Non-group gradings on simple Lie algebras

Alberto Elduque

Departamento de Matemáticas e Instituto Universitario de Matemáticas y Aplicaciones, Universidad de Zaragoza, Spain

elduque@unizar.es

A set grading on the split simple Lie algebra of type D_{13}, that cannot be realized as a group-grading, is constructed by splitting the set of positive roots into a disjoint union of pairs of orthogonal roots, following a pattern provided by the lines of the projective plane over $GF(3)$. This answers in the negative a question posed in 2013.

Similar non-group gradings are obtained for types D_n with $n \equiv 1 \pmod{12}$, by substituting the lines in the projective plane by blocks of suitable Steiner systems.