Symmetric polynomials in three noncommuting variables

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In 1936 Margarete Caroline Wolf published a paper where she proved that the symmetric polynomials in the free associative algebra form a free subalgebra and described the system of free generators. The aim of the talk is to present these results from modern point of view. Their relations with other results in the frames of commutative and noncommutative invariant theory are considered.

If symmetric group of degree $n > 2$ acts as permuting the variables on the free algebra of the same number of variables, then the algebra of symmetric polynomials in noncommuting variables is not finite generating ([2, 3, 5]). It has turned out that the analogue of the theorem of Emmy Noether for the finite generation of $K[X_d]^G$ for finite groups G holds for $K\langle X_d\rangle^G$ in very special case only. Koryukin ([4]) proved that if there is an extra action of symmetric group of degree n, then algebra of invariants of every reductive group (in particular finite) is finite generating.

We present our results in the case of symmetric polynomials in three noncommuting variables.

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