R E P O R T

by Prof. Velichka Milousheva

Institute of Mathematics and Informatics, Bulgarian Academy of Sciences

of the Dissertation Thesis

entitled

“Differential Geometry of Timelike Surfaces in the Four-dimensional Minkowski Space”

by

Victoria Gerasimova Bencheva-Petrova

submitted for obtaining the educational and scientific degree “Doctor”

in the Area of Higher Education: 4. Natural Sciences, Mathematics and Informatics,

Professional Field 4.5 Mathematics, Doctoral Program “Geometry and Topology”

I am a member of the Scientific Jury for the defense of this dissertation according to Order № 191 / 02.07.2024 of the Director of the Institute of Mathematics and Informatics at the Bulgarian Academy of Sciences (IMI-BAS). The report is prepared in accordance with the requirements of the Act on Development of the Academic Staff in the Republic of Bulgaria, the Regulations for its implementation, the Rules for the conditions and regulations for acquiring scientific degrees and occupying academic positions in the Bulgarian Academy of Sciences and the relevant regulations of IMI-BAS.

From the required documents and papers submitted by Victoria Bencheva-Petrova, I have made sure that she meets the requirements of the Act on Development of the Academic Staff in the Republic of Bulgaria, the Regulations for its implementation, the Rules for the conditions and regulations for acquiring scientific degrees and occupying academic positions in the Bulgarian Academy of Sciences and IMI-BAS, as well as the specific requirements for obtaining the educational and scientific degree "Doctor", given in the Rules for the conditions and regulations for acquiring scientific degrees and occupying academic positions in the Institute of Mathematics and Informatics at the Bulgarian Academy of Sciences.

1. General characteristics of the dissertation thesis

The dissertation thesis contains **112** pages and consists of an introduction, two chapters of 5 and 2 sections respectively, a bibliography with **67** references and a content. The research is focused on current problems in modern differential geometry, related to the local theory of two-dimensional timelike surfaces in the four-dimensional Minkowski space and the characterization of basic classes of such surfaces in terms of minimum number of functions satisfying a minimum number of partial differential equations.

Essentially, the dissertation is theoretical in nature and meets the generally accepted requirements for obtaining the educational and scientific degree “Doctor”. The bibliography shows that Victoria Bencheva has a good knowledge of the field in which she works.

2. Major scientific and scientific-applied achievements

In the first chapter, the local theory of timelike surfaces in the four-dimensional Minkowski space is developed. The theory for the class of surfaces, admitting parameterization with respect to principal lines, is developed analogously to the theory of spacelike surfaces by using a geometrically determined moving frame field defined by the principal lines. This class of surfaces is characterized by the condition that the Weingarten map is diagonalizable. For the class of timelike surfaces that do not have principal lines (or equivalently, the Weingarten map is non-diagonalizable), a new approach is applied which is based on the isotropic directions of the surface and uses parametrization with respect to isotropic parameters. In both cases, a geometrically determined moving frame field is introduced, the integrability conditions with respect to this frame field are derived, and existence and uniqueness theorems (fundamental theorems) for timelike surfaces are proved. These fundamental theorems are the main scientific achievements in the local theory of timelike surfaces.

In a separate section with several subsections, the class of timelike surfaces with parallel normalized mean curvature vector field is studied. Special isotropic parameters, called canonical, are introduced for this class of surfaces, with the help of which the number of functions and the number of partial differential equations, determining the surface up to a motion, is reduced to three. The fundamental theorems for the timelike surfaces with parallel normalized mean curvature vector field, proved in this section, constitute the most substantial contribution of the dissertation.

The developed local theory of timelike surfaces in the four-dimensional Minkowski space is applied in Chapter 2 by constructing two large classes of surfaces, one of which allows parametrization with respect to principal lines, and the other one does not allow such parametrization and therefore isotropic parameters are used in this case. Timelike general rotational surfaces are considered as a class of surfaces for which the Weingarten map is diagonalizable. Formulas for the main invariants of these surfaces are derived and different classes of general rotational surfaces are described analytically, namely: flat surfaces, surfaces with flat normal connection, minimal surfaces, surfaces with constant mean curvature vector field, and surfaces with parallel normalized mean curvature vector field.

As examples of surfaces for which the Weingarten map is non-diagonalizable, the so-called meridian surfaces of elliptical type have been constructed – these are two-dimensional surfaces which are 1-parameter systems of meridians of a rotational hypersurface with timelike axis. Isotropic parameters are introduced for these surfaces and the main invariants are expressed in terms of the isotropic parametrization. Some main classes of timelike meridian surfaces of elliptical type are described analytically.

The methods and results in this dissertation thesis represent an original contribution to the mathematical science and show that Victoria Bencheva has an in-depth knowledge in the field of the dissertation.

3. Approbation of the results

The results presented in the dissertation have been published in three papers in journals with impact factor:

- Bencheva V., Milousheva, V., *Basic Classes of Timelike General Rotational Surfaces in the Four-dimensional Minkowski Space*, Filomat, Vol. 37, no. 25 (2023), 8505-8519, ISSN: 0354-5180 (Print), ISSN: 2406-0933 (Online), **IF: 0.8, (Q2)**.

- Bencheva V., Milousheva, V., *Timelike Surfaces with Parallel Normalized Mean Curvature Vector Field*, Turkish Journal of Mathematics (2024), Vol. 48: no. 2, Article 15, ISSN:1300-0098, **IF: 1.0 (Q2)**.

- Bencheva V., Milousheva, V., *Fundamental Theorems for Timelike Surfaces in the Minkowski 4-Space*, C. R. Acad. Bulg. Sci., Vol. 77, no. 2 (2024), 167-178, ISSN: 1310–1331 (Print), 2367–5535 (Online), **IF: 0.3, (Q4)**, **SJR: 0.182 (Q3)**.

4. Critical notes and recommendations

I have no critical notes and recommendations.

5. Qualities of the dissertation summary

The dissertation summary contains **36** pages. It presents the relevance and motivation for work on the selected topic and describes in detail the results included in the dissertation. The main contributions of the dissertation thesis are properly reflected in the summary.

6. Conclusion

The above analysis shows that the submitted dissertation fully meets the requirements of the Act on Development of the Academic Staff in the Republic of Bulgaria, the Regulations for its implementation, the Rules for the conditions and regulations for acquiring scientific degrees and occupying academic positions in the Bulgarian Academy of Sciences and IMI-BAS, as well as the specific requirements for obtaining the educational and scientific degree "Doctor", given in the Rules for the conditions and regulations for acquiring scientific degrees and occupying academic positions in the Institute of Mathematics and Informatics at the Bulgarian Academy of Sciences.

All of the above gives me a good reason to confidently give my **positive evaluation** of the dissertation thesis **“Differential Geometry of Timelike Surfaces in the Four-dimensional Minkowski Space”** and strongly recommend the Scientific Jury to award **Victoria Gerasimova Bencheva-Petrova** the educational and scientific degree “Doctor” in the area of higher education 4. Natural Sciences, Mathematics and Informatics, professional field 4.5 Mathematics, doctoral program “Geometry and Topology”.

.

August 31, 2024 Member of the Scientific Jury:

/Prof. Velichka Milousheva/