Functions holomorphic over finite-dimensional commutative associative unital $\mathbb{C}$-algebras

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Let $\mathcal{A}$ be a finite-dimensional commutative associative unital $\mathbb{R}$-algebra and let $U \subseteq \mathcal{A}$ be an open subset. A function $f : U \to \mathcal{A}$ is called $\mathcal{A}$-differentiable at the point $Z_0 \in U$ iff

$$f'(Z_0) := \lim_{\substack{H \to 0 \\ H \in \mathcal{A}^\times}} \frac{f(Z_0 + H) - f(Z_0)}{H}$$

exists. When $\mathcal{A}$ carries a complex structure, $\mathcal{A}$-*holomorphic* functions exhibit a theory very similar to the classical theory of a single complex variable despite being functions of several complex variables. In particular, such functions admit:

- generalized Cauchy-Riemann PDEs with *complex* coeffcients;
- one-variable Cauchy's and Morera's Integral Theorems;
- one-variable Homological Cauchy's Integral Formula;
- analyticity over $\mathcal{A}$.

We then discuss $\mathcal{A}$-meromorphic functions, their singularities, and finally go on to introduce Riemann "Surfaces" over $\mathcal{A}$.