On existence of PI-exponent of codimension growth

Mikhail V. Zaicev

Faculty of Mathematics and Mechanics, Moscow State University, Moscow, Russia

zaicevmv@mail.ru

Given an algebra A, one can associate the sequence $c_n(A)$ of non-negative integers called *codimension sequence* that measures the number of polynomial identities of A. For many classes of algebras the sequence $\{c_n(A)\}$ is exponentially bounded. For example, if A is a finite dimensional algebra, $\dim A = d$, then $c_n(A) \leq d^{n+1}$. Codimension sequence is also exponentially bounded for any associative PI-algebra, for any infinite dimensional simple Lie algebra of Cartan type, for any Novikov algebra, etc.

At the end of 1980's S. Amitsur conjectured that so-called PI-exponent

$$exp(A) = \lim_{n\to\infty} \sqrt[n]{c_n(A)}$$

exists for any associative PI-algebra A. Amitsur's conjecture was confirmed not only for associative algebras but also for all finite dimensional Lie algebras, for all finite dimensional nonassociative simple algebras and many others. Unlike finite dimensional case, there are counterexamples to Amitsur's conjecture in the class of infinite dimensional nonassociative algebras.

If an algebra A is equipped with involution $* : A \to A$ then the same problem concerning $*$-identities and $*$-codimensions $c_n^*(A)$ can be considered. It is known that for an associative PI-algebra A the corresponding limit

$$exp^*(A) = \lim_{n\to\infty} \sqrt[n]{c_n^*(A)}$$

always exists. Nevertheless, in general nonassociative case we have the following result.

THEOREM. *For any real $\alpha > 1$ there exists an algebra C_α with involution $* : C_\alpha \to C_\alpha$ such that*

$$\liminf_{n\to\infty} \sqrt[n]{c_n^*(A)} = 1, \qquad \text{whereas} \qquad \limsup_{n\to\infty} \sqrt[n]{c_n^*(A)} = \alpha.$$