Quantum cluster algebras, quantum groups, discriminants and Azumaya loci

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Abstract

We will describe a ring-theoretic construction of quantum cluster algebra structures and will apply it in Lie theory. Its core result is that every symmetric CGL extension, or quantum nilpotent algebra, after rescaling its homogeneous prime elements, equals both the quantum and the upper quantum cluster algebra of an explicit seed; an integral version is obtained too. This rests on a structure theory of quantum groups. The dual canonical forms of all quantum unipotent cells will be proved to posses integral cluster structures, and the Berenstein-Zelevinsky conjecture is proved for all double Bruhat cells, with cluster and upper cluster algebras equal, via a bicrossed product and a quantum twist map. The proofs of further conjectures will be sketched: Andruskiewitsch-Dumas, via a rigidity theorem for completed quantum tori, and Goodearl-Lenagan polynormality, extended to all quantum Schubert cell algebras. A Poisson-geometric method for the computation of discriminants of specializations via Poisson primes will be given as well as a correspondence between maximal symplectic leaves and the Azumaya loci of root of unity quantum cluster algebras.