrates that this result remains valid under the extended conditions. for every $p\in(1,\infty)$ and $\gamma \in \left(0,3^{1/q}\right)$, where 1/p+1/q=1.

Chapter Five is devoted to the study of bilipschitz embeddings of Laakso graphs into Banach spaces. The chapter provides estimates for the distortions of these embeddings. Specifically, embeddings of $\mathcal{L}_n$ in arbitrary Banach spaces that are not superreflexive, with distortion 2+ε, ε>0, as well as in $L_1[0,1]$ with distortion 4/3 are constructed. In addition, it is proved that $\mathcal{L}_2$ can not be embedded in $L_1[0,1]$ with distortion less than 9/8. The figures in this chapter (depicting the diamond graph D_2, as well as the Laaxo graphs $\mathcal{L}_1$, $\mathcal{L}_2$and $\mathcal{L}_n$) make a good impression, as they effectively illustrate the exposition. Also presented is the code of a computer program designed to calculate a lower bound for the distortion in Laakso graph embeddings and diamond graphs within the Banach space $L_1[0,1]$.

3. Approbation of the results

The results presented in this dissertation have been published in three peer-reviewed articles with impact factors:

1. Stephen J. Dilworth, Denka Kutzarova, B…unyamin Sarı, Svetozar Stankov, Duals of Tirilman spaces have unique subsymmetric basic sequences, Bulletin of the London Mathematical Society Volume 56, 150-158, https://doi.org/10.1112/blms.12920, **JCR-IF (Web of Science): 0.90 (2022) Q2 (Mathematics)**

2. S. J. Dilworth, Denka Kutzarova, Svetozar Stankov, Metric embeddings of Laakso graphs into Banach spaces, Banach Journal of Mathematical Analysis 16 (2022), no. 4, Paper No. 60, 14 pp., http://doi.org/10.1007/s43037-022-00212-7 **JCR-IF (Web of Science): 1.20 (2022) Q2 (Mathematics)**

3. Svetozar Stankov, On the symmetrized dual of Schlumprecht's space, C. R. Acad. Bulg. Sci., 78, No 1, 2025, https://doi.org/10.7546/CRABS.2025.01.02 **JCR-IF (Web of Science): 0.30 (2023) Q4 (Multidisciplinary sciences)**

The results of the dissertation have also been presented in the following reports:

1. Stephen J. Dilworth, Denka Kutzarova, Svetozar Stankov, Metric embeddings of Laakso graphs into Banach spaces, Week of Mathematics and Informatics 2024, September 23-27, 2024, Duni Royal Resort, Bulgaria,
https://www.fmi.uni-sofia.bg/bg/wmi-2024-program

2. Stephen. J. Dilworth, Denka Kutzarova, Svetozar Stankov, Metric embeddings of Laakso graphs into Banach spaces, Annual Scientific Session of Analysis, Geometry and Topology Department, December 5, 2024, IMI-BAS, https://math.bas.bg/evento/отчетна-сесия-на-секция-анализ-геометрия-и-топология

4. Critical notes and recommendations

I have not found any significant critical remarks or recommendations concerning the content or layout of the dissertation.

5. Qualities of the dissertation summary

The dissertation summary spans 21 pages, briefly outlining the motivation behind the chosen research topic and providing a detailed description of the main results. The scientific contributions are accurately and clearly presented in the author's overview.

6. Conclusion

The above analysis demonstrates that the submitted dissertation fully complies with the requirements of the Act on the Development of the Academic Staff in the Republic of Bulgaria, its implementing regulations, the Rules for acquiring scientific degrees and holding academic positions at the Bulgarian Academy of Sciences and IMI-BAS, as well as the specific criteria for obtaining the educational and scientific degree "Doctor" outlined in the Rules of the Institute of Mathematics and Informatics at the Bulgarian Academy of Sciences.

All of the above provides ample grounds for me to confidently give a positive evaluation of the PhD thesis ***"Symmetry and Metric Geometry in Banach Spaces"*** and to strongly recommend that the Scientific Jury award **Svetozar Zlatkov Stankov** the educational and scientific degree of **"Doctor"** in the area of higher education 4. Natural Sciences, Mathematics and Informatics, professional field 4.5 Mathematics, within the doctoral program **"Mathematical Analysis"**.

May 22, 2025

Member of the Scientific Jury:

` Prof. Mikhail Krastanov