Enhanced degree of parallelism when solving optimal control problems constrained by evolution equations

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The recent development of the high performance computer platforms shows a clear trend towards heterogeneity and hierarchy. In order to utilize the computational power, particular attention must be paid to finding new algorithms or adjust existing ones so that they better match the computer architecture of the nowadays available high performance computer platforms.

In this work we consider an alternative to classical time-stepping methods based on use of time-harmonic properties and discuss solution approaches that allow efficient utilization of modern HPC resources.

The idea is as follows. Assume that we deal with a problem with a solution that is periodic. To be specific, let the solution function be time-dependent and periodic in time. Then the problem becomes time-harmonic and the solution can be expressed in terms of a Fourier series. The Fourier expansion offers the possibility to construct an approximation of the solution by truncating the infinite series appropriately.

Further, due to the orthogonality of the trigonometric functions in the expansion, for linear problems the computation of the Fourier coefficients separate and one can compute the solution for each period (frequency) separately, fully in parallel. Hence, the solution process is perfectly parallelizable across the different frequencies.

To overcome the restriction on the solution to be periodic, we show an approach to first symmetrically extend a non-periodic to a periodic function and then use the truncated Fourier expansion.

This approach is used to solve optimal control problems, constrained by a (non-periodic) evolutionary equation. We define the algebraic systems. resulting from a suitable discretization and describe the solution method. Due to the large size of the discretized systems one must use preconditioned iterative solution methods. We show some efficient preconditioning techniques, expected also to be well parallelizable, and illustrate the overall performance with some (so far serial) numerical tests. The structure of the proposed preconditioning technique The detailed description is to be found in [1], containing also various relevant publications, in perticular, [2].

References

[1] O. Axelsson, M. Neytcheva, Z.-Z. Liang, Parallel solution methods and preconditioners for evolution equations. Department of Information Technology, Uppsala University, TR 2017-017 August 2017, http://www.it.uu.se/research/publications/reports/2017-017/

[2] U. Langer. M. Wolfmayr, Multiharmonic finite element analysis of a time-periodic parabolic optimal control problem, *J. Numer. Math.*, 21 (2013), 265–300.