Referee's report

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About: Thesis for awarding the degree Doctor of Sciences

Topic: AN APPROACH FOR PRICING AMERICAN-STYLE DERIVATIVES

Author: Assoc. Prof. Tsvetelin Stefanov Zaevski

Scientific field: 4. Natural sciences, mathematics and informatics

Professional field: 4.5. Mathematics

1. Context and relevance of the dissertation.

I will begin this review with an introduction to the historical and contemporary context of the problem.

First of all, we must say that one of the main reasons for the opening of the Institute of Mathematics in Bulgaria in the distant post-war 1947 was the fact that due to the numerous loans and reparations payments, it was necessary to calculate the profitability of securities, hence, a special Commission for the financial and mathematical study of state and state-guaranteed foreign bond loans was created, with Academician Kiril Popov as chairman. On this occasion, see the article by Prof. Nikolay Yanev, "Fifty Years of the Institute of Mathematics and Informatics at the Bulgarian Academy of Sciences". BAS Magazine, Booklet 3-4/1998.

So, today the question arises: What is "modern finance" and does it have a place in Bulgaria?

Things are very complicated today due to the presence of many and diverse financial instruments, but the essence of the problem remains: The main question is, in fact, whether in a self-respecting country it is necessary to have not just competent people, but scientists with world-class academic publications who are competent in the field of modern finance? By the way, let us note the fact that only a few truly large banks in the world can afford to maintain a group of quants

who can swim in the modern sea of financial instruments - to possess and create in a very short time a modern arsenal of weapons with which the modern war in finance can be waged.

I leave the answer to the above question hanging.

On the other hand, within the IMI, over the past 15-20 years, a group of scientists has emerged who have proven their mastery of the modern apparatus of mathematical finance. This shows that the long-term mission of the IMI in the field of finance has been continued. And this dissertation is one proof of this.

Let us note that mathematical finance is essentially a multidisciplinary modeling of financial markets, which uses a very rich set of modern mathematics: differential equations and stochastic processes, optimization, numerical methods and many other areas, although to a lesser extent. It is this multidisciplinary nature that makes the task of growing good specialists in this field non-trivial.

The present dissertation is dedicated to highly professional study and development in the field of American-type Options. For the uninitiated reader, we must remind what the place of options is in modern financial markets, especially since this important point (important from a financial, but not from a mathematical-scientific point of view) is omitted in the dissertation under consideration.

Where are Options in the Financial Instruments Ecosystem today?

Without going into too much detail, let's say that options are one of the most basic financial instrument for hedging the risk of price fluctuations. Let's mention that Thales of Miletus used them to protect himself from the risks (of prices) of the olive harvest, i.e. from the volatility of the olive markets. Options also occupy a special place in modern finance, since, among other things, they are used as a thermometer to measure the temperature (volatility) of the sea of financial instruments.

However, the main evidence of the importance of a financial instrument is the size of its market niche: for example, as of February 7, 2025, the market share for options trading in the United States was distributed among several major exchanges: • Cboe: 30.02% ; • NASDAQ: 27.98% ; • MIAX: 15.94% ; • NYSE: 15.85% ; • BOX Options: 6.95% ; • Members Exchange: 3.26%

The above percentages represent the volume of options traded on each exchange relative to the total market. For comparison, we will also provide data from the stock markets, namely, the market share for some of the major stock exchanges as of the beginning of 2025: NYSE (New York Stock Exchange): 22.53% ; NASDAQ: 14.84% ; Cboe: 16.22% ; Members Exchange: 2.90% ; Investors Exchange: 1.53% ; MIAX Pearl: 0.62% ; LTSE: 0.22%

From the above data, we see that the options have a considerable volume on the market.

The above data come to convince us of **the relevance of the topic** discussed in the dissertation work and its importance for society.

2. Main scientific results of the dissertation work

The dissertation is in English and is 511 pages long, consists of introductory chapters, with introductory chapter 1, and chapters 2 and 3 providing basic facts from stochastic processes, models and numerical methods (Brownian motion, Laplace transform, Black-Scholes model, Crank-Nicholson method, etc.), which are used further in the dissertation, and a long Bibliography.

We will briefly describe the results by chapter:

Chapter 4: A new approach for pricing discounted American options;

One of the most basic contributions of the dissertation is the **Hybrid method for solving the equation with a free boundary (HMSEFB)**, which is described in this chapter.

This method seems to be a novelty in the field: Further in the dissertation the method is applied in various specific situations for the cases of final maturity: for American put -- pp.78-79 ; for American call -- pp. 84-85 ; for Generalized Payoffs -- p. 144 ; for strangles -- p. 181-182 ; for quadratic strangles -- p. 207 ; for cancelable options -- p. 347-348 ;

On the other hand, the method has also been developed for the case of infinite maturity (chapter 8), when the bounds are independent of time and are found in explicit form, again maximizing the investor's financial result.

It can be said that from a financial point of view, the **HMSEFB** method is based on maximizing the financial results and expectations of investors.

Technically speaking, the novelty lies in the numerical finding of the optimal boundary, by dividing it into two tasks: first the boundary is approximated and then the boundary problem is solved numerically, and then the Monte Carlo method (MC) for Brownian motion is applied. A second alternative method to MC is by using finite differences.

Chapter 5: Pricing discounted American capped options.

In this chapter, the solutions based on the **HMSEFB** method are found explicitly (only in some cases).

Chapter 6: On some generalized American style derivatives.

In this chapter, a generalized payoff function is introduced, which has smoothness C^2 (twice smoothly differentiable), by applying the **HMSEFB** method again, and in particular, payoffs of the

type of polynomial futures are considered. Then, an approximation of non-smooth payoffs with smooth ones is made and analogous results are obtained. A sufficient condition is found for the derivative to behave like a call or put option.

Chapter 7: American strangle strategies with arbitrary strikes, (Combined put-call options)

For the options considered in this chapter, a problem similar to that in Chapter 6 appears, but with two optimal boundaries. Another generalization in standard strangle options is that the call strike may be below the put strike. To solve the calculation problem, the **HMSEFB** method is modified for problems with two free boundaries.

Chapter 8: Quadratic American strangles in the light of two-sided optimal stopping problems.

The specificity of the options in this chapter is that they have a very special payoff, which is a quadratic polynomial, and the options are considered as a special case of more general payoffs, for which a sufficient condition is found to have the form of strangle options.

Chapter 9: Cancellable call options under perpetual assumptions.

The options considered in this chapter are without maturity restrictions, and the important thing with them is that the issuer has the right to terminate them after paying a penalty. Equations for the optimal boundaries of both the issuer and the owner are found. The formulas are explicit, since the equations are of polynomial form and it is proved that they have a unique solution.

Chapter 10: Cancellable put options without maturities.

The result here is analogous to the previous chapter, except for the case of put options. The penalty is a constant in physical time.

Chapter 11: Perpetual cancellable options with a proportional penalty.

In this chapter, unlike the previous chapters, the penalty is proportional to the usual payoff. Here too, the optimal bounds are found, as well as the prices to which they lead.

Chapter 12: Perpetual cancellable options with convertible features,

Here we have payoffs that are composed of three parts: a proportion of the payoff, the number of shares of the underlying asset, and a fixed amount. Here too, polynomial equations for the optimal bounds and prices are found in explicit form.

(I will note that there are new results for these options obtained by the doctoral candidate after writing this work. It is about the duality between call and put options of this type.)

Chapter 13: Pricing cancellable American put options on the finite time horizon.

This chapter deals with put cancellable options, with finite maturity. Since there is no obvious solution for the optimal boundaries here, the **HMSEFB** method is used. While for strings a double maximum is sought, here a min-max is sought.

Chapter 14: This chapter presents a large amount of basic Matlab codes that implement the results presented in the previous chapters. The graphs and figures and tables that are given in the text were made with them.

Chapter 15: Concluding remarks and further works

A range of future problems is outlined here.

Chapter 16: A brief summary of the main results of the dissertation has been provided.

3. Publications related to the dissertation

The results of the dissertation work have been published in 13 articles, and their scientometrics are as follows:

- 7 publications are in journals with Q_1 in WoS ,
- 1 publication is in Q_2 WoS ,
- 1 publication is in a journal with Q_3 WoS ,
- 3 publications are in journals with Q_4 WoS ,
- 1 publication is in Serdica Mathematical Journal

The number of citations of the publications on the dissertation noted by the dissertationist is 35. On the other hand, all citations in WoS and/or in Scopus of the publications on the dissertation are 29.

At this point, we should note the **productivity of the dissertationist**: he has a total of 43 publications, and also a considerable number of citations in the field of mathematical sciences - 123 (84 in WoS and/or in Scopus).

The submitted publications and citations meet and even exceed the minimum national requirements described in the Law ZRASRB, the regulations for its implementation and the specific regulations of the Bulgarian Academy of Sciences and IMI-BAS.

No plagiarism has been found in the submitted publications.

4. Notes, comments and recommendations.

In relation to the dissertation, I have the following comments and recommendations:

- The dissertation is excellently structured. The presentation is clear and understandable. The majority of chapters are indicated on the basis of which article they were made, namely from chapter 4 to chapter 13, while chapter 14 (from page 359) contains Matlab code - with over 200 independent programs.
- Numerous graphs/figures are attached to the text, which illustrate the obtained results.
- A recommendation: it is nice to have a numbered bibliography, which is easy to get with Latex today.
- Taking into account the comprehensiveness and novelty of the results obtained in the dissertation, it can be recommended to the dissertationist to publish it as a monograph - completely suitable for a reputable publishing house, such as Wiley, in their series "The Wiley Finance Series". Of course, this will require a lot of additional work. The problem is that in this form the dissertation is not oriented towards the end user from the financial sphere, but is rather a mathematical study, although with great potential for implementation.
- The above recommendation can also be extended to the future projects of the dissertationist: to emphasize in the future publications in journals that are recognizable to people from practical mathematical finance (the financial industry); the problem here is, mainly, that they do not have, and do not at all strive for, the artificial scientometrics that has flourished recently. Publishing in such journals is very difficult, but it is the best path to real recognition of achievements in the field, since these are the journals read and used by practitioners in the banking/finance sector.

The remarks made do not change the overall positive assessment of the dissertation work.

5. Dissertation Abstract and author reference.

Both documents are properly formatted according to the requirements and correctly reflect the results and contributions of the dissertation work. There are minor spelling errors; the translation of terminology from English is often ambiguous – for example, “derivatives” has two possible Bulgarian versions?

6. Conclusion.

The presented dissertation and the documents related to it show that Assoc. Prof. Dr. Tsvetelin Zaevski is a well-established specialist and has made significant personal contributions in the field of mathematical finance.

The analysis of the dissertation shows that it contains results that have been published in professional journals and prove the skills and professionalism of the dissertation candidate in the

field of Mathematical Finance, including in various areas of mathematics, such as Stochastic Processes, Partial Differential Equations, and Numerical Analysis.

According to the applied documents the candidate Assoc. Prof. Tsvetelin Zaevski satisfies all the minimal requirements set by the Act for the Development of the Academic Staff in the Republic of Bulgaria, the Regulations for its implementation and the Regulations of the Institute of Mathematics and Informatics, Bulgarian Academy of Sciences, stipulating the specific additional requirements for the award of scientific titles and academic positions. My professional opinion concerning his work is fully confirmed by the applied documents which clearly demonstrate that Assoc. Prof. Tsvetelin Zaevski is a very good specialist in the research area of his dissertation.

The above gives me reason to give a positive assessment of the dissertation work

"METHOD FOR EVALUATION OF AMERICAN TYPE DERIVATIVES"

and to confidently recommend to the Scientific Jury of IMI-BAS to award Assoc. Prof. Dr. Tsvetelin Zaevski the scientific degree "Doctor of Sciences" in the field of higher education "4. Natural Sciences, Mathematics and Informatics", professional direction "4.5 Mathematics".

Sofia, 23.2.2025 Referee:

(Prof. Dr.Sci Ognyan Kounchev)