Hilbert series and invariant theory of symplectic and orthogonal groups

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Let $A = \bigoplus_{i \geq 0} A^i$ be a finitely generated graded algebra over $\mathbb{C}$ such that each homogeneous component is a polynomial $\mathrm{GL}(n, \mathbb{C})$-module. Let G be one of the complex groups $\mathrm{O}(n, \mathbb{C})$, $\mathrm{SO}(n, \mathbb{C})$, and $\mathrm{Sp}(2d, \mathbb{C})$ (the last in the case $n = 2d$). In this talk, we present a method for computing the Hilbert series of the algebra of invariants A^G. Then, we take explicit choices of A and apply our method to compute a lot of examples. The main examples we consider for A are the symmetric algebra $S(W)$ and the exterior algebra $\Lambda(W)$ of a polynomial $\mathrm{GL}(n, \mathbb{C})$-module W and certain relatively free algebras in varieties of associative algebras. In some of the examples, we use the computed Hilbert series to determine a set of generators for the respective algebra of invariants. As a further application, we consider the question of regularity of the algebra $S(W)^{\mathrm{O}(n)}$. For $n = 2$ and $n = 3$ we give a complete list of modules W, so that if $S(W)^{\mathrm{O}(n)}$ is regular then W is in this list. The talk is based on a joint work with Vesselin Drensky.

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