Algebras defined by Lyndon words and Artin-Schelter regularity

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Let $X = \{x_1, x_2, \cdots, x_n\}$ be a finite alphabet, and let K be a field. We study classes $\mathfrak{C}(X, W)$ of graded K-algebras $A = K\langle X\rangle/I$, generated by X and with *a fixed set of obstructions* W. Initially we do not impose restrictions on W and investigate the case when the algebras in $\mathfrak{C}(X, W)$ have polynomial growth and finite global dimension d. Next we consider classes $\mathfrak{C}(X, W)$ of algebras whose sets of obstructions W are antichains of Lyndon words. The central question is "*when a class* $\mathfrak{C}(X, W)$ *contains Artin-Schelter regular algebras?*" We show that each class $\mathfrak{C}(X, W)$ defines a Lyndon pair (N, W) which, if N is finite, determines uniquely the Gelfand-Kirillov dimension, $GK\dim A$ and the global dimension, $gl\dim A$, for every $A \in \mathfrak{C}(X, W)$. More precisely, we prove that A has polynomial growth of degree d if and only if its sets of Lyndon atoms N has order d. In this case A has global dimension d and is standard finitely presented, with $d - 1 \leq |W| \leq d(d-1)/2$. We find a combinatorial condition in terms of (N, W), so that the class $\mathfrak{C}(X, W)$ contains the enveloping algebra $U\mathfrak{g}$, of a Lie algebra $\mathfrak{g}$. We introduce *monomial Lie algebras defined by Lyndon words*, and prove results on Gröbner-Shirshov bases of Lie ideals generated by Lyndon-Lie monomials. Finally we classify all two-generated Artin-Schelter regular algebras of global dimension 6 and 7 occurring as enveloping $U = U\mathfrak{g}$ of *standard monomial Lie algebras*. The classification is made in terms of their Lyndon pairs (N, W), each of which determines also the explicit relations of U.

References

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