Secant planes and nonspecial line bundles on curves

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A line bundle $\mathcal{L}$ on a smooth curve X is nonspecial if and only if $\mathcal{L}$ admits a presentation $\mathcal{L} \simeq \mathcal{K}_X - D + E$ for some divisors $D \geq 0,\ E > 0$ on X with $\gcd(D, E) = 0$ and $h^0(X, \mathcal{O}_X(D)) = 1$. Since a nonspecial line bundle can have several presentations, we are interested in a minimal presentation $\mathcal{L} \simeq \mathcal{K}_X - D + E$ (minimal with respect to $\deg E$). If $\mathcal{L} \simeq \mathcal{K}_X - D + E$ is a minimal presentation with $\deg E \geq 3$, then $\mathcal{L}$ is very ample and embeds the curve X with a $(\deg E)-$secant $(\deg E - 2)-$plane (spanned by $\varphi_{\mathcal{L}}(E)$), but with no s-secant $(s-2)-$planes for any $s \leq \deg E - 1$. Accordingly, a major issue in this study is how to determine the minimality of a presentation $\mathcal{L} \simeq \mathcal{K}_X - D + E$.

In this work, we investigate sufficient conditions on D, E for $\mathcal{L} \simeq \mathcal{K}_X - D + E$ to be a minimal presentation. Through this, we can construct a nonspecial very ample line bundle $\mathcal{L} \simeq \mathcal{K}_X - D + E$ on X which embeds X with/ without an n-secant $(n-2)$-plane for a given number n. Using this, we show the sharpness of Green-Lazarsfeld's conjecture on property (N_p).