R E P O R T

on the competition for academic position "Associate Professor" in professional field 4.5. Mathematics (Geometry and Topology),

for the needs of the Institute for Mathematics and Informatics, the Bulgarian Academy of Sciences,

announced in SN, issue 25/ 02.08.2024.

This report is prepared by Prof. **Gueo Valdimmirov Grantcharov** from the Florida International University, as a member of the Scientific Jury on this procedure.

Only one applicant has submitted documents for participation in the announced competition: **Researcher PhD Valdemar Vasilev Tsanov**, Institute of Mathematics and Informatics, Bulgarian Academy of Sciences.

I. *GENERAL DESCRIPTION OF THE PRESENTED DOCUMENTS*

1. *Information about the documentation*
The documents presented by the applicant for the competition satisfy the requirements of the Act on Development of the Academic Staff in the Republic of Bulgaria, the Regulations for its application, and the Rules for the conditions and regulations for acquiring scientific degrees and occupying academic positions at the Bulgarian Academy of Sciences. For participation in the competition, the applicant Valdemar Vasilev Tsanov has presented a list of 10 scientific papers, 5 of them being publications in prestigious journals with impact factor, 2 of them in refereed journals without impact factor, 2 in the proceedings conferences, and 1 habilitation from Ruhr University, Germany. A list of the applicant's citations is also presented, as well as all other required documents (official notes, certificates for working experience, references for participation in projects and other relevant evidence), supporting the applicant's achievements.

2. *Information for the applicant*
According to the submitted CV, Valdemar Tsanov is born in 1980 and graduate with MS degree from Sofia University in 2006, then defended his PhD in 2011 in Queens University, Canada, under the supervision of I. Dimitrov. He has held research and teaching positions at Goetingen University and Ruhr University, where he defended habilitation, and since January 2023 is a Researcher at the Institute of Mathematics and Informatics

3. *General characteristic of the scientific work and achievements of the applicant*
The main research interests of Valdemar Tsanov are in the area of geometric representation theory. The presented works are also contributions to 3-dimensional topology and categorical approach to representation theory. He has papers in Q1, Q2, Q3, and Q4 quartiles and a list of citations. His results extend classical theorems in representation theory using the geometry of homogeneous spaces.
4. *Analysis of the scientific and scientific-applied achievements of the applicant contained in the documents and publications presented for the competition.*
4a. The papers [2]-[9] are the main part of the research of the candidate which is in the geometric approach to representation theory. The paper [2] relates spherical representations of some reductive groups with the ranks of tensors on projective algebraic varieties invariant under an action of these groups. The cases which are considered correspond to physical quantum entanglement of distinguishable, bosonic, and fermionic particles. The corresponding representations with a state space $\mathcal{H}$ are
i) $G = GL(\mathcal{H}) \times GL(\mathcal{H}) \ldots \times GL(\mathcal{H})$ on $H_D = \mathcal{H} \otimes \mathcal{H} \ldots \otimes \mathcal{H}$ ii) $G = GL(\mathcal{H})$ on $H = S^d\mathcal{H}$ iii) $G = GL(\mathcal{H})$ on $H = \Lambda^s\mathcal{H}$

The G- varieties for this representations are the Segre embedded $P(\mathcal{H}) \times P(\mathcal{H}) \times \dots P(\mathcal{H})$ in $P(H_D)$, the embedded Veronese $P(\mathcal{H})$ and Plücker-embedded Grassmanian $Gr(d, \mathcal{H})$, accordingly. The main result is that the representation of G is spherical if and only if there are no exceptional states in the corresponding projective space $P(H_D)$ or $P(H)$. Here exceptional states are spaces on which the rank function over the variety is not semi-continuous. Such rank can be interpreted in terms of moment maps and geometry of osculating and secant varieties. In paper [4] are considered the images of the moment map of osculating spaces to the unique complex $K-$-orbit in the projective space of an irreducible representation of K. The spaces for which the moment polytope is covered by the image of a unique osculating space of order two are classified.

The paper [6] and the habilitation [9] are considering the question of the minimal d for which the space spanned by a generating set of the ring of invariant polynomials over an irreducible representation of a connected semi-simple complex Lie group contains polynomials of degree at most d . Such number is called Noether number of the representation, and the main result in [6] provides bounds of this number in terms of the rank over the projective orbit of the highest weight vector in some particular cases. The habilitation [9] also considers the possible divisors of the degrees of the all generating elements of the ring of invariant polynomials. Moreover in [3], [6], and [9] the degrees of minimal generating sets are given for the spaces with semi-continuous rank function. In paper [8] more inequalities for the degrees of the invariant polynomials of an irreducible representation of a compact semi-simple Lie group are proven. They are based on 2 invariants and use of partial convex hulls of coadjoint orbits.

The papers [5] and [7] consider in general terms a generalization of the question of decomposing an irreducible representation of a complex semi-simple Lie group into factors under an action of a subgroup. Paper [5] includes ideas and a partial case of [7] in the case where the subgroup is the main SL_2 subgroup. The standard approach to this problem is through branching. The paper [7] focuses on the study of the actions of semi-simple subgroups of a simply-connected semi-simple Lie group G on a flag manifold G/B for Borel subgroup $B \in G$ and uses the variation of Geometric Invariant Theory. Fixing a character lattice in the maximal torus Λ and B-dominant Weil chamber Λ^+, one defines $\Lambda^{++} \in \Lambda^+$ to be the set of strictly dominant weights which correspond to ample bundles. For $\lambda \in \Lambda^{++}$ the action of a subgroup $\hat{G}$ on the bundle L_λ has stable and unstable locus $X^{ss}_{\hat{G}}(\lambda)$ and $X^{us}_{\hat{G}}(\lambda)$ respectively. The authors of [7] define for each k

$$C_k = \overline{\{\lambda \in \Lambda^{++} : \mathrm{codim}_X X^{us}_{\hat{G}}(\lambda) \geq k\}}$$

which is a polyhedral cone in Λ^{++}, possibly empty.
The main results of [7] assert that for non-empty C_k, C_{k+1} is in the interior C_k, and when C_3 is non-empty, C_2 has non-empty interior. Also the cones C_k are determined by a finite set of inequalities explicitly written in terms of combinatorial data and dimensions of the orbits of $\hat{G}$. Moreover it is shown that for a GIT quotient $Y = X//\hat{G}$ $Pic(X)_R = Pic(Y)_R$.

4b. The paper [1] considers particular discrete subgroups Γ of the universal cover $SL(\tilde{2}, R)$ of SL(2,R). They lead to a biholomorphism of the quotient $T\mathbf{H} - \{0\}/\Gamma$ of the tangent bundle to the Poincare upper half-plane minus the zero section, and the complement of a singular affine curve in $\mathbb{C}^2$ The construction leads a correspondence between the group Γ and the complement of a knot in the unit sphere in $\mathbb{C}^2$. The construction generalizes the classical identification between $PSL(2, \mathbf{R})/PSL(2, \mathbf{Z})$ and the complement of the know $K_{2,3}$ in the 3-sphere.

4c. The paper [10] studies universal abelian tensor category modeled on a tensor category of representations of specific infinite-dimensional Lie algebras called Mackey Lie algebras. The paper is joint with I. Penkov and is a continuation of the previous research by I. Penkov and his collaborators. For a vector space V over a field K, its restricted dual V_*, and a pairing

$p : V \times V_* \to K$ the Mackey Lie algebra $\mathfrak{gl}^M(V, V_*)$ consists of the endomorphisms of V which dual preserves V_*. The category $\mathbf{T}_t$ studied in [10] is generated by two objects equipped with filtrations of order $t+1$ and a pairing. The study uses the tensor category $\mathbb{T}$ generated by the algebraic duals V^* and $(V_*)^*$ and $\mathbf{T}_t$ is defined as the category of its free I-modules, where I is the injective span of the trivial module K in $\mathbb{T}$. A description of the simple objects in $\mathbf{T}_t$ is given, among other results.

5. *Critical remarks and recommendations*
I have no critical remarks on the applicant's scientific work. The review of the documents presented at the competition shows that Valdemar Tsanov has deep understanding and contributions in an active area of current mathematics. I noticed numerous typos in the texts, as well as unexplained notations in his Authors Reference.

6. *Conclusion for the application*
After my careful and critical reading of the documentation and the publications presented for the competition and my analysis of their significance and the scientific and scientific-applied contributions, I confirm that the scientific contributions of Valdemar Vasilev Tsanov meet the requirements of the Act on Development of the Academic Staff in the Republic of Bulgaria, the Regulations for its application, and the Rules for the conditions and regulations for acquiring scientific degrees and occupying academic positions in the Bulgarian Academy of Sciences for occupying the academic position "Associate Professor" in the scientific field and the professional field of the competition. In particular, the applicant meets the minimal national requirements in the professional field and no plagiarism has been established in the scientific papers submitted for the competition. I give my positive evaluation for the application.

II. *GENERAL CONCLUSION*
Based on the above, I recommend the Scientific Jury to propose to elect Valdemar Vasilev Tsanov to occupy the academic position "Associate Professor" in the professional field 4.5 Mathematics (Geometry and Topology).

November 17, 2024

Referee: .....
(Prof. Gueo Grancharov)