Solving equations in the symmetric group

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We investigate the solutions of the general "cubic" equation

$$\alpha_1 \circ x^{r_1} \circ \alpha_2 \circ x^{r_2} \circ \alpha_3 \circ x^{r_3} = 1$$

(with $r_1, r_2, r_3 \in \{1, -1\}$) in the symmetric group S_n. In certain cases this equation can be rewritten as a conjugate equation of the form $\alpha \circ y \circ \alpha^{-1} = y^2$, where $\alpha = \alpha_1^{-1}$ and $y = x \circ \alpha_2$. It turns out that the existence of a non-trivial solution $y \neq 1$ of the above conjugate equation heavily depends on the structure of the cycles in α.

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