Phoenix restricted Lie algebras

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Different versions of Burnside Problem ask what one can say about finitely generated periodic groups under additional assumptions. For associative algebras, Kurosh type problems ask similar questions about properties of finitely generated nil (more generally, algebraic) algebras. Similarly, one considers finitely generated restricted Lie algebras with a nil p-mapping. Now we study an oscillating intermediate growth in the class of nil restricted Lie algebras.

Namely, for any field of positive characteristic, we construct a family of 3-generated restricted Lie algebras of intermediate oscillating growth. We call them *Phoenix algebras*, because of the following. a) For infinitely many periods of time the algebra is "almost dying" by having a *quasi-linear* growth, namely the lower Gelfand-Kirillov dimension is one, more precisely, the growth is of type $n\big(\underbrace{\ln\cdots\ln}_{q \text{ times}} n\big)^{\kappa}$, where $q \in \mathbb{N}$, $\kappa > 0$ are constants. b) On the other hand, for infinitely many n the growth function has a rather fast intermediate behaviour of type $\exp(n/(\ln n)^{\lambda})$, λ being a constant determined by characteristic, for such periods the algebra is "resuscitating". c) Moreover, the growth function is bounded and oscillating between these two types of behaviour. d) These restricted Lie algebras have a nil p-mapping.