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BGSIAM'25

EXTENDED ABSTRACTS

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BGSIAM'25 Extended abstracts

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PREFACE

The Bulgarian Section of SIAM (BGSIAM) was formed in 2007 with the purpose to promote and support the application of mathematics to science, engineering and technology in Republic of Bulgaria. The goals of BGSIAM follow the general goals of SIAM:

- To advance the application of mathematics and computational science to engineering, industry, science, and society;
- To promote research that will lead to effective new mathematical and computational methods and techniques for science, engineering, industry, and society;
- To provide media for the exchange of information and ideas among mathematicians, engineers, and scientists.

During the BGSIAM'25 conference a wide range of problems concerning recent achievements in the field of industrial and applied mathematics will be presented and discussed. The meeting provides a forum for exchange of ideas between scientists, who develop and study mathematical methods and algorithms, and researchers, who apply them for solving real life problems.

The strongest research groups in Bulgaria in the field of industrial and applied mathematics, advanced computing, mathematical modelling and applications will be presented at the meeting according to the accepted extended abstracts. Many of the participants are young scientists and PhD students.

LIST OF INVITED SPEAKERS:

- **Prof. Feodor M. Borodich**, College of Aerospace Engineering, Chongqing University, China, *"Synthesis of Engineering Surfaces and Models of Friction"*
- **Dr. Chwastyk Mateusz**, Institute of Physics of the Polish Academy of Sciences, Poland, *"Numerical Techniques for Studying Intrinsically Disordered Regions and Phase Behavior in Proteins"*
- **Dr. Genko Vasilev**, Faculty of Physics, Sofia University "St. Kliment Ohridski" & Advanced Data Science and Process Automation Department, United Bulgarian Bank, Bulgaria, *"Time Dependent Heston Model"*

The present volume contains extended abstracts of the presentations (Part A) and list of participants (Part B).

Assoc. Prof. Elena Lilkova
Chair of BGSIAM Section

Prof. Maria Datcheva
Vice-Chair of BGSIAM Section

Assoc. Prof. Milen Borisov
Secretary of BGSIAM Section

Sofia, December 2025

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Part A
Extended abstracts[1]

[1] Arranged alphabetically according to the family name of the first author.

Dynamic Port Knocking – A Cryptographic Hash-Based Approach to Time-Dependent Access Control

I. Atanasov, S. Fidanova

Port Knocking is a method for allowing access to ports that are blocked by default by the firewall. The purpose of this method is to implement an additional level of port protection by providing controlled access to services such as SSH, Virtual Private Network (VPN), etc. The port knocking method provides additional protection, especially for Linux servers. Access is granted after performing a precisely defined sequence of knocks (by sending TCP or UDP packets) to specific port numbers ranging from 1 to 65535. The port ”knock” itself is like a secret handshake and can consist of any number of TCP, UDP, ICMP, and other lower-level protocols such as ICMP to numbered ports on the target machine. The complexity of the knock can be anything from a simple ordered list (e.g. TCP port 7020, TCP port 4899, UDP port 3101) to a complex encrypted hash, time-dependent, based on source IP and other factors. TCP and UDP are most used, but it is possible to implement the method both through protocols lower than the transport layer and higher - for example, L7 (Layer 7 - Application Layer). This paper presents an enhanced implementation of port knocking security by using a dynamic port generation algorithm based on cryptographic hash function and time parameters. Traditional port knocking methods are quite effective for basic access control but suffer from a number of limitations due to their inherently static nature and possible vulnerability to replay attacks. Our approach introduces a new combination of static and dynamically generated ports, where the dynamic ones are computed using SHA-256 cryptographic hash function combined with temporal parameters including day, month, year, hour, and a secret salt value. This implementation guarantees that attackers cannot derive input parameters from observed port numbers, providing cryptographic level security without the overhead of key exchange protocols.

Keywords Network security, Port sequence generation, Time-based authentication, Prime number algorithm, SHA-256, Cryptographic hashing, Port randomization, Cybersecurity.

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Synthesis of Engineering Surfaces and Models of Friction

F.M. Borodich, Z. Gao, A. Pepelyshev, X. Jin

Tribology is the science devoted to studies of interactions between rubbing surfaces. It includes contact mechanics and research of friction, wear and lubrication. It is well known that topography of surfaces may have a great influence on the resulting stress-strain fields, while all engineering surfaces possesses some roughness, regardless of their preparation method. The surface roughness and asperity deformations have very significant influence on friction,

wear, energy dissipation during relative sliding of surfaces, and other processes of tribology. Questions related to modelling of rough surfaces have been intensively studied. In spite of many attempts to involve surface topography into problems of tribology, up to now there is no clear understanding how roughness affects contact and dry friction phenomena [1, 2].

This study presents a brief review of currently used approaches to surface roughness characterisation [1, 2]; a discussion of novel algorithms for synthesis of rough surfaces at nano/micro-scales [2, 3], and a further development of the Borodich-Savencu (BS) model of friction [4]. The synthesis algorithms are based on introduction and development of two new concepts, namely the representative elementary pattern of roughness (REPR) and the statistically representative pattern of surface roughness (SRPSR). From the statistical point of view, the REPR is the smallest interval (or area) over which a measurement can be made that represents statistically the whole surface. However, synthesis of surfaces by the direct use of the REPR may cause some artificial singularities. To avoid this drawback and to incorporate the synthetic surface in a numerical scheme of the contact solver, one needs to extend the REPR to a non-singular SRPSR that satisfies additional conditions of the scheme used. The representativeness may be justified by the use of the Kolmogorov-Smirnov statistic. Extraction of REPRs of surfaces and constructions of appropriate SRPSRs are demonstrated on experimental data obtained by stylus and Atomic-Force Microscopy at micro and atomic/nano scales respectively. It is shown that some statistical techniques of time series analysis, e.g. the moving window technique, may be effectively employed for extraction of the REPR from the experimental data.

The BS model combines the multiscale hierarchical representation of rough surface of a slider elastic-plastic metallic coating, the statistical synthesis of the rough base surface, and numerical simulations. The extended versions of the CB profile models [5] are used to model the surface of the slider. The simulations focus on the energy dissipation mechanisms during the sliding of elastic-plastic solids. It is assumed that atomic- and nanoscale asperities exhibit rigid-elastic behaviour described by the Borodich-Savencu approximation of the interface potentials and no plastic deformations at these scales due to the Polonsky-Keer effect. The model explains the cross-scale coupling of different energy dissipation mechanisms. It is considered chemical bonding at the atomic scale, adhesion at the nanoscale, and elastic-plastic deformations of asperities at the microscale. This work establishes a quantitative correlation between the contact behavior of engineering surfaces during sliding and their dry friction performance, providing theoretical support for overcoming the limitations of traditional contact models.

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PGT-Toolkit: Advancing Photogrammetry Workflows with Novel Human-Readable Coded Targets (HRCTs)

M. Borisov, S. Harizanov

The PGT-Toolkit (Photogrammetry Target Toolkit) is an open-source Python application designed for generation and automatic detection of photogrammetry coded targets (also known as control markers). Developed by our project team, it supports the established Circular Coded Target (CCT) and our novel Human-Readable Coded Target (HRCT). The HRCT was specifically developed to address the need for durable, universal control markers in challenging environments, including close-range, aerial, and underwater photogrammetry. Importantly, its use significantly enhances photogrammetry workflow reliability by embedding a clear, numeric identifier directly into the coding pattern, ensuring verifiable decoding by both automated software and human operators.
The PGT-Toolkit offers a comprehensive suite of tools for target processing. It generates high-resolution printable patterns of targets in both raster and vector formats, automatically finds and decodes targets within captured images, and visualizes detections by overlaying target borders and IDs on the images for fast verification. Finally, it exports structured CSV files containing decoded IDs of the targets and their image coordinates for seamless integration with commercial software like *Agisoft Metashape Professional* and open-source alternatives like *AliceVision Meshroom*.
This presentation will detail the capabilities and technical implementation of the PGT-Toolkit, offering a deeper look into the targets generation methodology, as well as their automatic detection and decoding algorithms.

Acknowledgements This work is supported by the Bulgarian National Science Fund, under the project, "An Integral Approach to Creating Digital Twins of Archaeological Immovable Monuments Using Innovative Technologies", contract No. KP-06-N82/1 (dated 06.12.2024).

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The Tubal Arnoldi Method for Tensor Function Approximation

F. Bouyghf, M. El Ghomari, A. El Ichi

Computing the approximation of $f(A)b$ where $A \in \mathbb{R}^{n\times n}, b \in \mathbb{R}^n$ and f is a function defined on the convex hull of the spectrum of A is an important task in various applications. These include network analysis ($f(x) := \exp(x)$), Markov model analysis ($f(x) := \log(x)$), lattice quantum chromodynamics ($f(x) = sign(x)$), exponential integrators in differential equations ($f_t(x) = \exp(-tx)$), and Quantum Chromodynamics theory ($f(x) = \sqrt{x}$). When the matrix A is small, the matrix function $f(A)$ can be defined in terms of its spectral decomposition or the Jordan canonical form. In many applications, the matrix A is so large that it is impractical to use one of these techniques in. In this case, several Krylov subspace methods have been developed. The first method introduced in this context is the standard Arnoldi process, which uses subspaces with nonnegative powers of A. Consider the Krylov subspace

$$K_m(A, b) = span\left\{b, Ab, \ldots, A^{m-1}b\right\}, \tag{1}$$

and applying m steps of the Arnoldi method to A with initial vector b yields the following decomposition:

$$AV_m = V_m H_m + h_{m+1,m} v_{m+1} e_m^T$$

where $V_m = [v_1, \ldots, v_m] \in \mathbb{R}^{n\times m}$ is an orthonormal basis for the Krylov subspace (1), and $H_m \in \mathbb{R}^{m\times m}$ is a Hessenberg matrix. The evaluation of $f(A)b$ via the Arnoldi process is given by

$$f(A)b = \|b\| V_m f(H_m) e_1.$$

A tensor is a multidimensional array of data that generalizes matrices. Tensors play essential roles in, e.g., network analysis and multidifferential equations. Kilmer and all [1] introduced the tensor t-product, which generalizes matrix-matrix and matrix-vector products to tensors and has many useful properties. The t-product has found applications in image processing, signal processing, tensor function approximation and data completion and denoising. Lund [2] defined tensor functions based on the t-product of third-order f-square tensors. The frontal slices of these tensors are square matrices and can be defined in terms of their spectral decomposition [2] or the Jordan canonical form. In this work, we introduce the tensor t-tubal Arnoldi method to evaluate the following tensor problem:

$$\mathcal{I}(f) := f(\mathcal{A}) \star \mathcal{B} \tag{2}$$

where $\mathcal{A} \in \mathbb{R}^{n\times n\times n_3}$ and $\mathcal{B} \in \mathbb{R}^{n\times 1\times n_3}$, and the function f is such that the matrix $f(\texttt{bcirc}(\mathcal{A}))$ is well-defined. This method is based on projecting the initial problem (2) onto an increasing sequence of Krylov subspaces via the t-product, transforming the large-scale problem into a reduced problem that can be easily solved by direct methods, with the solution serving as an

approximation to the solution of the initial problem. The proposed method is also applied to approximate parameter-dependent tensor functions of the form

$$\mathcal{U}(t) := \exp(-t\mathcal{A}) \star \mathcal{B}, \quad t > 0. \tag{3}$$

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Application of Computational Strategies and Numerical Methods for Mechanical Characterization of Micro-Scale Volumes and Thin-Layered Materials

G. Chalakova, M. Datcheva, R. Iankov

Based on previous studies focused on the mechanical characterisation of thin and multilayered materials, the present work applies computational strategies and numerical methods to analyze the behaviour of micro-scale volumes under local loading conditions. Using available experimental data and a depth-dependent material model proposed by the authors, key characteristics of the mechanical response are examined. The simulation results reveal trends in the system's behaviour and support the identification of essential material parameters, thereby providing a foundation for further refinement of the model.

* * *

Evaluation and Projection of Heating and Cooling Demand Period Parameters in Southeast Europe Using EURO-CORDEX

H. Chervenkov, K. Slavov

One of the most significant problems facing humanity today is climate change, a major environmental phenomenon on Earth. Climate change has both direct and indirect effects on industries such as the heating, ventilation, and air conditioning (HVAC) sector, as well as many other risks. Household use accounts for about one-third of global energy consumption, and HVAC systems consume a sizable share of that. Additionally, over 30% of the world's yearly carbon dioxide emissions are attributed to buildings. This energy demand has a direct impact on grid load and on the use of fossil fuels and electricity. The temperature outside is the leading factor in the energy consumption of HVAC systems in enclosed spaces, and as a result, much research is devoted to the meteorological aspects of the problem. The present study, which is a continuation of our previous one, is dedicated to the evaluation of the near past (1976–2005) and projection for the AR5 RCP4.5 and RCP8.5 scenario-driven future (2066–2095) of heating and cooling demand period parameters in Southeast Europe. The study is based on the widely used CIBSE methodology and utilizes fine-resolution input data for daily minimum and maximum temperatures from a large multi-model ensemble composed of 19 combinations of driving global circulation models and regional climate models in the EURO-CORDEX framework. The relatively large amount of data makes the considered problem computationally challenging. We provide evidence that the EURO-CORDEX ensemble is generally capable of simulating the spatiotemporal patterns of the considered parameters for the near past, but with some non-negligible biases in the cooling demand indicators. Consistent with the well-documented general temperature trend in the gradually warming climate of Southeast Europe, we demonstrate a steady increase in cooling and a decrease in heating demand period parameters, with the change in the former being significantly larger. However, there is significant temporal and spatial heterogeneity in the anticipated long-term changes in the cooling demand period parameters. In the scenario-driven future, it is expected that the overall shift and fragmentation of temperature-sensitive energy needs for heating and cooling will have operational and financial impacts on the energy system's balance at the national and EU-wide levels. Although the study is not exhaustive, stakeholders and policymakers could use the results as a scientific basis for implementing optimization and mitigation strategies.

Keywords heating and cooling demand period parameters; AR5 RCP4.5 and RCP8.5; EURO-CORDEX multimodel ensemble

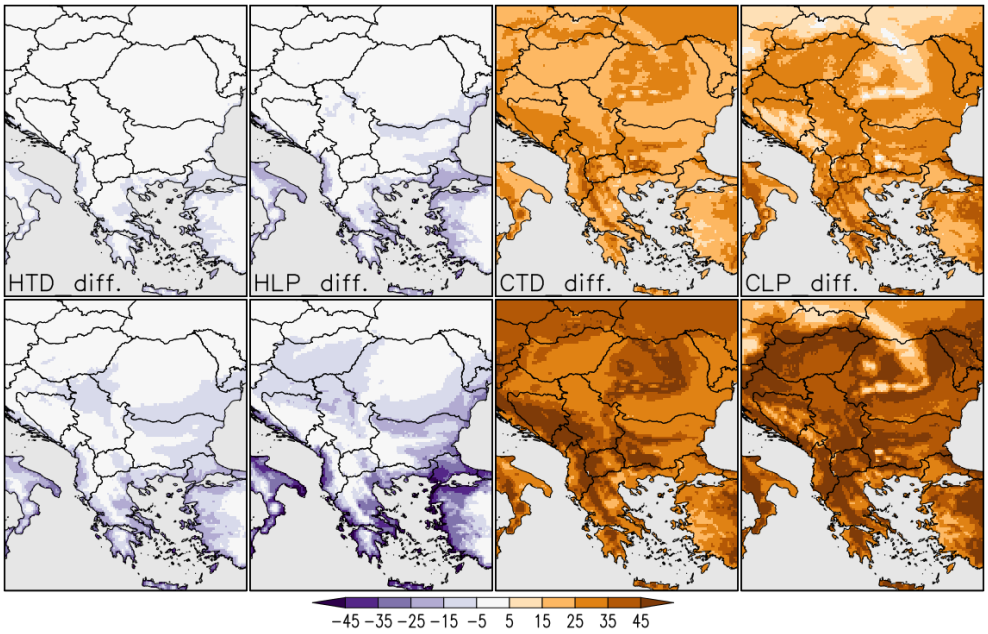

Figure 1: Maps of the absolute changes (unit: days) of the multiyear means for the future 2066–2095 in respect to the near past 1976–2005 of the total count of days of heating or cooling demand during the heating or cooling season correspondingly (labeled as HTD and CTD) and the longest continuous calendar period of heating or cooling demand during the corresponding season (HLP and CLP). The maps on the first row are valid for the RCP4.5 scenario, and these on the second — for the RCP8.5 scenario.

* * *

On a continuity of Solutions in Parametric Optimization Problems

D. Georgiev, S. Apostolov

Parametric optimization problems are tasks in which the objective function and/or the domain over which optimization occurs change depending on a parameter. The main subjects of study in these problems are the optimal value function and the multivalued mapping of the solutions to the optimization problem, which depend on the parameter.
Examining such problems is significant in the general theory of optimization and operations research due to their direct connection with the sensitivity and stability of optimization tasks under various disturbances. In practical problems, inaccuracies inevitably arise in observing

a given process, as well as errors in the numerical processing of the obtained data. Therefore, it is necessary to know whether the obtained numerical solution to the disturbed optimization problem is close (in some sense) to the solution of the original problem.
Such tasks naturally arise in game theory and mathematical economics. In a standard setup, the optimal consumption of a given agent is sought depending on their income and market prices. Another typical situation is when an optimal investment strategy is considered as a function of market indices.
A key tool in parametric optimization problems is Berge's maximum theorem. This theorem provides a sufficient condition for the continuity of the optimal value function and the upper semicontinuity of the solutions to the optimization problem. One important application of this theorem is in obtaining equilibrium existence results in game theory.
A substantial part of the study of such problems involves working with multivalued mappings. The sets over which we optimize are given as values of a multivalued mapping with the parameter as its argument. The set of solutions is also a multivalued mapping that has the parameter as its argument. We are mainly interested in the continuity properties of these multivalued mappings, specifically in what assumptions to impose on the mapping and what conclusions we would like to obtain for the set of solutions.
The continuity of multivalued mappings is a significantly more complex and ambiguous concept than the continuity of functions. The definition and study of the notion of continuity initially belonged to theoretical mathematics. Continuously developing fields of applied mathematics, such as optimization, economics, and others, further stimulate the expansion of these constructions, addressing the need for such a mathematical apparatus through which broader concepts can be established and more global results can be derived. Analogous to the relationship between limits of sequences and limits of functions, theories of convergence of sequences of sets and continuity of multivalued mappings have developed in parallel. The concepts of internal and external limits of a sequence of sets were first introduced by the French mathematician Paul Painlevé (1863-1933) in 1902, with the convergence of a sequence of sets characterized by the coincidence of its internal and external limits. Later, after the first quarter of the 20th century, scholars like Felix Hausdorff (1868-1942) and Kazimierz Kuratowski (1896-1980) developed and popularized Painlevé's concept, basing their work on it. This is why today this convergence is known as Painlevé-Kuratowski convergence.
With sets as values, a multivalued mapping allows for a similar construction to be applied to it. Thus, the concept of Painlevé-Kuratowski continuity of a multivalued mapping is reached. Another construction, which combines well with the metric in a metric space, was introduced and studied by mathematicians Pompeiu (1873-1954) and Hausdorff.
Another type of continuity we will consider in our work aligns well with the topological structure of the space and can be defined for mappings between arbitrary topological spaces. In our work, we will call this type of continuity topological.
In our study, we deal with this issue when the space over which we optimize is metric. Our main focus is proving the upper semicontinuity of the multivalued mapping of the solutions to an optimization problem. We also examine the interrelationships between different conditions imposed to relax compactness. We obtain several results that we have not encountered in the literature, summarizing and placing in a single framework known theorems in this field. In the beginning we optimize the objective function over a finite-dimensional space, with a weak

type of continuity (Painlevé-Kuratowski) of the multivalued mapping additionally imposing a condition to restrict the behavior of the function at infinity. In a case where the objective function does not depend on the parameter, we introduce a new concept of well-posedness. We use it to obtain a general result for which we additionally require only the continuity of the multivalued mapping in the sense of Painlevé-Kuratowski and the continuity of the objective function. After this we provide sufficient conditions for the well-posedness of a pair. We show how known results are obtained as special cases. During our considerations, we introduce a new type of lower semicontinuity, stronger than all known to us so far.

Acknowledgments This research was supported in part by Sofia University Science Fund Grant No. 80-10-11/2025 and the Education Programme 2021-2027, co-financed by the European Union, contract No. BG05SFPR001-3.004-0012.

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2D Documentation from 3D Laser Scanning of Architectural Heritage - Ochusha Church

I. Georgiev, P. Georgiev, D. Mihaylova, S. Trifonova, G. Evtimov

The focus of our research is the Orthodox Church "All Saints" in the village of Ochusha (20 km from the town of Kostenets). The goal was to create 3D digitalisation and extract information from the created model in order to produce 2D documentation. The church is one of the oldest in the region, with preserved icons since 1859. It is a one-story building with stone walls and a wooden roof structure. The church is single-nave with one apse, with dimensions of 20 x 9.5 meters. On the west wall is an internal wooden balcony with one apse. At the west and southwest walls of the church, a vestibule was later added. The survey of the site covers three objects: the church, the bell tower, and the gate.

To achieve the goal, we needed to create a 3D digital twin of the site. To facilitate the creation of the twin, coded markers were used and placed before the survey. They were of two types, one were coded markers 1 fixed to the walls and floors of the site, and the others were of the checkerboard 2 type that were moved parallel to the new position of the scanning device. For the external survey, the point cloud was obtained by the use of digital photogrammetry.

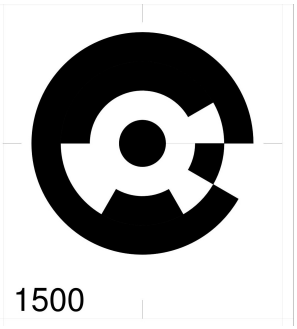

Figure 1: 2D Coded marker, diameter 160mm

Figure 2: Checkerboard marker

The photogrammetry survey was carried out with a DJI Mavic 3M drone, with a 20 MP camera, 4/3 CMOS sensor, and an equivalent focal length of 24mm. Due to the complicated geometry of the objects and the presence of numerous obstacles, the survey was carried out in "tripod" mode. To achieve an overlap between the photos of approximately 80%, the buildings were surveyed through consecutive horizontal and vertical flights, with a distance between 2 and 4 m from the surface of focus. To improve the alignment between the internal and external models, a survey flight was made in the vestibule of the church. The western and eastern entrance doors of the premises were used as points of entry. Thus, the total area surveyed is 1275 m2.

The photogrammetric model is composed of a total of 4902 photos, each with a resolution of 5280×3956 pixels. They were processed with the latest version of Agisoft Metashape software, with a standard workflow to create a three-dimensional model. As a result, a cloud of over 940 million points was created, which, after manual removal of vegetation and other unnecessary objects, was reduced to 400 million points. A polygonal model with over 250 million polygons with a resolution of 1.17 mm/pix was built from it.

For Georeferencing and sizing of the photogrammetric model, 10 ground control points from the facade of the church were used. The coordinates were measured with a GNSS Leica GS 18 I antenna with RTK correction, and a Leica TS13 robotised total station. Thus, a total error of the markers below 8 mm was achieved.

The 3D laser scanner "Faro Focus +" was used for the nave of the church and parts of the vestibule. To survey it, 20 base stations were used, three checkerboards, and ten coded markers. 3D laser scanning technology allows us to scan objects by obtaining a cloud of points.

A cloud of points represents multiple points with three coordinates x, y, z as well as three values for the color of each point R, G, B. So for one point the record is x, y, x, R, G, B. The set goal was achieved by combining the photogrammetry model of the exterior and laser scanning of the interior with common points in the vestibule. Thus, a complete model with subcentimetric accuracy of the church and the surrounding area was achieved, from which a 2D documentation was created. That will support further conservation works and ensure the preservation of this valuable architectural complex for future generations.

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Annual 3D Digital Documentation of the "Kazlacha" Neolithic Circular Enclosures: Integrating Multispectral Drone Photogrammetry into a Long-Term Archaeological Monitoring Workflow

P. Georgiev, V. Petrova, M. Raykovska, N. Petkov, G. Vasilev, G. Evtimov

Introduction The Neolithic site of Kazlacha, located in Southeastern Bulgaria, is one of the most remarkable circular ditch systems in the Balkans. Dating to the early 6th millennium BC, the complex spans about 6 hectares and includes five concentric enclosures formed by multiple V-shaped ditches. Because the site lies on privately farmed land, every excavation season ends with mandatory backfilling. This process erases surface traces, leaving long-term digital recording the only way to preserve its heritage. Since 2...

Methods and Materials The 2025 field campaign combined precision georeferencing with advanced aerial imaging. A multispectral drone equipped with four spectral bands (Green, Red, Red-Edge, and Near-Infrared) flew at low altitude to record fine surface variations. The

collected data were processed to generate vegetation and reflectance indices that highlight differences in plant vigor and soil moisture — both indicators of buried archaeological structures. All data were aligned with previous datasets from 2021–2024, including ...

Results Preliminary analysis of the 2025 multispectral data revealed clear spectral anomalies corresponding to known enclosure boundaries and ditches. In the Near-Infrared and Red-Edge bands, moisture-retaining zones outlined previously excavated and reburied features. Additional faint circular patterns, visible only through spectral analysis, suggest the presence of new, undocumented structures. By combining these multispectral layers with 3D terrain models, researchers created enhanced visualizations that corr...

Discussion The ongoing Kazlacha project demonstrates how annual 3D documentation can transform archaeological monitoring into a dynamic, data-rich process. Traditional laser scanning and RGB photogrammetry record geometry and color, but the new multispectral approach adds a powerful analytical dimension — revealing biological and chemical traces linked to past human activity. This integration of geometric and spectral data gives a more holistic view of the site's evolution over time. Because the site cannot be cons...

Keywords Neolithic Archaeology, Multispectral Drone Imaging, Photogrammetry, 3D Documentation, Cultural Heritage Monitoring, Digital Archaeology

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On Estimation Procedures of Odd Lindley–Half Logistic Model, its Properties and Applications

S. Gjorgiev, E. Stoimenova, A. Gacovska–Barandovska

Statistical modeling in survival analysis and reliability theory often relies on flexible probability distributions capable of capturing diverse patterns in lifetime data. Traditional lifetime distributions such as the exponential, Weibull, or log-normal often provide useful but limited representations. Motivated by this, statisticians propose a wide range of generalized families of distributions that allow greater adaptability. Estimating distribution parameters is central to statistical modeling with real world data. While some distributions admit simple estimators, for others, more advanced techniques are required.
In this paper, we study the Odd Lindley Half- Logistic (OLiHL) distribution and derive explicit closed-form maximum likelihood estimators for its parameter. We derive the asymptotic confidence interval of the parameter of the distribution using Fisher information. We also regard the moment-generating function and the moments of the distribution. To the best of our knowledge, this is the first work to present such derivations for OLiHL parameter estimation.

Keywords OLiHL, maximum likelihood estimator, asymptotic confidence interval, generating function

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Searching for Periodic Orbits of the Planar Three-Body Problem and Investigating their Linear Stability

I. Hristov, R. Hristova

An efficient approach for searching periodic orbits of the classical gravitational three-body problem is proposed and realized. Two traditionally studied types of initial conditions for a planar three-body motion are investigated, namely the Euler half-twist and the free-fall initial conditions. The bodies have equal masses, but some generalizations of the approach for the different mass case are also possible.

Newton's method is used to capture and compute the periodic orbits with high accuracy. The grid-search method is used to find initial approximations for Newton's method. The matrices for each iteration of Newton's method are computed using the high-order Taylor Series Method implemented with high precision floating-point arithmetic.

Usually, the grid-search method in combination with Newton's method (or another correction method) are applied to the standard periodicity equation considered on the entire period. The new approach uses the symmetries of the initial conditions and replaces the solving of the periodicity equation with the solving of a half-period equation, which also characterizes the periodic orbits. This leads to much greater efficiency, especially in the chaotic regions of the initial conditions' domains, and the discovery of many new periodic orbits. The efficiency of the new approach can be explained in terms of the Lyapunov exponents of the unstable periodic orbits.

The linear stability investigation is conducted by computing the corresponding monodromy matrices and finding their eigenvalues. The computing of the monodromy matrices does not raise any additional technical difficulties as they are of the same Jacobian type as the matrices in Newton's method and are computed in the same way - though the Taylor Series Method. The discovery of new unstable periodic orbits is important for the study of chaos in the three-body problem, while the new stable periodic orbits are of astronomical importance.

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Numerical and Experimental Evaluation of Natural Poroelastic Acoustic Materials Using COMSOL

R. Iankov, M. Datcheva

This work presents a finite element modeling strategy for predicting the acoustic performance of poroelastic sound-absorbing materials developed from natural, environmentally friendly constituents. The computational framework, implemented in COMSOL Multiphysics, enables the estimation of the sound absorption coefficient and provides detailed insight into the underlying wave–material interactions that govern acoustic attenuation. The numerical results are validated using impedance-tube measurements, demonstrating good agreement and confirming the reliability of the simulation methodology. The study emphasizes the benefits of microstructure-informed modeling for understanding and optimizing the behavior of sustainable acoustic materials, and it highlights the potential of such approaches for guiding the design of next-generation, eco-efficient sound absorbers.

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A Framework for Generation of Synthetic Samples Simulating RNA-seq Data from Steady State Distributions of Gene Regulatory Networks

I. Ivanov, K. Shegunov, N. Shegunov, P. Armyanov

One of the critical questions in the fields of Genomic Signal Processing and Functional Genomics is the problem of inferring gene regulatory structures from RNA-seq data (bulk or single cell) [1, 2]. Many of the currently existing inference algorithms are based on the concept of co-expression of genes which is essentially looking to establish the network connections based on correlations or more generally, regression models of gene interactions [3]. There are also examples of tree-based network inference [2]. However, such approaches represent a snapshot of the network connectivity and thus, preclude studies about the evolution of the gene expression profile over time and the related question of control of the dynamics of gene regulation. We propose a general framework for synthetic RNA-seq sample generation that allows for computational examination and comparison of the existing algorithms for network model inference. The underlying assumption of our framework is that data is generated from the steady-state (or a distribution close to it) of the gene regulatory network – an assumption which is usually met in many of the current experimental settings using live cells or animal models. This assumption also allows for the interpretation of the sample ordering as pseudo-time because it reflects the behavior of a dynamic model of gene regulation in a long run. Our framework is flexible and allows for simulating both bulk and single cell RNAseq data. Moreover, it is scalable in terms of using parallel computation. Our data generation

framework was applied to evaluate the performance of select network identification/recovery algorithms and software packages [1, 2].

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On the Effectiveness of Various Quantum-Computing-Based Algorithms for the MaxCut Problem

A. Kirilov, E. Atanassov , S. Ivanovska, S. Yordanov, I. Georgiev

The MaxCut problem is a fundamental problem in Combinatorial Optimization, with significant implications across diverse domains such as community detection, portfolio clustering, vehicle routing. Although the MaxCut problem on graphs is a classic NP-hard problem, efficient algorithms and software for finding approximate or exact solutions are widely studied. Two leading approaches that involve quantum computing in finding a solution are the Quantum Approximate Optimization Algorithm (QAOA) and the Variational Quantum Eigensolver (VQE). These algorithms need a classical optimizer as part of the computations, as they must find optimal values of some real-valued parameters of the respective quantum circuits.

In this work we study the performance of several optimization methods, coupled with QAOA and VQE, in solving MaxCut problems. One of these methods is proposed by us, using partical swarms on spheres, while the others are widely known, like COBYLA or SPSA. The computations used simulation on the HEMUS supercomputer, but the code has been tested also on actual quantum devices from the IBM Quantum platform. We also provide data about the usage of various system resources, like GPUs, memory, power, etc., with the aim of achieving a better understanding of the scalability issues.

Acknowledgments The work was partially supported by the Centre of Excellence in Informatics and ICT under the Grant No BG16RFPR002-1.014-0018-C01, financed by the Research, Innovation and Digitalization for Smart Transformation Programme 2021-2027 and co-financed by the European Union

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Key Challenges in Reconstructing Three-Dimensional Architecture from a Historical Photograph

V. Kolev, N. Petkov

Introduction Reconstructing three-dimensional architectural environments from historical photographs poses significant methodological and interpretative challenges. Archival images often serve as the only surviving evidence of past urban landscapes, yet they are frequently distorted by geometric deformations, scale inconsistencies, variations in lighting, camera angle, and physical deterioration of the photographic medium. These imperfections obscure accurate spatial and material information, making the reconstruction of authentic historical geometry highly complex. This study explores these challenges through a case study of Targovska Street in Sofia, aiming to establish a rigorous methodological framework that combines technological precision with expert architectural interpretation.

Methods and Materials The study systematizes the process of 3D reconstruction into several key stages: identifying the perspective parameters of the original photograph, extracting metric data from fragmentary visual and contextual sources, performing interpretative modeling to infer missing elements, and integrating all data into a coherent digital model. Perspective rectification techniques are employed to virtually "straighten" distorted images and align them with presumed real-world geometry. Metric reconstruction is based on known dimensions, architectural proportions, and analogous historical references. The approach also integrates catalogued architectural details to ensure stylistic and temporal accuracy. Modern computer vision and artificial intelligence tools — such as edge detection, texture reconstruction, and photogrammetric alignment — are applied to enhance image quality and assist in data interpretation.

Results Applying this methodology to archival photographs of Targovska Street allowed for the reconstruction of façades and urban segments with a high degree of geometric and visual accuracy. Cross-referencing multiple images, taken under varying conditions, helped to reveal hidden or lost architectural details and track changes in façade composition over time. While AI-assisted tools supported data extraction and visualization, they proved limited in reconstructing complex architectural forms or interpreting stylistic nuances when only a single image or low-resolution data was available. The results underscore the necessity of combining computational techniques with informed architectural reasoning to achieve credible reconstructions.

Discussion The study highlights that effective 3D reconstruction of historical architecture depends on the integration of technical expertise, historical knowledge, and interpretative skill. Digital tools and AI technologies serve as valuable aids but cannot substitute for the critical judgment of the architectural historian, who must interpret context, identify materials, and infer missing details with cultural and stylistic sensitivity. Consequently, the reconstruction of historical urban environments should be viewed as a hybrid methodology — one that

merges technological innovation with humanistic analysis. This interdisciplinary synergy ensures not only accurate geometric representation but also the preservation of historical authenticity and cultural meaning. Ultimately, such reconstructions contribute to research, education, and the preservation of architectural heritage in a digitally mediated cultural context.

* * *

Developing an AI-powered Tutor Using RAG

I. Lambov, C. Lambov

AI tutors have demonstrated their potential to improve student learning, yet general-purpose systems such as ChatGPT are poorly suited for exam preparation because their training on vast, heterogeneous data leads to curriculum misalignment and unnecessary complexity. This challenge is particularly acute in Bulgaria, where few AI tools are tailored to the national curriculum. In this study, we develop a prototype AI tutor based on Retrieval-Augmented Generation (RAG), trained on a synthetic Bulgarian 10th-grade history textbook. Through systematic and controlled experimentation, we identify optimal design parameters, the most effective retriever, and the best-performing language model. The resulting system provides accurate, curriculum-aligned answers without introducing external or irrelevant content. Our findings establish baseline benchmarks for RAG-based educational systems in under-resourced languages.

In recent years, artificial intelligence (AI) has surged in popularity across numerous domains, including education (Durak et al., 2024). AI-powered chatbots hold significant potential to support students by offering personalized homework assistance, explanations of complex topics, clarifications across different subjects, and tailored exam preparation materials (Labadze, Grigolia, & Machaidze, 2023). Despite these benefits, relatively few schools have established formal policies regarding AI use (United Nations Educational, Scientific and Cultural Organization, 2023), suggesting that school-level chatbot implementation remains rare. This issue is particularly pronounced in developing countries, where access to AI resources is often limited (Demaidi, 2023). As a result, students frequently rely on general-purpose systems such as ChatGPT. However, these models are trained on vast, diverse datasets and may provide answers that extend beyond official curricula, potentially confusing or discouraging learners. To restrict AI responses to school-relevant content, students must either craft elaborate prompts or supply extensive context—an approach that is both inefficient and burdensome. Although some specialized AI tutors exist, very few have been designed with the specific needs of countries such as Bulgaria in mind.

This paper addresses this gap by developing an AI tutor for Bulgarian students using Retrieval-Augmented Generation (RAG). RAG is a method in which a large language model (LLM) is provided with context retrieved from a curated knowledge base (Lewis et al., 2020). The model's responses are therefore constrained to the retrieved material, making RAG especially well-suited for AI tutors intended to deliver curriculum-specific content derived directly from

textbooks. The objectives of this paper are:

- Determine an appropriate chunk size for effective retrieval.
- Identify the most suitable embedding model for the tutor.
- Evaluate the best-performing LLM for this application.
- Develop a functional prototype of the AI tutor based on these parameters.

* * *

On parallel scalability and energy efficiency in solving dense linear systems

S. Margenov, D. Slavchev

Parallel scalability and energy efficiency are two key issues for high-performance computing (HPC). They are inextricably linked in optimizing performance, computational costs, and sustainability. We investigate the parallel solution of the system $\mathrm{A}\mathbf{x} = \mathbf{f}$, where $\mathrm{A} \in \mathbb{R}^{n \times n}$ is a dense matrix, on a shared memory computing platform. This is a basic problem of computational linear algebra with important implications for HPC emerging in a wide range of scientific and engineering applications. For large n the problem is computationally intensive, requires high memory bandwidth, and can benefit significantly from parallelism. This motivates our problem-oriented analysis. Here, parallel scalability refers to how effectively a block LU-factorization solver can use an increasing number of cores and threads to improve performance. In this case, energy efficiency means maximum computational work, measured by the number of unknowns n per unit of energy consumed. We discuss in particular limitations on increasing the number of cores/threads that lead to performance improvements and the associated communication overhead, noting that idle cores/threads also consume power. The experimental study characterizes the behaviour of a state-of-the-art hardware running a highly optimized parallel MKL solver with OpenMP communications. Numerical tests are conducted on a dual-processor HPE ProLiant XL220n Gen10 Plus server.

* * *

Continuous-Time Actor–Critic Control of the Mountain Car Benchmark Using Spiking Neural Networks in the NEST Simulator

B. Markov, P. Koprinkova-Hristova

This paper presents an experiment that combines ideas from reinforcement learning and spiking neural networks. The main goal is to show how a biologically inspired, continuous-time learning system can control a standard task from machine learning, the Mountain Car Continuous benchmark. The work uses the NEST simulator to model a simple actor–critic network that learns by interacting with the environment.

In our model, both the actor and critic are made of populations of spiking neurons. The critic learns to estimate the value of a state using a temporal-difference rule, and the actor changes its output based on a reward signal that works like dopamine in the brain. The inputs to the network are the car's position and velocity, which are converted into spikes using population coding. The network output represents the force applied to the car.

The network runs in continuous time inside NEST, but the Mountain Car environment from OpenAI Gym works in discrete time steps. To make them compatible, we use a simple translation method: the neural activity is simulated for a short period, and the resulting spike activity is averaged to produce one action for each Gym time step. In this way, we keep continuous-time dynamics inside the neural model but still communicate with the Gym environment that expects iterative updates.

We tested the system and observed that the spiking actor–critic model is able to learn to move the car up the hill and reach the goal. The learning process is slower than in standard artificial neural network models, but it is more biologically realistic and shows stable behavior once it learns the task. The trained network produces smooth and consistent control signals even with noise in the inputs.

This approach shows that continuous-time reinforcement learning can be implemented with spiking neurons in NEST and can work with standard machine learning environments. It also gives a way to connect ideas from computational neuroscience with more practical reinforcement learning research. The method could later be used for more complex control tasks or tested on neuromorphic hardware platforms.

Overall, this work tries to make a bridge between classical reinforcement learning and biologically inspired neural models. It is a small step toward understanding how brain-like systems can learn to control environments through continuous interaction and reward signals.

* * *

Presenting Research Project "Growing but not Getting Old: Semantic and 3D Technologies for the History of Streets and Buildings in Sofia"

I. Nachev, M. Raykovska, V. Kolev, N. Petkov, P. Osenova, Y. Konstantinova, D. Ivanov, A. Zlatkov

The research project "Growing but not getting old: semantic and 3D technologies for the history of streets and buildings in Sofia" is focussed on the study and innovative presentation of the historical development of the Bulgarian capital city in the period from the 19th century until to the middle of the 20th century. The planned activities during the three-year project period envisage active interaction between humanities, urban history and contemporary digital technologies as a means of creating and...

The project envisages researching and systematising original documentary material from a wide range of sources, guaranteeing the reliability of the study of iconic buildings, streets and public spaces. At the next level, the project's goals include the creation of a 3D architectural model - a graphic restoration in a virtual interactive environment of the emblematic, but demolished historical street "Targovska" in the heart of the Bulgarian capital city. The semantic technologies in the project include ...

The leading organization of the project is the Institute of Balkan Studies with a Center for Thracology at the Bulgarian Academy of Sciences (IBCT-BAS), partner organization is the Institute of Information and Communication Technologies at the Bulgarian Academy of Sciences (IICT-BAS), with experts from the Regional Historical Museum Sofia (RIM-Sofia) and the University of Civil Engineering, Architecture and Geodesy (UACEG) also participating in the project. The project "Growing but not getting old: sem...

Keywords Urban History, 3D Architectural Modelling, Semantic Technologies, Cultural Heritage, Sofia, Historical Reconstruction, Digital Humanities

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Towards Creating 3D Virtual Reality in a Wild Cave by Using LiDAR and Photogrammetric Technology with Dedicated Markers

T. Ostromsky, M. Raykovska, C. Kabadzhova, N. Petkov, P. Georgiev

Creating an accurate three-dimensional virtual reality (VR) representation of a rough underground natural environment is rather challenging problem. The main obstacles include

limited visibility or full absence of light, irregular geometry, absence of clear and stable reference points and unavailability of the GPS (satellite signals cannot penetrate underground). In this study innovative technology is applied for this purpose, combining terrestrial LiDAR scanning and fine-resolution photogrammetry. A system of dedicated artificial markers helps to enhance spatial control in case of monotonic environment and/or high blur level caused by high relative humidity. LiDAR, a high-precision, non-contact distance measurement technology, uses laser pulses to generate dense and geometrically reliable point clouds, converted after processing into detailed 3D map of the surrounding surfaces. Combined with photogrammetry, which provides high-quality surface textures, these technologies would be a powerful instrument in creation of digital twins of complex underground environments.
This study is intended to test the applicability and capabilities of the LiDAR and photogrammetry technologies in imaging, mapping, and creating 3D model of a particular underground object – Svirchovitsa Cave, located in Karlukovo karst region of Northwestern Bulgaria. Our research highlights the advantages of these methods for speleological studies, risk assessment, conservation, and tourism. By integrating advanced tools, researchers and visitors can better understand the geometry, structure, and evolution of karst systems. This study demonstrates the potential of these technologies to enhance exploration, monitoring, and sustainable management of fragile and challenging underground environments.
The presented results are, for now, modest in scope and not particularly impressive at this stage. However, they played an important role as a test of the capabilities of the novel technologies that are making their way into 3D modeling of hard-to-access outdoor objectd, as well as of our teams competence in this field. This experience gave our team confidence and assurance, and it opens up prospects for much more extensive and in-depth research in this area. It paves the way toward building an effective multidisciplinary system for modeling, non-destructive multidisciplinary investigation, and presentation of large, rugged karst terrains and hard-to-access underground sites.

Keywords 3D model; LiDAR; photogrammetry; cave; karst; speleology

Acknowledgments This research was supported in part by the Bulgarian NSF project Integrated approach to creating digital twins of archaeological immovable monuments using innovative technologies, contract KP-06-N82/1 with the Bulgarian National Science Fund. We acknowledge also the access provided to the E-Infrastructure of the Laboratory for 3D Digitization and Microstructural Analysis at the Institute of Information and Communication Technologies, Bulgarian Academy of Sciences, Grant No BG05M20P001-1.001-0003.

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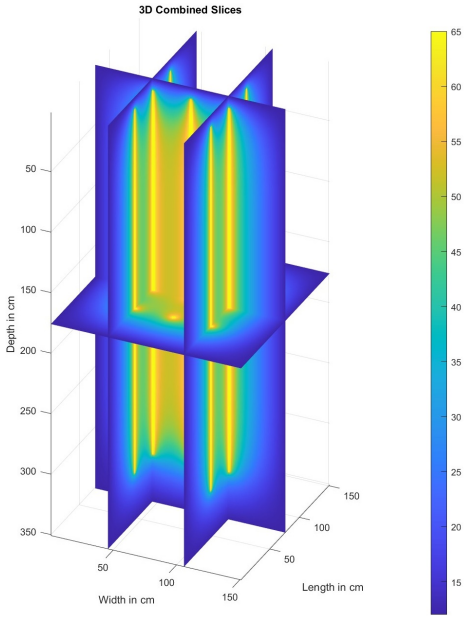
Numerical Modeling of the Temperature Field of Seasonal Borehole Thermal Energy Storage – Case Study

M. Rashevski, S. Slavtchev, M. Datcheva, R. Stoykov, M. Ganchev, D. Dzhonova, D. Kolev

Two problems related to the temperature field of a borehole thermal energy storage (BTES) are considered. First, numerical simulation based on the three-dimensional heat equation is performed to evaluate the temperature field of consisting of 9 boreholes located in a square grid. An insulating cap is modeled through the Fourier equation and some important parameters such as the soil mean temperature and the storage capacity are assessed. The simulation considers the seasonal character of the BTES and is performed over a period of 6 months. Different cap insulation thicknesses and lateral dimensions are evaluated. The parametric study of the results assists designers in choosing an appropriate heat source temperature in the borehole and to decide on the thickness and surface dimensions of the insulating cap.

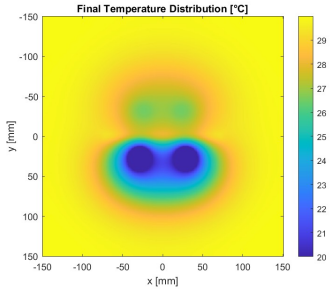


(a) (b)

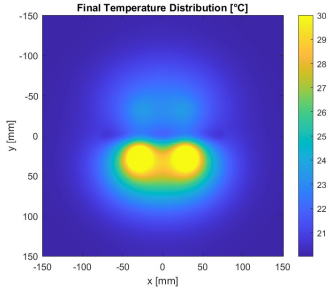
Figure 1: Three-dimensional temperature field of a BTES consisting of 9 boreholes in a square grid with constant inlet temperature of 80 C

The second problem deals with the temperature field of a horizontal 2D section of the borehole heat exchanger (BHE). An insulating strip, which prevents interference between the hot

and cold pipe-couples, is modeled, considering individual material properties of soil, grout, pipes and insulation. Heating and cooling modes are evaluated and the temperature difference between the inlet and outlet pipe-couples is calculated. Based on that, different strip thicknesses are assessed to determine the most suitable and practically viable scenario prior to real scale installation.



(a)



(b)

Figure 2: Two-dimensional temperature field in cooling (left) and heating (right) modes

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Integrating Blue Culture Technologies for the Protection, Monitoring, and Virtual Exploration of Underwater Cultural Heritage: The BCThubs Project

M. Raykovska, I. Georgiev, N. Prahov, Z. Zhelyazkov, K. Velkovsky, P. Georgiev, N. Petkov, F. Bruno, M. Cozza, F. Buffone, S. Krinidis, A.Dimara, F. Figurella, M. Pieri, I. Boujmil, S. Paolacci

Introduction The world's underwater cultural heritage — shipwrecks, ancient ports, and submerged settlements — holds extraordinary stories about our past, yet many of these sites are under constant threat. Climate change, pollution, and human activity continue to damage what lies beneath the surface. The BCThubs project introduces Blue Culture Technologies (BCT) to protect and study these sites in smarter, more sustainable ways. Combining robotics, artificial intelligence, and digital storytelling, the project bridges marine science and cultural heritage to make underwater archaeology safer, more precise, and more accessible.

Methods and Materials To meet these challenges, BCThubs has developed a suite of innovative tools. A robotic platform equipped with optical, acoustic, and navigation sensors allows researchers to create detailed 3D models in real time, even at depths of up to 600 meters. High-resolution electro-resistivity systems help detect buried structures, while a vibro-coring tool collects seabed samples for scientific and conservation analysis. Beyond exploration, the project focuses on sharing discoveries in engaging ways. Cloud-based virtual reality (VR) platforms allow users to explore underwater sites remotely, while augmented reality (AR) tools, synchronized with acoustic localization, enable divers to see digital reconstructions during dives — a feature known as Augmented Diving. For ongoing site care, AI-driven 4D modelling tracks environmental changes and material degradation. The team has even introduced a biometric buoy that monitors divers' heart rates in real time to improve safety during field operations.

Results The combined use of these technologies has transformed how underwater sites are explored and protected. The robotic and AI systems provide faster, more accurate mapping of archaeological environments. Subsurface detection now reveals hidden structures like shipwrecks that traditional sonar would miss. Immersive VR and AR experiences make it possible to "dive" into reconstructed sites without leaving the surface, offering new opportunities for research, education, and public outreach. Meanwhile, the continuous monitoring of site conditions and diver safety data ensures a more sustainable and secure approach to underwater fieldwork.

Discussion The BCThubs project demonstrates how digital innovation can reshape the study and preservation of underwater cultural heritage. By merging advanced engineering with the humanities, it creates a powerful balance between technology and storytelling —

between data and meaning. These Blue Culture Technologies not only safeguard fragile marine heritage but also invite the public to experience it in entirely new ways. In doing so, the project contributes to a broader vision of sustainable Blue Growth, where science, culture, and environmental responsibility work hand in hand to protect our shared underwater past.

Keywords Underwater Cultural Heritage, Blue Culture Technologies, Artificial Intelligence, 3D Modelling, Sustainable Blue Economy, Virtual Reality, Augmented Diving

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A Quantitative Micro-CT Protocol for Voxel-Based Evaluation of Regenerative Bone Volume

Mir. Raykovska, M. Tsvetkov, S. Trifonova, I. Georgiev, M. Kostadinova, M. Mourdjeva

This study presents a micro-computed tomography (micro-CT) workflow designed for the quantitative assessment of localized bone healing in small-animal models. The protocol integrates high-resolution scanning, voxel-level reconstruction, adaptive segmentation, and standardized volumetric measurements to provide a consistent analytical framework for regenerative studies. To evaluate the method, we examined 3-mm cylindrical cranial defects and compared them with matching reference volumes extracted from adjacent native bone, enabling direct geometric normalization.

Methods: Acquisition and Reconstruction Ex vivo scans were carried out using a Nikon micro-CT system at 100 kV and 110 μA, with a 500 ms exposure time and no filter. Continuous rotation generated 2880 projections, producing an isotropic voxel size of 9 μm. Reconstruction was performed with Nikon Reconstruction Software employing a Feldkamp-type filtered back-projection algorithm. No smoothing or post-processing corrections were applied so that original grayscale values and the linear relationship between attenuation and density were preserved.

Segmentation and Region Definition Mineralized tissue was segmented using adaptive thresholding, allowing local variations in grayscale intensity to be captured without Gaussian or median smoothing. To maintain voxel geometry, no morphological operations (opening, closing, despeckling) were applied. For each specimen, a cylindrical region of interest (ROI) with a 3 mm diameter—corresponding to the trephine defect—was defined. A second cylinder of identical size was extracted from intact bone adjacent to the defect and used as a subject-specific reference volume.

Mathematical Metrics Several volumetric descriptors were computed. The bone volume fraction was defined as:

$$\text{BV/TV} = \frac{V_{\text{segmented bone}}}{V_{\text{ROI}}}.$$

A normalized regeneration index (NRI) was calculated as:

$$\text{NRI} = \left(\frac{NBV}{RBV}\right) \times 100\%,$$

where NBV denotes newly formed bone volume within the defect, and RBV represents the reference bone volume from intact tissue of identical geometry.
To assess mineralization differences, voxelwise grayscale distributions from regenerated and native bone regions were compared.

Results: Protocol Evaluation New bone volumes ranged between 0.007 mm^3 and 1.72 mm^3. Reference bone volumes varied from 1.13 mm^3 to 1.72 mm^3. These values yielded regeneration indices between 0.48% and 123.74%. The results demonstrate that the protocol is sensitive to modest variations in bone formation and that geometric normalization using matched cylindrical ROIs is both feasible and reliable.

Conclusion The protocol provides a practical framework for voxel-level quantification of localized bone regeneration. Through standardized acquisition settings, adaptive segmentation, and matched geometric regions, the method enables consistent comparison between specimens. Its mathematical formulation and density-based evaluation make it suitable for both quantitative imaging analysis and computational modelling applications.

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Linear Congruent Generator Imitation in Slot Machine Gambling Games

V. Saykov, I. Dimitrov, R. Batalski, D. Tsvetanov, G. Doldurov, L. Doldurova, T. Balabanov

Introduction Slot machines are among the most popular gambling games. The concept behind them involves virtual strips of winning symbols. They are often referred to as fruit machines because, when first invented, they were mechanical devices that offered fruit bars as prizes for hitting specific combinations of drawn fruits on spinning reels.
Modern slot machines are computerized devices or part of an online entertainment system. Instead of mechanical reels, they utilize virtual reels, which are often represented as two-dimensional arrays of numbers that identify the symbols. The traditional spinning mechanism

has been replaced by a process that randomly selects the reels' stopping positions. This random selection relies on random number generators.
In most developed markets, the law mandates the use of cryptographically secure random number generators. These types of generators produce high-quality random numbers, although their generation speed may be slower. Additionally, legal requirements state that each randomly generated number must be stored in an event log for potential legal inspection.

Linear Congruential Generator LCG is one of the most popular pseudo-random number generators. Its greatest advantage is its simplicity, and its greatest disadvantage is its simplicity. The advantage is the simple linear formula used to generate the following number. The formula uses only three operations: multiplication, addition, and modulus. The most significant disadvantage of LCG is that the generated numbers do not meet moderate statistical quality requirements.

Imitating LCG in the Gambling Logging a large volume of random numbers on heavily loaded gambling servers can be resource-intensive. This overload, combined with the limited capacity of high-quality, cryptographically secure random number generators, drives the need for more efficient methods to calculate gambling outcomes. In this context, the idea of imitating an LCG becomes relevant.
In this approach, the seed value is generated using a cryptographically secure random number generator, and only this seed is logged in the server's storage. All subsequent values are then calculated from that seed through LCG-like mathematical transformations. This method complies with legal requirements to use only one RNG instance. Additionally, from the programmer's perspective, the numbers generated in this way appear to originate from an RNG.

Acknowledgments This research is supported by Velbazhd Software LLC.

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Artificial Intelligence Algorithm for Studying Scalar Field Propagation in the Space-Time of Schwarzschild Black Hole

A. Slavova, V. Ignatov

In this talk the wave equation describing a scalar field propagation in the space-time of Schwarzschild black hole with a horizon $r = 2M$ is studied. We shall find first the analytical solutions of this problem. Then we shall apply physics informed neural networks that are trained to solve supervised learning tasks while respecting any given laws of physics described by general nonlinear partial differential equations. Moreover, we shall introduce a new artificial intelligence algorithm based on physics informed cellular neural networks

in order to obtain the solutions in a fast way and in real time. Numerical simulations will illustrate the theoretical results proved in the paper.

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Optimizing the Outsourced Knitwear Manufacturing

J. Stanchov, S. Fidanova

This work addresses two key challenges in managing the outsourced production of knitted sweaters: (1) optimization of production schedules across subcontractors with limited capacity and variable deadlines, and (2) optimization of transportation logistics between the central factory and the logistics center. Due to restricted in-house capacity for finishing operations (linking) and the reliance on multiple subcontractors, traditional manual planning often results in uneven workload distribution, production delays, and non-optimal transportation routes.

A hybrid Ant Colony Optimization (ACO) algorithm is proposed to simultaneously model order allocation (Scheduling) and logistics routing (VRP). A multi-criteria model is formulated that combines both objectives-minimizing production delays and minimizing logistics costs. The overall objective function is:

$$\min Z = \alpha \sum_{i=1}^{N} \max(0, t_i - d_i) + \beta \sum_{k=1}^{K} \sum_{(u,v) \in R_k} c_{uv},$$

where t_i is the predicted completion time of order i, d_i is its deadline, R_k is the route of vehicle k, c_{uv} is the transportation cost between nodes u and v, and α and β are weights of the two criteria.

The ACO framework enables dynamic construction of solutions, where ants select subcontractors and transportation routes based on pheromone accumulation and evaluation of partial schedules. Real production data from a knitwear factory including Excel-based planning sheets, subcontractor capacities, and transportation distances are used in the simulations.

Experimental results show significant improvements over manual planning: reduction of accumulated delays, more balanced subcontractor workload, and shorter transportation routes. The method provides high flexibility, adapts to changes in order volumes, and is suitable for integration into MES/ERP systems used in textile manufacturing. The obtained results demonstrate both scientific contribution and strong practical applicability in real industrial conditions.

Acknowledgments The work was partially supported by the Centre of Excellence in Informatics and ICT under Grant No. BG16RFPR002-1.014-0018-C01, financed by the Research, Innovation and Digitalization for Smart Transformation Programme 2021–2027 and co-financed by the European Union.

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* * *

Minimal Colorings for Parallelization of Nearest-neighbour Operations on Cartesian Grids

N. Tchipev

Nearest-neighbour operations on Cartesian grids are ubiquitous in numerical simulations. Some examples include short-range kernels in Molecular Dynamics (MD) and Smoothed-particle Hydrodynamics [1], Gauss-Seidel smoothers for Multigrid [2, 3], and the streaming step in the Lattice-Boltzmann Method for Computational Fluid Dynamics [4] among others. In this work, we focus on compact kernels, where one point (or cell) on the grid interacts with its closest $3^D - 1$ points (or cells). When such operations are not embarrassingly parallel, one way to parallelize them for execution on shared-memory environments is coloring, with a classic example being the Red-Black Gauss-Seidel algorithm.

In the context of MD, a naive coloring of the dependencies of the Linked Cell algorithm results in $2 \cdot 3^D - 1 = 18$ colors in three dimensions [5]. In [6] we showed how those 18 colors can be reduced to $2^D = 8$. In [7], we further found a new scheme with just $D + 1 = 4$ colors and in [8] we found a highly practical $D + 1$-colored variant $c04\_hcp$, see Figure 1. This prompts the question - what is the minimal number of colors required to parallelize this type of operations?

The contributions of the current work are as follows. First, we present a proof that that $D + 1$ is the minimal number of colors for $D \leq 3$. Next, we demonstrate the flexibility of the $c04\_hcp$ scheme and its applicability beyond MD. Then, we show the applicability beyond a single layer of neighbours. Finally, we analyze properties of the scheme and when it is most advantageous.

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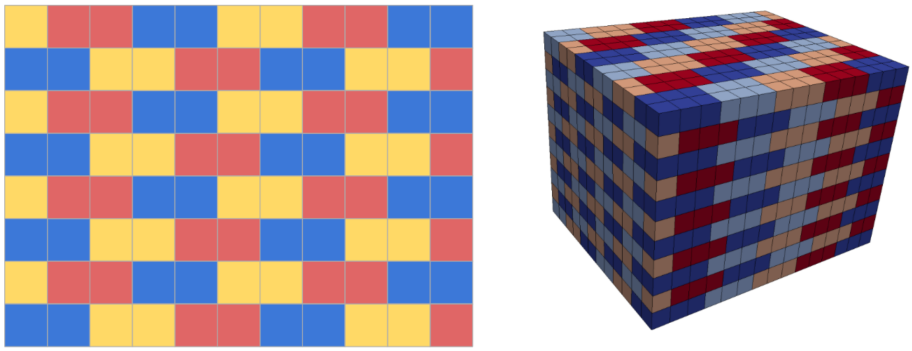

Figure 1: The $c04\_hcp$ scheme in two dimensions (left) and three dimensions (right).

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Improved Monte Carlo Approaches to Pricing Multi-Dimensional European Options

V. Todorov, V. Traneva, S. Tranev, Y. Dimitrov

This study investigates in detail the problem of pricing high-dimensional European-style options, a task that has become increasingly important in modern financial engineering and risk management. As the number of underlying assets grows, the dimensionality of the pricing problem rises sharply, making classical numerical techniques difficult to apply or even infeasible. In this context, Monte Carlo and quasi–Monte Carlo simulations have emerged as powerful tools because of their ability to handle complex, multidimensional structures that are typical in quantitative finance, such as basket options, index options, and multi-asset derivatives.

In our work, we systematically evaluate the efficiency of these stochastic simulation techniques for computing the fair value of European options written on multiple underlying assets. The proposed framework combines simulation-based optimization with low-discrepancy sequences and sophisticated variance reduction strategies in order to enhance the accuracy and stability of standard pricing models. By carefully tailoring these elements, we aim to reduce estimator variance, accelerate convergence, and improve the robustness of the numerical results. The resulting methodology yields more reliable valuation outcomes, especially in settings where traditional deterministic approaches (e.g., finite differences or tree methods) tend to struggle—such as in high-dimensional payoff structures, products with path-dependent or highly nonlinear features, and scenarios requiring frequent revaluation for risk management purposes. In this way, the study not only addresses a challenging computational problem but also contributes to the broader effort of developing scalable and trustworthy tools for high-dimensional financial modeling.

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Three-Dimensional Circular IF Zero Point Method for Transportation Problems

V. Traneva, S. Tranev

This paper presents a novel three-dimensional extension of the Zero Point Method for solving transportation-type optimization problems under Circular Intuitionistic Fuzzy (C-IF) uncertainty. We introduce the Three-Dimensional Circular Intuitionistic Fuzzy Transportation Problem (3D-CIFTP), in which every cost entry is represented as a C-IF triple and organized into a multi-layer structure. A new aggregation operator is defined to combine the layers into a unified C-IF cost matrix, preserving membership, non-membership, and circular hesitation information. Based on this structure, we formulate the Three-Dimensional Circular

IF Zero Point Method (3D-CIF-ZPM), extending the classical Zero Point approach to multidimensional C-IF data. The algorithm identifies circular zero points, constructs balancing cycles, and iteratively updates the solution until reaching optimality. A numerical example with synthetic three-dimensional C-IF data demonstrates the correctness and applicability of the method. The contribution lies in defining the 3D-CIFTP model and introducing the first Zero Point technique operating entirely in a circular intuitionistic fuzzy environment, providing a mathematically consistent framework for multidimensional uncertainty and offering a new direction for further generalizations in high-dimensional fuzzy optimisation. In addition, the proposed framework clarifies how circular hesitation radii influence the optimisation trajectory, thus revealing structural properties that remain hidden in classical IF and fuzzy approaches. The model's layered construction also makes it suitable for extensions to dynamic, scenario-based, or expert-weighted transportation systems, positioning it as a foundation for future developments in multi-dimensional fuzzy decision-making.

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Elliptic Intuitionistic Fuzzy Quad Model for Project Workforce Productivity Assessment

V. Traneva, S. Tranev, V. Todorov

This paper introduces an Elliptic Intuitionistic Fuzzy Quad (E-IFQ) model for evaluating workforce productivity in project-based environments under multidimensional uncertainty. The proposed framework extends previous developments in intuitionistic fuzzy evaluation by replacing the symmetric hesitation zone with an elliptic geometry capable of capturing directional and asymmetric uncertainty. Each productivity evaluation is represented through an elliptic intuitionistic fuzzy quadruple that encodes membership, non-membership, hesitation, and geometric dispersion, where the major and minor axes of the ellipse quantify the orientation and magnitude of evaluator disagreement. This geometric refinement allows the model to distinguish between positive-driven and negative-driven divergence, revealing patterns of asymmetry that remain invisible under circular or classical intuitionistic fuzzy approaches.
To structure the assessment process, an index-matrix representation is constructed to organise evaluations across multiple criteria and evaluators. A set of E-IFQ aggregation, normalisation, and comparison operators is defined, enabling a mathematically coherent productivity-scoring function that respects the underlying elliptic geometry. A synthetic numerical example illustrates how the E-IFQ model responds to imbalanced evidence and directional uncertainty across criteria, demonstrating its ability to capture subtle variations in evaluator judgments.
The contribution of the work lies in providing the first application of Elliptic Intuitionistic Fuzzy Quadruples to multi-criteria productivity assessment in project-oriented workforce settings, offering an interpretable and geometry-consistent framework for analysing performance in complex and uncertainty-rich project environments. This establishes a solid basis

for further extensions in high-dimensional fuzzy modelling, geometric uncertainty representation, and advanced decision-support methodologies.

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Development and Cross-Platform Validation of a Workflow for Recognizing New Generation Universal Ground Control Coded Targets in Photogrammetric Software

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This presentation discusses ongoing work within the project "An Integral Approach to Creating Digital Twins of Archaeological Immovable Monuments Using Innovative Technologies", which is funded by the Bulgarian National Science Fund. The initiative aims to improve the precision and flexibility of georeferenced 3D digitization workflows by developing universal coded Ground Control Point (GCP) markers. These markers are specifically designed to overcome the limitations of conventional photogrammetric and laser scanning techniques when applied to immovable heritage objects.

The markers being developed are engineered to be durable and suitable for a range of environments, including close-range, aerial, and underwater photogrammetry workflows. A primary objective is to ensure compatibility with widely used commercial platforms such as Agisoft Metashape Professional and open-source alternatives like Meshroom, despite their limited marker recognition algorithms.

As part of this initiative, the project team has developed Human Readable Coded Targets (HRCTs) and a custom Photogrammetry Target Toolkit (PGT-Toolkit) workflow to streamline the management of these markers throughout the photogrammetric process. The toolkit integrates a Python-based pipeline that generates high-resolution printable target patterns in both raster and vector formats. It also automatically finds and decodes targets in captured images, visualizes detections by overlaying their borders and identification numbers (IDs) for verification. Additionally, it exports structured CSV files containing decoded IDs, image coordinates, and size data, allowing for seamless integration with photogrammetry software. This unified solution ensures consistent, efficient, and cross-platform-compatible handling of coded targets, thereby enhancing the accuracy, reproducibility, and interoperability of multi-sensor digitization workflows in cultural heritage documentation. As part of the project's applied research phase, a pilot digitization of the "Prof. Marin Drinov" Publishing House at the Bulgarian Academy of Sciences was conducted. The workflow integrates close-range and UAV photogrammetry, terrestrial laser scanning, and GNSS-based coordinate acquisition using newly developed prototype markers, while their complete coding system is still being refined. This field test aims to evaluate the detectability, accuracy, and cross-platform

interoperability of the markers while generating a high-fidelity 3D model for heritage documentation.
The presentation will provide an overview of the project's methodology, experimental design, fieldwork, and its potential application to digital documentation of heritage sites and monuments. Special attention will be given to the role and performance of custom GCP markers, as well as the PGT-Toolkit's contribution to automating and optimizing their use. This case study contributes to the growing field of digital heritage by proposing a scalable and adaptable approach to accurate spatial referencing in challenging archaeological contexts. Findings from this pilot study are expected to contribute to the development of a full-length article focused on marker optimization and cross-platform compatibility. Furthermore, the next planned digitization campaign focuses on a genuine archaeological site: the underground temple-well in the village of Garlo (the Sacred Pit of Garlo), Pernik Province, Bulgaria. This second case study will employ the above-mentioned combined scanning methods to validate the practical applicability of the proposed techniques and the on-field performance of HRCTs for documenting a variety of archaeological objects and terrains.

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Exact Solutions to the Nonlinear Model Equation for Waves in Microtubules

K.N. Vitanov, N.K. Vitanov

We discuss a version of the z– model of microtubules with the possibility for large oscillations of the molecules which form the protophilaments of the tubule. The obtained model equation is solved by means of the version SEsM(1,1) of the Simple Equations Method. In order to obtain exact solutions to the model equation, we use a special function: the V-function, which contains, as specific cases, the trigonometric and the hyperbolic functions We study the influence of several forces and potentials on the velocity and other characteristics of the obtained waves

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An Improved Algorithm for Scattered Data Interpolation using Quartic Triangular Bézier Surfaces

K. Vlachkova

We revisit the problem of interpolation of scattered data in $\mathbb{R}^3$ and propose a solution based on Nielson's minimum norm network and triangular Bézier patches. We aimed at solving the problem using the least number of polynomial patches of the smallest possible degree. We propose an alternative to the previously known algorithms, see (Clough and Tocher [1]) and (Shirman and Séquin [2,3]). Although conceptually similar, our algorithm differs from the previous in all its steps. As a result the complexity of the resulting surface is reduced and its smoothness is improved.

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Part B

List of participants