Derivations and automorphisms of the endomorphism semiring of an infinite chain

Dimitrinka Vladeva[a], **Ivan Trendafilov**[b]

[a] Institute of Mathematics and Informatics, Bulgarian Academy of Sciences, Sofia, Bulgaria

d_vladeva@abv.bg

[b] Technical University - Sofia, Sofia, Bulgaria

ivan_d_trendafilov@tu-sofia.bg

We introduce the endomorphism semiring of an infinite chain and deals with the increasing endomorphisms which are endomorphisms with a right inverse. Two derivations such that any left ideal of the considered semiring is closed under the derivation δ_ℓ and any right ideal is closed under the derivation δ_r are constructed. We prove that the product of these two derivations is an automorphism and consider all positive and negative integer degrees of all maps. Furthermore we construct new derivations using the positive degrees of δ_ℓ. The main result is that the set of these new derivations is a commutative additively idempotent semiring. Analogous results we propose for the derivations constructed from the positive degrees of δ_r and for the mixed derivations constructed from the positive degrees of δ_ℓ and δ_r. At the last we obtain the similar result for automorphisms constructed from the degrees of $\delta_\ell.\delta_r$.