S T A T E M E N T

on the contest for the Acquisition of the Academic Position " Associate Professor"
in Professional Direction 4.5 Mathematics
(Homogeneous spaces and geometric invariant theory)
at the Institute of Mathematics and Informatics (IMI)
of Bulgarian Academy of Sciences (BAS)
announced in St. Gaz. 65/2.08.2024 and at the Website of IMI-BAS

The statement is written by Azniv Kirkor Kasparian, Section of Algebra, Faculty of Mathematics and Informatics, Sofia University "St. Kliment Ohridski", Professional direction 4.5 Mathematics, as a member of the scientific juri for the contest, according to Order 347/13.09.2024 of the Director of the Institute of Mathematics and Informatics at Bulgarian Academy of Sciences.

The only applicant for the announced contest is Ph.D. Valdemar Vasilev Tsanov from the International Centre for Mathematical Sciences (ICMS) at IMI-BAS. He participates in the contest with a Habilitation thesis and nine articles. Three of the articles are standalone and six are joined. To the best of my knowledge, all co-authors of the joint publications have equipollent contributions. Valdemar Tsanov has 30 noticed citations, from which 16 are provided for the contest. Besides copies of the diplomas for acquisition of Ms.D. in Mathematics from Sofia University "St. Kliment Ohridski", Ph.D. in Mathematics from Queen's University, Kingston, Canada and Habilitation from Ruhr-Universität Bochum, Germany, the transcripts include evidences for work experience in teaching and research, an author's summary of the scientific contributions, a list of noticed citations and a reference for compliance with the minimal national requirements of the Law on the Development of Academic Staff of Republic Bulgaria, the Rules on its implementation and the Rules on the Terms and Conditions for Acquisition of Academic Degrees and Occupation of Academic Positions at IMI-BAS.

Valdemar Tsanov has been a graduate student and a teaching fellow for 4 years at Queen's University. He held research positions at Ruhr-Universität Bochum for about 7 years, at Georg-August Universität Göttingen for three and a half years, at Jacobs University, Bremen for an year and at ICMS at IMI-BAS for more than two years. Valdemar Tsanov has an extensive teaching experience at Queen's University, Ruhr-Universität Bochum and Georg-August Universität Göttingen. There he taught, both, classical obligatory subjects like Real Analysis, Complex Analysis, Differential Equations, Algebra, Combinatorics, as well as lectures and seminars on advanced topics from Lie algebras, Algebraic Geometry, Representation Theory, Toric Varieties, Geometric Invariant Theory, Harmonic Analysis, etc., reflecting his own research. Valdemar Tsanov has supervised a Ph.D. student at Ruhr-Universität Bochum and a visiting Ph.D. student at Georg-August Universität Göttingen. He has participated in five DFG projects - two at Ruhr-Universität Bochum and one at each of Georg-August Universität Göttingen, Jacobs University Bremen and Constructor University Bremen. Valdemar Tsanov has held a personal two-years DFG-grant at Georg-August Universität Göttingen and is a part of the scientific programme enhancing the research capacity in mathematical sciences (PICOM) at ICMS, IMI-BAS. He has visited Max-Planck Institute of Mathematics in Bonn, the Centre for Theoretical Physics of the Polish Academy of Sciences in Warsaw and Ruhr-Universität Bochum. Valdemar Tsanov has delivered 50 talks and two minicourses at highly prestigious specialized conferences and seminars like Fields Institute and other mathematical forums in Germany, USA, Canada, Italy and Bulgaria. He has contributed to the preparation of the lecture notes of two courses from workshops at Jacobs University Bremen.

Valdemar Tsanov participates in the contest with a Habilitation work and nine articles, from which one with Impact Factor (IF) from the first quartile (Q1), two with IF from Q2 and two more articles with IF from Q3, respectively, Q4. He has also provided 16 citations and participation in two projects of DFG, one of which is personal. In such a way, he exceeds the minimal national requirements for the occupation of the academic position "Associate Professor" and the specific requirements of IMI-BAN.

Let us proceed with content analysis of the articles of Valdemar Tsanov from the competition. One of them reflects partially his Master's Thesis and realizes diffeomorphically $\mathrm{PSL}_2(\mathbb{R})/\Gamma_{p,q}$ for (p,q,∞)-triangular Fuchsian groups $\Gamma_{p,q}$, $\mathrm{GCD}(p,q)=1$ as knot complements in appropriate lens spaces. To this end, the automorphic forms with fractional degree and characters for the lifting $\widetilde{\Gamma_{p,q}}$ of $\Gamma_{p,q}$ to the universal cover $\widetilde{\mathrm{SL}_2(\mathbb{R})}$ of $\mathrm{SL}_2(\mathbb{R})$, are shown to have such two generators ω_a, ω_b that the cuspidal $\widetilde{\Gamma_{p,q}}$-modular form of minimal positive degree is of the form $\omega_\infty = c_a\omega_a^p + c_b\omega_b^q$ for some $c_a, c_b \in \mathbb{C}^*$. The form ω_∞ does not vanish on the universal cover $\widetilde{T'H^2}$ of the complement $T'H^2$ of the zero section in the tangent bundle TH^2 to the upper half-plane $H^2 \subset \mathbb{C}$. If the knot $K_{p,q} = S^3 \cap V_{p,q}$ is cut by the sphere $S^3 \subset \mathbb{C}^2$ and the affine curve $V_{p,q} = \left\{(z_1,z_2)\in\mathbb{C}^2 \,|\, c_a z_1^p + c_b z_2^q = 0\right\}$ then its complement $S^3 \setminus K_{p,q}$ is proved to be diffeomorphic to $\widetilde{\mathrm{SL}_2(\mathbb{R})}/G$ for an appropriate subgroup G of $\widetilde{\Gamma_{p,q}}$ of index $r = pq-p-q$. The identification of $\mathrm{PSL}_2(\mathbb{R})/\Gamma_{p,q}$ with the quotient $UH^2/\Gamma_{p,q}$ of the unit tangent bundle UH^2 of H^2, allows to embed it in $T'H^2/\Gamma_{p,q}$. The cyclic group of order r, generated by $h = \left(e^{\frac{2\pi i p}{r}}, e^{\frac{2\pi i q}{r}}\right)$ acts on $V_{p,q}$ and S^3, so that $\mathcal{K} = K_{p,q}/\langle h\rangle$ is a knot in the lens space $\mathcal{L} = S^3/\langle h\rangle$. The article establishes that $T'H^2/\Gamma_{p,q}$ is biholomorphic to the complement $\left(\mathbb{C}^2/\langle h\rangle\right)\setminus\left(V_{p,q}/\langle h\rangle\right)$ of the singular curve $V_{p,q}/\langle h\rangle$ in the singular complex surface $\mathbb{C}^2/\langle h\rangle$. That allows to conclude that $\mathrm{PSL}_2(\mathbb{R})/\Gamma_{p,q}$ is diffeomorphic to $\mathcal{L}\setminus\mathcal{K}$. The aforementioned article has 8 citations from Web of Science, provided for the competition.

Let $\mathcal{H}$ be a state space of a system of L distinguishable particles, L bosons or L fermions, $X \subset \mathbb{P}(\mathcal{H})$ be the algebraic variety of the coherent states and K be the compact symmetry group of X. In all of the aforementioned cases, X spans $\mathbb{P}(\mathcal{H})$ and the X-rank of $[v]\in\mathbb{P}(\mathcal{H})$ is the minimal number r of points of X, whose sum equals v. The states of X-rank r, which cannot be approximated by states of X-rank $< r$ are called exceptional. An article of Adam Sawicki and Valdemar Tsanov from 2013 establishes that if the $K^{\mathbb{C}}$-action on $\mathbb{P}(\mathcal{H})$ is spherical then there are no exceptional states. This article has 7 citations, provided for the competition. It establishes that for $L\geq 3$ and sufficiently large dimension of the state space of one particle there exist exceptional states, which occur in the second secant variety of X. As a consequence, the existence of exceptional states is a physical obstruction for deciding whether two states belong to one and a same K-orbit by the one-particle-reduced density matrices. A specific example shows that the lack of exceptional states does not require the sphericity of the action.

An article of Alexey Petukhov and Valdemar Tsanov from 2015 classifies all the projective varieties $X\subset\mathbb{P}(V)$, which are transitively acted by a linear automorphism group $G\leq \mathrm{Aut}(X) < \mathrm{SL}(V)$ and have lower semi-continuous rank function rk_X. This work has one citation, provided for the competition. Let V be a finite dimensional vector space over an algebraically closed field F of $\mathrm{char}(F)=0$ and $X\subset\mathbb{P}(V)$ be an algebraic variety, generating $\mathbb{P}(V)$. When the X-rank $\mathrm{rk}_X : \mathbb{P}(V)\to\mathbb{N}$ is lower semi-continuous, X is said to be rs-continuous. If $X = Gx_o$, $x_o\in X$ is acted transitively by $G<\mathrm{Aut}(X)<\mathrm{SL}(V)$ then $V=V(\lambda)$ is an irreducible G-module, determined by its highest weight λ. Moreover, $X = G\left[v^\lambda\right]$ is the G-orbit of the highest weight line $\left[v^\lambda\right]$. That allows to construct all such $X = G\left[v^\lambda\right]$, starting from a semisimple algebraic group G and an irreducible G-module $V(\lambda)$ with highest weight λ. There is a unique closed G-orbit $X(g,V(\lambda)) = G\left[v^\lambda\right]$ in $\mathbb{P}(V(\lambda))$. The article classifies all the irreducible representations $G\to\mathrm{SL}(V(\lambda))$ of $\dim V(\lambda)\geq 2$, for which $X(G,V(\lambda))$ is rs-continuous. It lists all the possible groups G, as well as the highest weights λ of their corresponding finite dimensional irreducible representations $V(\lambda)$. A homogeneous variety $X\subset\mathbb{P}(V)$ is subcominuscule if there is an irreducible Hermitian symmetric space S with semisimple part $K(S)$ of the isotropy subgroup of the isometry group $I(S)$ of S, such that $X = X(K(S)^{\mathbb{C}}, T_{\check{o}}S)$ for the tangent space $T_{\check{o}}S$ of S at the origin. The results of the article imply that a homogeneous projective variety $X\subset\mathbb{P}(V)$ is rs-continuous if and only if $X = (K(S)^{\mathbb{C}}, T_{\check{o}}S)$ is subcominuscule or X is a hyperplane in a subcominuscule variety $\widetilde{X} = (K(\widetilde{S})^{\mathbb{C}}, T_{\check{o}}\widetilde{S})$ in its minimal projective embedding. In the latter case, the rank function of X is the restriction of the rank function of $\widetilde{X}$ and $\sigma_r(X) = \sigma_r(\widetilde{X}) \cap \mathbb{P}(V)$ for the corresponding r-secant varieties.

An article of Tomasz Maciażek and Valdemar Tsanov, published in "Journal of Physics A: Mathematical and Theoretical" from Q1, studies the one-particle reduced density matrices, arising from a pure state. Their spectra are known to constitute the convex spectral polytope, which coincides with the image of the momentum map. The work constructs a lower and an upper bound of the spectral polytope, making use of a local description of the spectral polytope around the vertex, associated with the highest weight. The facets of the upper bound polytope bound sharply the spectral polytope. They provide some generalized Pauli constraints for L fermions in N modes and is of interest in quantum chemistry. The construction of the lower bound of the spectral polytope uses only doubly excited states. Making use of the representation theory of the compact semisimple Lie groups and their complexifications, the article establishes that the nondegeneracy of the Gaussian second fundamental form suffices for the coincidence of the lower bound with the entire spectral polytope. It classifies in four classes the loci of non-entangled states of the quantum systems, for which the lower and the upper bound coincide with the spectral polytope. The article of Tomasz Maciażek and Valdemar Tsanov studies for the first time the spectral polytope of a fermionic Fock space. Except the aforementioned classification result, it provides an example, in which the spectral polytope contains strictly the lower bound.

An article of Henrik Seppänen and Valdemar Tsanov from 2017 studies the action of principal SL_2-subgroups S of a connected, simply connected, semisimple complex Lie group G on a flag variety $X = G/B$, determined by a Borel subgroup $B < G$. It relates the properties of the S-invariant sections of line bundles on X to geometric properties of appropriate GIT (Geometric Invariant Theory) quotients by S. Let H be a Cartan subgroup of G, contained in B and W be the Weyl group of H. The Picard group $\mathrm{Pic}(X)$ of X is isomorphic to the weight lattice Λ of H, as far as any line bundle $\mathcal{L} \to X$ is of the form $\mathcal{L}_\lambda = G \times_B \mathbb{C}_{-\lambda}$ for some $\lambda \in \Lambda$. That allows to identify the set of the effective line bundles on X with the monoid Λ^+ of the dominant weights with respect to B. This is due to the fact that the global sections of an effective line bundle on X constitute an irreducible G-module and any irreducible G-module is realized in such a way. Similarly, any ample line bundle on X is very ample and corresponds to an element of the monoid Λ^{++} of the strictly dominant weights with respect to B. For any $\lambda \in \Lambda^{++}$ the Kirwan strata of the S-unstable locus in X with respect to an ample line bundle $\mathcal{L}_\lambda$ is shown to be the saturations of appropriate Schubert cells. The article concentrates on the generic case, in which any simple factor of G has at least 5 positive roots. Then the S-orbits of dimension < 3 are shown to be the ones through the H-fixed points. Namely, there is a single 1-dimensional orbit, isomorphic to $\mathbb{P}^1(\mathbb{C})$ and $\frac{1}{2}|W| - 1$ orbits of dimension 2, isomorphic to the complement of the diagonal in $\mathbb{P}^1(\mathbb{C}) \times \mathbb{P}^1(\mathbb{C})$. The rest of the orbits turn to be 3-dimensional, with trivial or finite abelian isotropy groups. The article proves that any ample line bundle on X has some power, which admits S-invariant sections. The S-unstable locus of an arbitrary line bundle on X is shown to be of codimension ≥ 2. The GIT-equivalence classes of S-ample line bundles on X are defined by the subdivision of the dominant Weyl chamber $\Lambda^+_{\mathbb{R}}$ by the system of hyperplanes $\mathcal{H}_w = \{\lambda \in \Lambda_{\mathbb{R}} \,|\, \lambda(wh) = 0\}$, where h is an arbitrary fixed non-zero element of the Lie algebra of $S \cap H$ and w varies over the Weyl group W of H. The semistable loci $X_{ss}(\mathcal{C})$ of the GIT-equivalence classes of the connected components $\mathcal{C}$ of $\Lambda^{++}_{\mathbb{R}} \setminus (\cup_{w \in W} \mathcal{H}_w)$ are shown to consist of 3-dimensional orbits. The GIT quotient $Y = X_{ss}(\mathcal{C})//S$ turns to be a Mori dream space, whose Picard group is a lattice of the same rank as Λ. The $\mathbb{Q}$-Picard groups of X and Y are isomorphic to each other. The pseudo-effective cone and the movable cone in $\mathrm{Pic}(Y)$ are isomorphic to each other and to $\Lambda^+_{\mathbb{R}}$. The nef-cone in $\mathrm{Pic}(Y)$ is isomorphic to the closure of $\mathcal{C}$. An arbitrary nef line bundle on Y admits a fixed point free power. For any $\lambda \in \Lambda^+$ there exist $k \in \mathbb{N}$ and a line bundle $\mathcal{L} \to Y$, such that $V_{k\lambda}$ has a non-zero S-invariant element and the global sections

$H^0(Y, \mathcal{L}^j) \simeq (V_{jk\lambda})^S$ of $\mathcal{L}^j$ coincide with the S-invariants of $V_{jk\lambda}$ for all $j \in \mathbb{N}$.

An article of Valdemar Tsanov from 2019 studies the polynomial invariants $\mathbb{C}[V]^G$ of a finite dimensional module V of a connected complex reductive algebraic group G. The ring $\mathbb{C}[V]^G$ is said to admit a generator of degree $d \in \mathbb{N}$ if the homogeneous component $\mathbb{C}[V]^G_d$ of $\mathbb{C}[V]^G$ of degree d is not contained in the ring, generated by the homogeneous components of degree $< d$. Let $H < G$ be a Cartan subgroup with weight lattice Λ and root system $\Delta \subset \Lambda$, $\Lambda(V)$ be the set of the H-weights of V and $M \subset \Lambda(V)$ be a subset with $M \cap (M + \Delta) = \emptyset$, which is linearly dependent over $\mathbb{N}$ and minimal with respect to this property. If $\sum_{\nu \in M} b_\nu \nu = 0$ is the unique expression with relatively prime coefficients $b_\nu \in \mathbb{N}$ then $\mathbb{C}[V]^G$ admits a generator of degree $k \left(\sum_{\nu \in M} b_\nu \right)$ for some $k \in \mathbb{N}$. Let G be semisimple, $V = V(\lambda)$ be an irreducible G-module with highest weight λ with respect to a Borel subgroup $B < G$ and $X \subset \mathbb{P}(V)$ be the unique closed G-orbit in the projectivization $\mathbb{P}(V)$ of V. Denote by J the homogeneous augmentation ideal of $\mathbb{C}[V]^G$, set $I(J)$ for the homogeneous ideal of $\mathbb{C}[V]$, generated by J and put $\sqrt{I(J)}$ for the radical of $I(J)$. The projective variety $\mathbb{P}^{\mathrm{us}} = \mathbb{V}(J) \subset \mathbb{P}(V)$ of J is called the unstable locus and the maximal $r \in \mathbb{N}$, for which the secant variety $\sigma_r(X)$ is contained in $\mathbb{P}^{\mathrm{us}}$ is the rank of instability r_{us}. The article shows that if a non-constant homogeneous invariant $f \in \mathbb{C}[V(\lambda)]^G$ vanishes on a secant variety $\sigma_r(X)$ then $\deg(f) > r$. Moreover, if $\mathbb{C}[V(\lambda)]^G \neq \mathbb{C}$ then $d_1 > r_{us}$ for the minimal positive degree d_1 of a homogeneous invariant polynomial. In the special case of an rs-continuous $X \subset \mathbb{P}(V(\lambda))$, the generators of $\mathbb{C}[V(\lambda)]^G$ correspond to the secant varieties, intersecting $\mathbb{P}(V(\lambda)) \setminus \mathbb{P}^{\mathrm{us}}$.

Let G be a connected, simply connected, semisimple complex algebraic group, B be a Borel subgroup of G, Λ be the character lattice of a maximal torus of B and Λ^+ be the B-dominant Weyl chamber of Λ. The finite dimensional irreducible G-modules $V(\lambda)$ are known to be parameterized by $\lambda \in \Lambda^+$ and associated with line bundles $\mathcal{L}_\lambda = G \times_B \mathbb{C}_{-\lambda}$ on the complete flag variety $X = G/B$, whose global sections are $H^0(X, \mathcal{L}_\lambda) = V_\lambda^*$. Any semisimple complex subgroup $\widehat{G}$ of G acts on $H^0(X, \mathcal{L}_\lambda^k)$ and $X_{\mathrm{us}} := \{x \in X \,|\, s(x) = 0, \forall s \in H^0(X, \mathcal{L}_\lambda^k)^{\widehat{G}}, \forall k \in \mathbb{N}\}$ is called the unstable locus on λ. If $X_{ss}(\lambda) := X \setminus X_{us}(\lambda)$ is the semistable locus and $Y = X_{ss}(\lambda)//\widehat{G}$ is its GIT quotient then there exist $m \in \mathbb{N}$ and a line bundle $L_\lambda \to Y$, such that the $\widehat{G}$-invariants of the section ring of $\mathcal{L}_\lambda^m$ coincide with the section ring of L_λ. An article of Henrik Seppänen and Valdemar Tsanov from 2022 studies the existence and some properties of such Y, for which the $\widehat{G}$-invariants $\mathrm{Cox}(X)^{\widehat{G}} = \mathrm{Cox}(Y)$ of the Cox ring of X coincide with the Cox ring of Y. The $\widehat{G}$-ample cone $C^{\widehat{G}}(X)$ is the convex cone in $\mathrm{Pic}(X)_\mathbb{R}$, generated by the ample line bundles with $\widehat{G}$-invariant sections. For any $k \in \mathbb{N}$ let $C_k^{\widehat{G}}(X)$ be the set of those $\lambda \in \Lambda_\mathbb{R}^{++}$, which belong to the closure of $\{\lambda \in \Lambda_\mathbb{Q}^{++} \,|\, \mathrm{codim}_X X^{\mathrm{us}}(\lambda) \geq k\}$. Then $C_1^{\widehat{G}}(X) = C^{\widehat{G}}(X)$ is the $\widehat{G}$-ample cone and $\mathrm{Mov}^{\widehat{G}}(X) := C_2^{\widehat{G}}(X)$ is called the $\widehat{G}$-movable cone. A movable chamber consists of $\lambda \in \Lambda^+$, for which the $\widehat{G}$-orbits on $X^{\mathrm{ss}}(\lambda) = X \setminus X^{\mathrm{us}}(\lambda)$ are infinitesimally free and $X^{\mathrm{us}}(\lambda)$ is of codimension ≥ 2 in X. The article shows that for all $k \in \mathbb{N}$ the sets $C_k^{\widehat{G}}(X)$ are convex rational polyhedral cones in the open Weyl chamber $\Lambda_\mathbb{R}^{++}$. The cone $C_{k+1}^{\widehat{G}}$ is contained in the interior of $C_k^{\widehat{G}}(X)$ with respect to the relative topology of $\Lambda_\mathbb{R}^{++}$. If $C_3^{\widehat{G}}(X) \neq \emptyset$ is non-empty then there exist $\widehat{G}$-movable chambers. The work describes explicitly the $\widehat{G}$-ample cone $C^{\widehat{G}}(X) \subset \Lambda_\mathbb{R}^+$ by the inequalities $\lambda(w^{-1}\xi) \leq 0$ for any dominant weight ξ, corresponding to a maximal parabolic subgroup $P_\xi < G$ and any w from the Weyl group W, such that $\dim \widehat{G} P_\xi(wB) = \dim \left(\widehat{G}/P_\xi \cap \widehat{G} \right) + \dim P_\xi(wB) = \dim(G/B)$. Similarly, $C_k^{\widehat{G}}(X) \subset \Lambda_\mathbb{R}^+$ is cut by $\lambda(w^{-1}\xi) \leq 0$ for all (ξ, w) with $\dim \widehat{G} P_\xi(wB) = \dim \left(\widehat{G}/P_\xi \cap \widehat{G} \right) + \dim P_\xi(wB) = dim(G/B) - k + 1$. For an arbitrary $\widehat{G}$-movable chamber $\mathcal{C} \subset C^{\widehat{G}}(X)$ let $Y = X^{\mathrm{ss}}(\mathcal{C})//\widehat{G}$ be the corresponding GIT quotient. The article proves that Y is a Mori dream space and there is a canonical isomorphism $\mathrm{Pic}(X)_\mathbb{R} \simeq \mathrm{Pic}(Y)_\mathbb{R}$ of the corresponding $\mathbb{R}$-Picard groups. This isomorphism identifies the $\widehat{G}$-

ample cone $C^{\widehat{G}}(X)$ of X with the effective cone $\overline{\mathrm{Eff}}(Y)$ of Y. The GIT chambers are shown to correspond to Mori chambers. The $\widehat{G}$-invariants $\mathrm{Mov}^{\widehat{G}}(X)$ of the movable cone of X are identified with the movable cone $\mathrm{Mov}(Y)$ of Y. The closure $\overline{\mathcal{C}}$ of the $\widehat{G}$-movable chamber $\mathcal{C}$ is isomorphic to the nef cone $\mathrm{Nef}(Y)$ of Y and the $\widehat{G}$-invariants $\mathrm{Cox}(X)^{\widehat{G}}$ of the Cox ring $\mathrm{Cox}(X)$ of X form a finite extension of the Cox ring $\mathrm{Cox}(Y)$ of Y.

An article of Valdemar Tsanov from 2024 introduces and studies partial convex hulls of co-adjoint orbits of connected semisimple compact Lie groups K. The co-adjoint orbits $K\lambda \subset \mathrm{Lie}K$ are parameterized by the entries λ of a Weyl chamber $\mathfrak{t}_+$ in a maximal abelian subalgebra $\mathfrak{t} \subset \mathrm{Lie}K$. For any $r \in \mathbb{N}$ the r-th partial convex hull $C_r(K\lambda) \subset \mathrm{Lie}K$ of $K\lambda$ is the union of the convex hulls of arbitrary r-tuples of points from $K\lambda$. After showing that the partial convex hull $C_{\dim \mathfrak{t}+1}(K\lambda)$ is convex and coincides with the convex hull $\mathrm{Conv}(K\lambda) \subset \mathrm{Lie}K$ of $K\lambda$, the article defines $r(\lambda)$ to be the minimal $r \in \mathbb{N}$, for which the r-th partial convex hull $C_r(K\lambda) = \mathrm{Conv}(K\lambda)$ coincides with the convex hull of $K\lambda$. Bearing in mind that $0 \in \mathrm{Conv}(K\lambda)$, $\forall \lambda \in \mathfrak{t}_+$, the work defines $r_o(\lambda)$ as the minimal $r \in \mathbb{N}$, for which $0 \in C_r(K\lambda)$. Let Λ be the weight lattice of $T = \exp(\mathfrak{t}) < K$. The irreducible complex representations $V(\lambda)$ of K are parameterized by the dominant weights $\Lambda^+ = \Lambda \cap \mathfrak{t}_+$. Let $d_1(\lambda)$ be the minimal positive degree of an entry of a minimal homogeneous generating set of the ring $\mathbb{C}[V(\lambda)]^K$ of the K-invariant polynomials in $V(\lambda)$. For any $\lambda \in \Lambda^+ \setminus \{0\}$ and $q \in \mathbb{Q}_{>0}$ the article shows that $d_1(q\lambda) \geq r_o(\lambda)$. Let $\mathcal{CLR}_r(K)$ be the r-th Littlewood-Richardson's cone and $i_r : \mathrm{Lie}K \to (\mathrm{Lie}K)^{\oplus r}$ be the diagonal embedding. Then the sets $\mathfrak{A}_r = \{\lambda \in \mathfrak{t}_+ \,|\, r_o(\lambda) \leq r\}$, $\mathcal{C}_r = \{\lambda \in \mathfrak{t}_+ \,|\, r(\lambda) \leq r\}$ are proved to be rational convex polyhedral cones in $\mathfrak{t}_+$ and $i_r(\mathfrak{A}_r) = \mathcal{CLR}_r(K) \cap i_r(\mathfrak{t}_+)$ For any $\xi \in \mathfrak{t}_+$ let K'_ξ be the derived group of the centralizer of ξ. Then $C_r(K\lambda) = \mathrm{Conv}(K\lambda)$ if and only if for any $\xi \in \mathfrak{t}_+$ the orthogonal projection of λ on $\mathrm{Lie}(K'_\xi)$ belongs to $\mathcal{CLR}_r(K'_\xi)$.

An article of Ivan Penkov and Valdemar Tsanov from 2024 constructs a tensor category T_t, generated by two objects X, Y with filtrations of length $t+2$ and a pairing $X \otimes Y \to \mathbf{1}$ to the monoidal unit, which is universal in the following sense: for any objects X', Y' of a tensor category T' with filtrations of length $t'+2$ for some $t' \leq t$ and a pairing $X' \otimes Y' \to \mathbf{1}'$, there is a left exact monoidal functor $F : T_t \to T'$ with $F(X) = X'$, $F(Y) = Y'$, which is compatible with the corresponding filtrations. The construction of T_t starts with a tensor category $\mathbb{T}_t$ over the Mackey Lie algebra $\mathfrak{gl}^M = \mathfrak{gl}^M(V, V_*)$ of a diagonalizable pairing $V \otimes V_* \to K$ of a vector space V of dimension $\aleph_t$ over an algebraically closed field K of $\mathrm{char}(K) = 0$ and the span V_* of a dual basis of V in $V^* = \mathrm{Hom}(V, K)$. The algebra $\mathfrak{gl}^M$ consists of the linear operators $\varphi : V \to V$ with $\varphi^*(V_*) \subset V_*$. Making use of the filtrations of the dual spaces V, V_*, the article computes the socle and describes all ideals of $\mathfrak{gl}^M$. The category $\mathbb{T}_t$ is defined as the full tensor subcategory of the category of $\mathfrak{gl}^M$-modules, generated by V^* and $(V_*)^*$, which is closed under arbitrary direct sums. After classifying the simple objects of $\mathbb{T}_t$ and parameterizing them by pairs $\lambda_\bullet$, $\mu_\bullet$ of sequences of Young diagrams of length $t+2$, the article describes the indecomposable injective objects of $\mathbb{T}_t$ and computes explicitly the layers of their socle filtrations. The simple objects of $\mathbb{T}_t$ are shown to have infinite injective length and the injective hull I of the trivial 1-dimensional $\mathfrak{gl}^M$-module K turns to be a commutative associative algebra. After providing explicit injective resolutions of the simple objects, the work obtains explicit formulae for all Exts between simple objects of $\mathbb{T}_t$. By definition, the objects of T_t are those objects of $\mathbb{T}_t$, which are free I-modules. The morphisms of T_t are those morphisms of $\mathfrak{gl}^M$-modules, which are morphisms of I-modules, as well. The simple objects of T_t are the simple objects of $\mathbb{T}_t$, tenosred by I, and they have finite injective length in T_t. As an object of T_t, the module I is, both, simple and injective. All Exts between simple objects of T_t are found explicitly, after constructing explicit canonical injective resolutions of the simples.

The habilitation work of Valdemar Tsanov studies the polynomial invariants of reductive complex linear algebraic groups by the geometry of the corresponding orbits. It is defended at Ruhr-Universität Bochum and published by Lap-Lambert Academic Publishing. After an introduction and a preliminary chapter, the third chapter reflects the results of the article of Alexey Petukhov and Valdemar Tsanov from 2015, as well as the article of Valdemar Tsanov from 2019. Besides some extra examples, this chapter shows that in the set up of Valdemar Tsanov's article from 2019, if a set $M \subseteq W\lambda \subseteq \Lambda(V(\lambda))$ of extremal weights of an irreducible G-module $V(\lambda)$ is linearly dependent over $\mathbb{N}$ then there exists such $k \in \mathbb{N}$ that any minimal set of generators of $\mathbb{C}[V(k\lambda)]^G$ has an element of degree $\sum_{\nu \in M} b_\nu$, where $\sum_{\nu \in M} b_\nu \nu = 0$ is the unique non-trivial linear combination of $\nu \in M$ with relatively prime coefficients $b_\nu \in \mathbb{N}$. The fourth chapter of the habilitation work is devoted to the subgroup actions on flag varieties. It reflects the results of the article of Henrik Seppänen and Valdemar Tsanov from 2017, relating the subgroup invariants with appropriate GIT quotients.

After getting acquainted with the materials and the scientific works, presented for the competition, and based upon the aforementioned analysis of their scientific significance and applicability, I confirm that the scientific contributions comply with the Law on Development of the Academic Staff of Republic Bulgaria, the Rules on its Implementation and the Rules on the Terms and Conditions for Acquisition of Academic Degrees and Occupation of Academic Positions at the Institute of Mathematics and Informatics of Bulgarian Academy of Sciences, for occupation by the applicant of the academic position " Associate Professor" in the scientific field and the professional direction of the contest. In particular, the applicant satisfies the minimal national requirements in the professional direction and no plagiarism was found in the presented scientific works. That is why, **I evaluate positively the application of Valdemar Vasilev Tsanov.**

Based upon the aforementioned, **I strongly recommend** the Scientific Juri to propose the appropriate election authority of the Institute of Mathematics and Informatics at Bulgarian Academy of Sciences to elect **Valdemar Vasilev Tsanov as an " Associate Professor" in Professional Direction 4.5 Mathematics (Homogeneous spaces and geometric invariant theory) at the Institute of Mathematics and Informatics of Bulgarian Academy of Sciences.**

November 12, 2024

Statement written by:

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