The *commutator-degree* of a polynomial and images of multilinear polynomials

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Let K be a field, $X = \{x_1, x_2, \dots\}$ be an infinite set and $K\langle X\rangle$ be the free associative algebra freely generated by X.

We have a strictly descending chain of T-ideals of $K\langle X\rangle$,

$$K\langle X\rangle \supsetneqq \langle [x_1, x_2]\rangle^T \supsetneqq \langle [x_1, x_2][x_3, x_4]\rangle^T \supsetneqq \langle [x_1, x_2][x_3, x_4][x_5, x_6]\rangle^T \supsetneqq \cdots$$

We say that a polynomial $p \in K\langle X\rangle$ has *commutator-degree* r if $f \in \langle [x_1, x_2][x_3, x_4]\cdots[x_{2r-1}, x_{2r}]\rangle^T$ and $f \notin \langle [x_1, x_2][x_3, x_4]\cdots[x_{2r+1}, x_{2r+2}]\rangle^T$.

In this talk we show how multilinear polynomials of a given commutator-degree r can be characterized in terms of its coefficients.

We apply the above characterization to show that the image of a multilinear polynomial evaluated on the algebra of $n \times n$ upper triangular matrices is a vector space (a solution to the analogous of the Lvov-Kaplansky conjecture for the algebra of $n \times n$ upper triangular matrices).

This is a joint work with Ivan Gonzales Gargate.