OPINION

on the dissertation for the degree of Doctor of Science,
field of higher education: 4. Natural Sciences, Mathematics and Informatics,
professional field: 4.5. Mathematics,
Author: Tsvetelin Stefanov Zaevski,
Title: "An approach for pricing American-style derivatives"

This opinion is prepared by **prof. PhD Pavlina Kalcheva Jordanova (Yordanova),** from the Department of Economics and mathematical modeling, Faculty of Mathematics and Computer Science, Konstantin Preslavsky University of Shumen, area and professional field 4.5. Mathematics (Theory of probability and mathematical statistics).

By Order No. 457/ 03.12.2024 of the Director of the Institute of Mathematics and Informatics (IMI) at the Bulgarian Academy of Sciences (BAS), Corresponding Member, D.Sc. Petar Boyvalenkov, I was appointed as a member of the scientific jury for the procedure for the defense of Assoc. Prof. Dr. Tsvetelin Stefanov Zaevski's dissertation. As a member of the scientific jury, I received electronically all the documents required by the Law for Development of the Academic Staff in the Republic of Bulgaria (LDASRB), the Regulations for implementation of LDASRB and the Rules on the Terms and Conditions for Acquisition of Academic Degrees and Holding Academic Positions (RTCAADHAP) at BAS and at IMI of BAS.

1.Analysis of the publications and the main scientific and applied scientific contributions in the dissertation

The dissertation is devoted to research on modern methods for the pricing of American options and derivatives. The research is based on the Black and Scholes model, i.e. it is assumed that the dynamics of the price of the underlying asset is described by a Log-normal random process, also called (and more precisely) Geometric Brownian Motion. Here we should note that the second concept is more accurate, since there can be many Log-normal random processes, but the term "Geometric Brownian Motion" fixes the corresponding dependence structure of the process. This condition is too restrictive, because it requires the existence of all moments. The author focuses on working with discounted derivatives, valuation of option prices when limiting the price of the underlying asset, strangle strategies and valuation of cancelable options. Some of the problems considered in his work lead to solving a one-sided problem for hitting a level, or a two-sided problem for first exit from a strip.

The dissertation is written in English. The total number of pages is 497. The first chapter is an introduction. Twelve theoretical chapters follow. The fourteenth chapter contains MATLAB codes.

The work ends with a conclusion, a description of the contributions, and a bibliography. The literature consists of 312 titles. Assoc. Prof. Dr. Tsv. Zaevski is the author of 19 of them. In the procedure, the author applies with 13 titles. They were published in the period 2020 to 2024 inclusive. He is the sole author of all of these works. As the author himself said in his summary, articles with numbers 1, 9, 10 and 11 had been used in a previous procedure for an associate professor, therefore, I will exclude them from the analysis, i.e. only 9 titles can be acknowledged in the current procedure. Of these: 3 are in Q1 of WoS, 2 are in Q2 of WoS, 2 are in Q4 of WoS, 1 is in Scopus and 1 is in the Bulgarian journal *Serdica* and at the time of writing this review has not been indexed in Scopus yet. The total impact factor of these works as of the year of publication is 19.658. Due to the increasingly strong penetration of financial markets in our country, the topic is contemporary.

So, I accept that the dissertation of Assoc. Prof. Dr. Tsv. Zaevski is based **on 9 publications that were not used for his habilitation.** All articles are in English and are published. One of them, namely publication [3], at the time of writing this Opinion has not been indexed in Scopus yet.

As I already mentioned, the first chapter is an introduction. It contains mainly a history of the problem, and consists of 4 pages.

In Chapter 2, the present value of the final payment of a call option is considered. First, a financial derivative with a bounded conservation domain on both sides is considered. Theorems from the above-mentioned articles with numbers 1, 9, 10 and 11 are included at the beginning of this chapter. The new results describe the limits of the Laplace transforms of the Brownian motion at the moment of its first hitting partially linear boundaries. In particular, the moment of exercise is the moment of the first leaving of a strip. The case when one of the boundaries disappears after some time is also considered. These results are published in article [2] from the list of publications related to the dissertation.

Chapter 3 is devoted to the problem of maximizing the profit of the derivative holder. The principle of risk-neutral pricing of the derivative price is used. The Black-Scholes equation is modified by changing the payment function. This part begins with a result from [10], which, as I explained, I will exclude from the author's contributions since it is included in his habilitation procedure. Next, an approach is proposed to solve the corresponding problem with a free boundary (i.e., in which both the solution and the domain in which it is implemented must be found) with improved computational time. From an applied point of view, it answers the question of whether it is optimal to exercise the option immediately or not. The solution to this problem is divided into two parts. In order to reach the PDE on a known set, first the optimal boundary is approximated. Then this PDE is solved by the expectation. The chapter ends with a comparison of the results of several numerical methods. These results are published in article [3] from the list of publications related to the dissertation.

In Chapter 4, the problem of approximating the optimal boundary for an American put option with an exponent of partially linear functions is first solved. Then, an algorithm is sought to find the largest optimal value that would make the non-immediate exercise optimal. To achieve higher accuracy, it is proposed to apply the algorithm several times with different time intervals. Thus, the boundary is approximated on a denser grid. Finally, a Monte Carlo method is applied to find the expectation. This algorithm allows the extraction of the prices of the usual put option for different values of the underlying asset at once. It is one of the main results in the dissertation. This chapter consists of 31 pages and presents the results of paper [4] from the list of publications.

Chapter 5 deals with restricted options, which limit the spot price at which the holder can exercise the option to a fixed level. This chapter consists of 30 pages. Accordingly, the payment functions become more complicated, including this new fixed level. The main result in this chapter is the formula for the price of a put option at finite maturity. The corresponding infinitesimal generators are used to prove it. The numerical method in the previous chapter allows for a quick evaluation of the price of the American put option, which leads to a quick evaluation of the restricted options in this chapter. These results are published in article [5].

In Chapter 6, call and put derivatives are considered. Here the payoff function becomes more complicated. This time, the state space can be divided into two connected parts, which represent an optimal region and a conservation region. Conditions are found for the differential operator that lead to such instruments. Then, call derivatives are considered for an infinite time interval. It is proved that the financial result of the first-to-the-bound strategy has no more than one local maximum in this interval. Conditions are found under which an optimal boundary exists, or when it is zero, i.e. when all points are optimal. The third option is when the optimal boundary is infinity. In the latter case, early exercise is never optimal. Analogous results are obtained for put derivatives. Cases are indicated when the derivative is a combined put-call and immediate exercise is always optimal. Or, when immediate exercise is never optimal. Various results from this chapter are published in [6]. This chapter is also one of the main ones in the evaluated work and consists of 30 pages.

Chapter 7 discusses the so-called American strangle strategies. The author gets rid of the condition that the call strike is above the put strike. He allows the weights of the call and put rights to be different, and reduces the task of determining the optimal moment when to exercise the derivative to that of the first exit from a strip, which represents the area for preserving the derivative. The payment function is now a set of those of the put and call characteristics, depending on the client's preferences at the respective moment. For this reason, there are also two optimal limits. It turns out that here immediate exercise is never optimal as a call if the additional discount factor is zero. Assuming that it is greater than zero, the optimal limits at infinite maturity are determined and it is proven that they are unique. Finally, in order to evaluate the strangle combination, the corresponding

PDE can be solved via appropriate numerical method. These results are published in [7]. This chapter is one of the main ones and consists of 30 pages.

Chapter 8 considers some American-type financial instruments that lead to a strip exit. Special attention is paid to strangle-like derivatives but with a quadratic payoff function. These financial instruments are considered in the light of much more general payoff structures that guarantee that the optimal strategy is a strip exit. A necessary condition is found for the differential operator, which is related to the infinitesimal applied to the payoff function, to lead to such a problem. First, the optimal conservation set is described. The problem is reduced to the corresponding one for European derivatives. In the case where the maturity is finite, the approach from the previous chapter is used. In this case, the expectations are obtained by using the results of Chapter 2. In the other cases, Chapter 6 is used. This part of the dissertation is published in [8].

Chapters 9, 10 and 11 are devoted to a strategy that maximizes the utility of the holder and the writer in cancellable American options, also known as game or Israeli options. These chapters present results from articles [9], [10], [11], which were used for the author's habilitation, so I will exclude them from my assessment. They consider the problem when the writer has the right to cancel the contract prematurely by paying some penalty.

Chapter 12 summarizes the results of the previous three chapters. The author studies the amount that the writer owes if the holder exercises the option at a given time, and the amount that the writer must pay if he cancels the contract. Unlike strangles, where one searches a maximum of a two-dimensional functional on the space of random Markov moments, here the author searches a saddle point. The space of states is divided into three parts, and the set optimal for the holder, the set optimal for the writer, and the set for retention are described. The author works under the condition that the issuer does not cancel the contract immediately, even when it is optimal for him, assuming that some future strategy provides the same result. This part of the dissertation is published in [12].

Chapter 13 is devoted to the valuation of cancellable American put options over a finite time interval. This part of the dissertation is published in [13].

The last Chapter 14 contains codes that are intended for personal use, for implementing and testing the Monte Carlo methods used in the dissertation. They were prepared in MATLAB and, as the author himself has indicated, they do not pretend to be exceptional, which is why I exclude them from the author's contributions.

Finally, conclusions are drawn and the author's main contributions are described.

2. Citations

Assoc. Prof. Dr. Tsvetelin Stefanov Zaevski has presented a list of 35 citations to these articles, 29 of which are published in 8 articles in Scopus and/or Web of Science.

3. Summary and author's references.

The Summary is prepared according to the requirements, and correctly reflects the content of the considered work.

4. Critical remarks and recommendations

I have no significant critical remarks regarding the level of the scientific results of Assoc. Prof. Dr. Tsvetelin Zaevski, with which he participated in this procedure.

In my opinion, it would be appropriate to exclude articles [1], [9], [10], [11] from the competition altogether because they were used in a previous procedure for an associate professor. These works could simply be cited without participating in this procedure.

Assoc. Prof. Dr. Tsvetelin Zaevski should think more about the reader in his published works. For example, for improving readability, it would be better when using strictly specific terms and notations in his articles if the author defined them at the place of their first usage or in additional material. See, for example, p. 13.

Quite a great number of the references are simply mentioned in the text, without describing their contributions or shortcomings, and without being used in the work. This is, for example, the case with many of the articles cited on p. 4 of the dissertation, where about 53 titles are cited. The situation is similar on pages 2, 3, 5, 6 and 8. That is the reason why the list of references is too long.

5. Conclusion

After I got acquainted with the materials and scientific papers presented in the procedure, and based on the analysis of their significance, and the scientific, and applied contributions contained therein, I confirm that the presented dissertation meets the set of criteria and indicators according to the LDASRB, its Regulations, and the Regulations for the Implementation of the LDASRB at the BAS, and IMI. The achieved results allow me to suggest that the scientific degree "Doctor of Sciences" be awarded to Assoc. Prof. Dr. Tsvetelin Zaevski in the field of higher education: 4. Natural Sciences, Mathematics and Informatics, professional field: 4.5. Mathematics. In particular, the candidate meets the minimum national requirements in this professional field, and no plagiarism has been established in the scientific papers presented for the procedure.

I give my positive assessment of the candidacy.

Febr. 20th, 2025 Prepared by: .....

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