Recursion operators and hierarchies of NLEE equations related to Kac-Moody algebras

V. S. Gerdjikov[a], A.A. Stefanov[b], I. D. Iliev[c]

[a] Institute of Mathematics and Informatics, Bulgarian Academy of Sciences and Institute for Advanced Physical Studies, Sofia, Bulgaria

vgerdjikov@math.bas.bg

[b] Institute of Mathematics and Informatics, Bulgarian Academy of Sciences and Faculty of Mathematics and Informatics, Sofia University, Sofia, Bulgaria

aleksander.a.stefanov@gmail.com

[c] Institute of Mathematics and Informatics, Bulgarian Academy of Sciences, Sofia, Bulgaria

iliya@math.bas.bg

We start with a brief review of our recent results [1, 2, 3] on the nonlinear evolution equations (NLEE) of mKDV type related to Kac–Moody algebras. In [1] we constructed the three nonequivalent gradings in the algebra $D_4 \simeq so(8)$. The first one is the standard one obtained with the Coxeter automorphism $C_1 = S_{\alpha_2} S_{\alpha_1} S_{\alpha_3} S_{\alpha_4}$ using its dihedral realization; here S_{α_j} is the Weyl reflection related to the simple root α_j of D_4. In the second one we use $C_2 = S_{\alpha_1} S_{\alpha_3} R$ where R is the mirror automorphism. The third one is $C_3 = S_{\alpha_2} S_{\alpha_1} T$ where T is the external automorphism of order 3. For each of these gradings we constructed the basis in the corresponding linear subspaces $\mathfrak{g}^{(k)}$, the orbits of the Coxeter automorphisms and the related Lax pairs generating the corresponding mKdV hierarchies. We found compact expressions for each of the hierarchies in terms of the recursion operators. At the end we wrote explicitly the first nontrivial mKdV equations and their Hamiltonians.

In [2] the same approach has been applied to the Kac–Moody algebras $A_5^{(1)}$ and $A_5^{(2)}$. Again we construct explicitly the gradings of these algebras and derive the corresponding systems of mKdV equations. In [3] we analyze the spectral theory of the relevant Lax operators related to these algebras.

In addition we formulate also the well known 2-dimensional Toda field theories which must be considered as negative flows of the corresponding mKdV hierarchies. We will also briefly discuss the construction of the soliton solutions of these equations.

References

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