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ABSTRACTS

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Positive Harmonic Functions on Denjoy Domains in the Complex Plane

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Let Ω be a domain in the complex plane $\mathbb{C}$ whose complement $E = \overline{\mathbb{C}} \setminus \Omega$, where $\overline{\mathbb{C}} = \mathbb{C} \cup \{\infty\}$ is a subset of the real line (i.e. Ω is a Denjoy domain). If each point of E is regular for the Dirichlet problem in Ω, we provide a geometric description of the structure of E near infinity such that the Martin boundary of Ω has one or two "infinite" points.

Double Complex Laplace Operator

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The commutative non-division algebra of double complex numbers consist on the couples of complex numbers $(z + jw)$, $z, w \in \mathbb{C}$, $j^2 = i$. The double complex Laplace operator

$$\Delta_+ = \frac{\partial^2}{\partial z^2} + i\frac{\partial^2}{\partial w^2}$$

is considered. The problem $\Delta_+ u(z,w) = 0$ is solved on the unit four dimensional cube, represented as a cartesian product of two squares D_1, D_2 in $\mathbb{C}$ for double complex functions of double complex variable.

The boundary conditions are the following ones:

$u(z,w) = u_0(z,w) + ju_1(z,w)$ with $u_0(z,w), u_1(z,w) : \mathbb{C} \to \mathbb{C}$
$u_1(z,0) = \varphi_1(z)$, where $\varphi_1(z)$ is an odd, double periodic antiholomorphic function on the square D_1
$\varphi_1(x) = \varphi_1(x+1), \varphi_1(iy) = \varphi_1(iy+i)$ for $0 \le x \le 1, 0 \le y \le 1$.

The method of separation of the variables is used.

The Support of the Equilibrium Measure

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On the real line we consider the equilibrium problem with an external field. The equilibrium measure is a probability measure which minimizes the weighted energy integral. Conditions which guarantee that the support of the equilibrium measure is an interval are of importance in potential theory. We will provide such conditions in the talk. The main technique used is the iterated balayage algorithm. This is a joint work with S. Damelin and P. Dragnev.

On Asymptotics of Eigenpolynomials of Exactly-Solvable Operators

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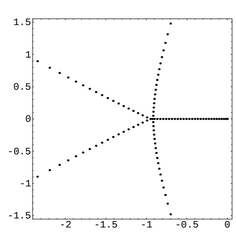
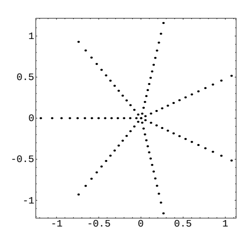
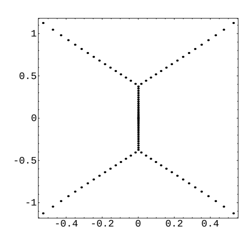
We will discuss properties of zeros in families of polynomials satisfying certain linear differential equations. Namely, consider a differential operator $T = \sum_{j=1}^{k} Q_j \frac{d^j}{dz^j}$ where the Q_j are complex polynomials in one variable satisfying the condition $\deg Q_j \leq j$ for all j, with equality for at least one j. Such operators are referred to as degenerate exactly-solvable operators (ES-operators). If $\deg Q_k = k$ for the leading term we call T a non-degenerate ES-operator, whereas if $\deg Q_k < k$ we call T a degenerate ES-operator. We show that in the former case the zeros of the unique and monic eigenpolynomial (as its degree tends to infinity) are distributed according to a certain probability measure which is compactly supported on a tree and which depends only on the leading polynomial Q_k. As for the degenerate operators, computer experiments indicate the existence of a limiting root measure in this case too, but that it has compact support (conjecturally on a tree) only after an appropriate scaling. We conjecture (and partially prove) the growth of the largest modulus of the roots in this case and deduce the algebraic equatio n satisfied by the Cauchy transform of the properly scaled eigenpolynomial as its degree tends to infinity. Operators of the type we consider occur in the theory of Bochner-Krall systems, which are systems of polynomials which are both eigenpolynomials of some finite-order differential operator and orthogonal with respect to some suitable inner product. Our study can thus be considered as a natural generalization of the behaviour of the roots of classical orthogonal polynomials. By comparing our results on eigenpolynomials of ES-operators with known results on orthogonal polynomials we believe it will be possible to gain new insight into the nature of Bochner-Krall systems. Below some typical pictures of the zero distribution of the eigenpolynomial for three different degenerate ES-operators after an appropriate scaling.







Lorentz Counterexample for Rational Approximants of Arbitrary Degrees

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Best real rational approximants on $[0;1]$ segment are considered. Weak convergence of counting measures of their alternance point sets is investigated. Lorentz-type counterexample is provided: it is shown, that for any given nondecreasing sequence of denominator degrees there exist function (which is analytical at $[0;1]$) and sequence of numerator degrees such that corresponding sequence of measures does not converge to equilibrium measure. The generalizations for the cases of arbitrary compact set and p-norms are discussed.

Pluripotential Theory and a Multivariate Bernstein Inequality

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Classical potential theory in $\mathbb{R}^2 \simeq \mathbb{C}$ provides a useful tool to study polynomial inequalities in one variable: the Green function with pole at infinity associated to a compact set. In particular, using the Green function associated to the interval $[a,b] \subset \mathbb{R} \subset \mathbb{C}$, one can prove the classical Bernstein inequality

$$|p'(x)| \leq \frac{n\sqrt{\|p\|^2_{[a,b]} - p^2(x)}}{\sqrt{(b-x)(x-a)}} \qquad (a < x < b).$$

Pluripotential theory is a non-linear generalization to $\mathbb{C}^N$ $(N > 1)$ of potential theory in $\mathbb{C}$. The basic objects of study are plurisubharmonic (psh) functions and closed positive currents, and the complex Monge-Ampére operator is a nonlinear generalization of the Laplacian to this setting. One can define an analogue of the classical Green function. This is the Siciak-Zaharjuta extremal function, V_K, associated to a compact set $K \subset \mathbb{C}^N$. When $K \subset \mathbb{R}^N \subset \mathbb{C}^N$ is a compact, convex body, one can use V_K to prove a Bernstein-type inequality for multivariate polynomials. This was done by M. Baran.

In this talk I will introduce the necessary tools to give Baran's proof of a Bernstein-type inequality for a compact, convex body in $\mathbb{R}^N \subset \mathbb{C}^N$. For convex *symmetric* sets $(K = -K)$, this inequality is sharp. If time permits we will also discuss the complex equilibrium measure.

Foliations for the Extremal Function

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The Siciak-Zaharjuta extremal function, V_K, associated to a compact set $K \subset \mathbb{C}^N$ is the generalization to pluripotential theory of the classical green function in $\mathbb{C}$. It is an example of a maximal function in $\mathbb{C}^N \setminus K$; maximal functions are multivariate generalizations of harmonic functions, but unlike the latter they are not necessarily smooth. One reason for a psh function u to be maximal on a domain is the local existence, through each point, of a one-dimensional complex submanifold on which u is harmonic. For the function V_K, this property holds if K is

1. the closure of a smooth, strictly linearly convex domain in $\mathbb{C}^N$ (Lempert)

2. a convex body in $\mathbb{R}^N$ (joint work with Burns and Levenberg)

I will discuss in some detail the situation for convex bodies in $\mathbb{R}^N$; it turns out that the manifolds on which V_K is harmonic are complexifications of real ellipses inscribed in K.

Applications: The Inscribed Ellipse Method, and the Complex Equilibrium Measure

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A geometric approach to proving a Bernstein-type inequality for a compact, convex body $K \subset \mathbb{R}^N$ is the inscribed ellipse method of Sarantopoulos. Here the polynomial inequality is obtained in terms of an ellipse parameter. For convex *symmetric* sets, the inscribed ellipse method gives a sharp inequality. Since both methods (pluripotential theory and inscribed ellipse) give a sharp result for symmetric convex bodies, how do they compare in the nonsymmetric case? For the standard triangle (or simplex) in $\mathbb{R}^2$, the exact yields of both methods have been computed, and found to be equivalent.

The equivalence of the Bernstein factors obtained by the inscribed ellipse and pluripotential-theoretic methods is no coincidence. We show that the ellipses that give the best bound for the inscribed ellipse method turn out to be identical to the ellipses whose complexifications give the one-dimensional manifolds on which V_K is harmonic. This is used to show that indeed, the two methods are equivalent for all convex bodies $K \subset \mathbb{R}^N$.

We also use the equivalence of these ellipses to prove a result about the complex equilibrium measure. That is, a formula due to Bedford/Taylor and Baran for the complex equilibrium measure associated to a symmetric convex body holds in fact for all (not necessarily symmetric) convex bodies.

This is joint work with Dan Burns, Norm Levenberg and Szilard Revesz.

The Role of Hermite Polynomials in Asymptotic Analysis

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My research base with Hermite polynomials which are considered as approximants in asymptotic representations of certain other polynomials. Examples are given for polynomials from the Askey scheme of hypergeometric orthogonal polynomials. I also mention that Hermite polynomials can be used as main approximants in uniform asymptotic representations of certain types of integrals and differential equations.

Elliptic Equations on Almost Complex Manifolds

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My talk concerns a paper of Donald Spencer published in 1955 under the title "POTENTIAL THEORY AND ALMOST COMPLEX MANIFOLDS" (See: Lectures on Functions of Complex Variables, ed. Wilfred Kaplan et al, the University of Michigan Press, Ann Arbor, 1955). According to Spencer most of the potential theory usually associated with complex structure is carried out through the Laplacian Δ defined in terms of the derivatives ∂ and $\bar{\partial}$ and their adjoint operators. A deep analysis for the possible interconnection between topological and analytic nature of the arised problems is given in the mentioned paper.

Our purpose is more concrete. We shall consider $2n$-dimensional manifolds M equipped with an almost complex structure J, i.e. $J : TM \to TM$ is a fiberwise linear automorphism of the fiber bundle TM such that $J^2 = -id$, and the operator $J_p : T_pM \to T_pM$ depends smoothly of the point $p \in M$.

In the case of real coordinates (x_k) on M, we construct a concrete elliptic operator DJ such that the local solutions of the equation $D_J u = 0$ to be the almost pluriharmonic functions u on M. i.e. $dJdu = 0$ (see [2]). If J coincides with the standard almost complex structure S the operator D_S is the ordinary Laplace operator.

Instead of real coordinates we can consider self-conjugates coordinates $(z_k, \bar{z}_k)$. In the case of positive type $m > 0$ (a notion introduced by Spencer) the fundamental system $Jdf = idf$ of almost-holomorphic functions $f = u + iv$ seems as follows

$\partial f/\partial \bar{z}_j = 0, \quad j = 1, \ldots, m,$
Other linear equations with respect to the partial derivatives of f
and the coefficients of the local matrix representation of J.

In the 4-dimensional case of constant type 1, for instance, for every point $p \in M$ there exists an open Spencer local coordinate system $(U, (z_1, \bar{z}_1, z_2, \bar{z}_2))$. Then the system mentioned above seems as follows

$$\partial f/\partial z_1 - = 0$$
$$J_4^1 \partial f/\partial z_1 + J_4^2 \partial f/\partial z_2 + (J_4^4 - i)\partial f/\partial \bar{z}_2 - = 0.$$

Calculating the symbol $A = A(z_1, z_2, \xi_1, \xi_2)$ we conclude that the above written system is an overdetermined elliptic system.

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Transfinite Diameter, Chebyshev Constant and Energy on Locally Compact Spaces

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We study the relationship between transfinite diameter, Chebyshev constant and Wiener energy in the abstract linear potential analytic setting pioneered by Choquet, Fuglede and Ohtsuka. It turns out that, whenever the potential theoretic kernel has the maximum principle, then all these quantities are equal for all compact sets. For continuous kernels even the converse statement is true: if the Chebyshev constant of any compact set coincides with its transfinite diameter, the kernel must satisfy the maximum principle. An abundance of examples is provided to show the sharpness of the results.

Approximations of Orthogonal Polynomials in Terms of Hermite Polynomials

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My talk will be on Several orthogonal polynomials have limit forms in which Hermite polynomials show up. Examples are limits with respect to certain parameters of the Jacobi and Laguerre polynomials. In this paper am interested in more details of these limits and I we give asymptotic representations of several orthogonal polynomials in terms of Hermite polynomials. In fact i we give finite exact representations that have an asymptotic character. From these representations the well-known limits can be derived easily. Approximations of the zeros of the Gegenbauer polynomials $C_n^a(x)$ and Laguerre polynomials $L_n^a(x)$ are derived (for large values of a and a, respectively) in terms of zeros of the Hermite polynomials and compared with numerical values. I we also consider the Jacobi polynomials and the so-called Tricomi-Carlitz polynomials.

Rough Solutions for the Maxwell-Schrödinger System

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The work treats the nonlinear Maxwell - Schrödinger equation with initial data

$$(\psi_0, A|_{t=0}, \partial_t A|_{t=0}) \in H^s \times H^{1+2\delta} \times H^{2\delta},$$

where ψ is the spinor field, A is the magnetic potential and Coulomb gauge is imposed. Existence of local solutions is shown, when $7/8 < s \leq 2$ and $0 < \delta \ll 1$. Large data global existence result is proved for $5/4 \leq s \leq 2$ and $0 < \delta \ll 1$.

Extension Property of Cantor-type Sets in Terms of Potential Theory

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Given a compact set $K \subset \mathbb{R}$, let $\mathcal{E}(K)$ denote the space of Whitney jets on K, that is traces on K of functions from $C^\infty(\mathbb{R})$. The compact set K is said to have the extension property if there exists a linear continuous extension operator $L : \mathcal{E}(K) \longrightarrow C^\infty(\mathbb{R})$. We discuss a longstanding problem of a geometric characterization of the extension property and give such a characterization for the case of Cantor-type sets.

The criterion can also be given in terms of the rate of growth of the values of the minimal discrete energy of compact sets that locally form K. Examples of polar Cantor-type sets with the extension property as well as non-polar Cantor-type sets without the extension property are presented.

Around Newtonś Theorem on the Gravitational Attraction Induced by Ellipsoids and Null Quadrature Domains

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Let E be an ellipsoid in $\mathbb{R}^3$ centered at the origin. We call $\lambda E \setminus E$ $(\lambda > 1)$ a *homoeoidal ellipsoid*. A remarkable theorem due to Newton asserts that the gravitational attraction at any internal point of a homogeneous homoeoidal ellipsoid is zero. Are ellipsoids the only bodies having the property that gravitational force induced by a homoeoid is zero at all internal points?

So let $K \subset \mathbb{R}^3$ be a closed bounded set containing a neighborhood of the origin. If the homogeneous homoeoid $\lambda K \setminus K$ $(\lambda > 1)$ produces no gravity force in the cavity K, then

$$\int_{\mathbb{R}^3 \setminus K} h dx = 0 \tag{1}$$

for all harmonic and integrable functions h in $\mathbb{R}^3 \setminus K$. An open set Ω which satisfies (1) is called a *null quadrature domain*. In this talk, I will link between Newtonś Theorem of no gravitational force in the cavity and null quadrature domains. I will also discuss the classification problem, in particular in the case where K is unbounded.

Double-Complex Harmonic Functions

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The term double-complex variables is used in [1]. In fact the double-complex algebra $C(1, j)$, i.e. the algebra of double-complex variables, is algebraically isomorphic to the algebra of bi-complex numbers BC. (see for instance [2]).

Double-complex function theory is an isomorphic version of the the former bi-complex function theory initiated by Corado Segre (1892), and developed by different authors like G. Baley Price, Stefen Roon, A.F.Turbin, A.A.Pogorui and others. The elements of the double-complex algebra, just denoted by $C(1, j)$, are represented as follows: $a = z + jw$, where $j^2 = i$, and z, w, i are complex numbers, a is a double-complex number, j is a hypercomplex number, not complex. The algebra $C(1, j)$ is not a division algebra. It is a complex pair holomorphic with respect to a kind of Cauchy-Riemann system. Let $f(a) = f_0(z, w) + jf_1(z, w)$, then $f(a)$ is a double-complex holomorphic function iff

$$\partial f_0/\partial z = \partial f_1/\partial w \text{ and } \partial f_0/\partial w = i\partial f_1/\partial z.$$

In this note we develop some basic notions of differential forms on $C(1, j)$ and we study different quadratic geometries (Q-geometries) over the double-complex algebra $C(1, j)$, and over the bi-complex algebra BC.

We introduce the algebra $C^n(1, j)$ of double complex n-vectors (for further details see [3]). The operators ∂, ∂^*, d and $\partial\partial^*$ are defined. We study the solutions of the equation $\partial\partial^* f = 0$, i.e. the double-complex pluriharmonic functions, and respectively the solutions of the double-complex Laplace equations $\Delta_n f = 0$, $\Delta_{kk} f = 0$, i.e. the double-complex harmonic functions f and complex harmonic ones. Examples of double-complex harmonic surfaces are given.

References

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Ostrowski Gaps in Fourier Series, Zeros and Overconvergence

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We first consider overconvergent power series. The main idea is to relate the distribution of the zeros of subsequences of partial sums and the phenomenom of overconvergence. Sufficient conditions in terms of the distribution of the zeros of a subsequence for a power series to be overconvergent as well as to possess Ostrowski gaps are provided. Also, results of Jentzsch-Szegö type about the asymptotic distribution of the zeros of overconvergent subsequences are stated. The results are extended to Fourier series of polynomials orthogonal with respect to regular Borel measures.

Remark on the Double-Complex Laplacian

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We recall the double-complex variable's algebra $C(1,j)$, $j^2 = i$, where j is a not complex hyper complex number, $a \in C(1,j)$ iff $a = z + jw$, $z, w \in C$. The well known bicomplex algebra BC is constituted by bicomplex numbers, namely $z + iw$, where z, w and i are complex numbers. The complex Laplacian operator is naturally defined on BC

$$\Delta u = \partial^2 u/\partial z^2 + \partial^2 u/\partial w^2.$$

Recently new second order complex differential operators have been introduced. In connection with the double-complex algebra $C(1,j)$, namely

$$D_+ u = \partial^2 u/\partial z^2 + i\partial^2 u/\partial w^2 \ \text{ and } \Delta_- u = \partial^2 u/\partial z^2 - i\partial^2 u/\partial w^2,$$

the first one called double-complex Laplacian.

Clearly, the complex Laplacian Δu is invariant under the symmetry

$$(z, w) \to (w, z) \ \text{ on } \ CxC,$$

but $\Delta_+ u$ and $\Delta_- u$ are related as follows $\Delta_-^{wz} u = (-i)\Delta_+^{zw} u$. So, both of these PDO can be considered as a double-complex Laplacians.

In this note we consider the simplest parabolic equation $\partial u/\partial t = \partial^2 u/\partial x^2$ in terms of double-complex variables, namely the equation

$$\partial u/\partial t = \partial^2 u/\partial a^2 \ \text{ with } \ a \in C(1,j),\ u(t,\alpha) = u_0(t,z,w) + ju_1(t,z,w), \alpha = z + jw, t \in R.$$

We show that the study of the above double-complex PDE is related with the Laplacian $D_- u = \partial^2 u/\partial z^2 - i\partial^2 u/\partial w^2$. More precisely the considered equation $\partial u/\partial t = \partial^2 u/\partial a^2$ is equivalent to the following system

$$\begin{aligned} \partial u_0/\partial t &= 1/4\Delta_- u_0 - i/2\partial^2 u_0/\partial z\partial w, \\ \partial u_1/\partial t &= 1/4\Delta_- u_1 + i/2\partial^2 u_1/\partial z\partial_w. \end{aligned}$$

We apply the method of the separation of the variables related with the search of a function

$$u * (t, z, w) = \sum_{k=1}^{\infty} T_k(t) X_k(z, w).$$

References

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Solvability of Nonlinear Elliptic Systems Generating Minimal Foliated Semi-Symmetric Hypersurfaces

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Using the characterization of a foliated semi-symmetric hypersurface in Euclidean space as the envelope of a two-parameter family of hyperplanes, each such hypersurface can be determined by a pair of a unit vector-valued function l and a scalar function r. The classes of the minimal and the bi-umbilical foliated semi-symmetric hypersurfaces are characterized analytically by systems of partial differential equations for the functions l and r. The minimal foliated semi-symmetric hypersurfaces are determined by nonlinear elliptic systems. We study the local classical solvability of the elliptic systems, generating minimal foliated semi-symmetric hypersurfaces in the four-dimensional Euclidean space $\mathbb{E}^4$. Since we consider the two-dimensional surface, determined by the vector-valued function l, parameterized locally by isothermal parameters, we look for solutions satisfying the Plateau boundary conditions. This problem is deeply connected with the eikonal equation. In the case of surfaces with constant Riemannian curvature we prove that the problem has a unique solution with zero Riemannian curvature, which is the standard flat torus in $\mathbb{E}^4$. As for the surfaces with non-zero Riemannian curvature we consider the three possible typical cases based on the classification of the eikonal equation.

About Gauss's Theorem in the Space of Fractional Dimension

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In the case of space of fractional Hausdorf dimension the Gauss's theorem has to be modified. The modification of this theorem is proposed. Some useful remarks about fields and potentials are obtained and applications are discussed.

Jackson Inequality in $\mathbb{R}^N$

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Let E be a compact subset of the space $\mathbb{R}^N$ such that $E = \overline{\mathrm{int}E}$. We shall say that E admits ***Jackson's inequality*** $(\mathcal{J})$ if for each $k = 0, 1, \ldots$ there exist a positive constant C_k and a positive integer m_k such that for all $f \in C^{\infty}_{\mathrm{int}}(E)$ and all $n > k$ we have

$$n^k \mathrm{dist}_E(f, \mathcal{P}_n) \leq C_k \|f\|_{E,m_k} \tag{$\mathcal{J}$}$$

where

$$\|f\|_{E,m} := \sum_{|\alpha| \leq m} \sup_{x \in E} |D^{\alpha} f(x)|$$

and

$$\mathrm{dist}_E(f, \mathcal{P}_n) := \inf\{\sup_{x \in E} |f(x) - p(x)| : p \in \mathcal{P}_n\}.$$

Here $C^{\infty}_{\mathrm{int}}(E)$ denotes the space of all $\mathcal{C}^{\infty}$ functions in $\mathrm{int}E$ which can be continuously extended together with all their partial derivatives to E and $\mathcal{P}_n$ is the space of all polynomials of degree at most n. By known multivariate versions of the classical Jackson theorem, every compact cube P in $\mathbb{R}^N$ admits Jackson's inequality with $m_k = k + 1$. The purpose of our talk is to deliver other examples of Jackson sets in $\mathbb{R}^N$. We shall show, in particular, that a finite union of disjoint Jackson compact sets in $\mathbb{R}^N$ is also a Jackson set and that this in general fails to hold for an infinite union of Jackson sets. We also give a characterization of Jackson sets in the family of Markov compact sets in $\mathbb{R}^N$ which together with a Bierstone result permits to show that Whitney regular compact subsets of $\mathbb{R}^N$ are Jackson.

On the Tangential Oblique Derivative Problem

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This talk deals with the oblique derivative problem for second derivative problem for second order elliptic partial differential operator in a bounded domain. The smooth realvalued nonvanishing vector field l is tangential to some part E of the boundery. This boundary value problem (bvp) was considered for the first time by Poincaréwhile studying the tides of the ocean.

Depending on the behaviour of L near E three different cases appear: (i) bvp of Friedholm type, (ii) bvp having in finite dimansional kernel, (iii) bvp having infinite dimensional cokernel.

In order to gaurantee the Friedholm property of our bvp in case (ii), the values of the solution u should be prescribed on E. Even under this additional condition we have loss of regularity of the solution both in Sobolev and Hölder spaces. In the case (iii) our bvp is not even locally solvable in a neighborhood of each point of E. This strange effect can be explained by the discontinuity of u (with finite jump) on E. The main ideas and methods in investigating the Poincaré bvp are illustrated by model examples. The result here proposed are based on subelliptic pseudodifferential equations satisfied by the restriction of u on the boundary.

Recognition of the Coulomb Potential via "Approximations to the Identity"

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Let Φ be a smooth function on $\mathbb{R}^3$ that is appropriately small at infinity and satisfies $\|\Phi\|_{L^1(\mathbb{R}^3)} = 1$. From such a Φ, we fashion an "approximations to the identity" via convolution operator as follows: For $t > 0$, using the appropriate scaling in $\mathbb{R}^3$, we define $\Phi_t(x) = t^{-3}\Phi(x/t)$. It is a well-known result that for any $f \in L^p(\mathbb{R}^3)$, $1 \le p \le \infty$, we have

$$\lim_{t \to 0} (\Phi_t * f)(x) = f(x),$$

for a.e. $x \in \mathbb{R}^3$. We shall consider the convolution operator with Coulombian kernel $k(x) = \frac{1}{|x|}$ and we aim at establishing the rate of convergence of $\|\Phi_t * \frac{1}{|x|} - \frac{1}{|x|}\|_{L^p(\mathbb{R}^3)}$ for $t \to 0$. More precisely, we shall prove the following estimate

$$\left\| \Phi_t * \frac{1}{|x|} - \frac{1}{|x|} \right\|_{L^p(\mathbb{R}^3)} \le Ct^s \| |x|^s \Phi\|_{L^1(\mathbb{R}^3)}, \quad s = \frac{3}{p} - 1, \quad \frac{3}{2} < p < 3.$$

Finally, we present vivid examples from the theory of the nonlinear wave and Schrödinger equations, where the above result plays crucial role.

Efficient Boundary Element Method in Solid Mechanics

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Boundary integral methods for existence and regularity analysis of elliptic boundary value problems have a long history which goes back to the basic ideas of Green, Gauss and Poincaré in the 19th century. Till these days they serve as decisive tools for intrinsic analysis of elliptic problems. Their numerical employment began during the 50s of the last century when electronic computation opened the opportunity of numerical simulation of larger complexity.

The reformulation of three–dimensional elliptic boundary value problems in terms of the nonlocal boundary integral equations reduces interior as well as exterior problems to equations on the two–dimensional boundary. A finite element approximation of the boundary charges requires the triangulation of the boundary surface only which is a great advantage in practice. The Galerkin (as well as collocation) discretization of the boundary integral equations results in large systems of equations involving fully populated matrices which was rather disadvantageous up to about 30 years ago. More recently, however, the combination of degenerate kernel and low rank approximation with a hierarchical structure of the boundary triangulation has lead to a drastic improvement of the performance of boundary element methods. For boundary charges with N degrees of freedom now the matrix times vector product can be executed with $N \log^2 N$ complexity. With appropriate preconditioning, nowadays large, enormous systems can be solved very efficiently. In combination with nonoverlapping domain decomposition, boundary element methods became a very efficient simulation instrument with a wide variety of industrial applications.

Extremal Problems for Potentials with External Fields. Applications to the Theory of Capacities of Condensers

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We shall be concerned with the well-known Gauss variational problem on the minimum of the energy in the presence of an external field, the infimum being taken over fairly general classes of signed Radon measures in a locally compact space. We shall show that in the noncompact case the problem is in general unsolvable, and this occurs even under extremely natural assumptions (in particular, for the Newtonian, Green, or Riesz kernels in an Euclidean space). Necessary and sufficient conditions for the problem to be solvable will be given. Some related extremal problems in the theory of capacities of condensers are also supposed to be discussed.

How the 2-capacity of a Space Condenser Can Be Written in Terms of the Newtonian Energies? A Solution to F. W. Gehring's Problem

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In $\mathbb{R}^n$, $n \geqslant 2$, let us consider a condenser, that is, a pair of closed disjoint sets A and B, A being compact. By T. Bagby, the 2-capacity of a plane condenser turns out to be reciprocal (up to a constant factor) with the infimum of the logarithmic energies over the class of all Borel measures ν such that ν^+ and ν^- are unit measures on A and B, respectively. F. W. Gehring asked whether this still holds for a space condenser, but with the Newtonian energies instead of the logarithmic ones. We have proved that the answer to F. W. Gehring's problem is, generally speaking, **no**, and obtained necessary and sufficient conditions for that conjecture to be valid. Besides, an actual description of the 2-capacity of an arbitrary space condenser in terms of the Newtonian energies has been given. Some related problems are also supposed to be discussed.

Divergence of rational approximants of non-analytic functions

Hans-Peter Blatt

We investigate the convergence behaviour of best uniform approximants $r^*_{n,m}$ with numerator degree n and denominator degree m to functions $f \in C[-1,1]$ which are not analytic on the interval $[-1,1]$.

Examples are $f(x) = |x|^\alpha$, $\alpha > 0$, for ray sequences in the lower triangle of the Walsh table, i. e. for sequences $\{(n, m(n))\}$ indices with

$$\frac{n}{m(n)} \longrightarrow c \in (1, \infty) \text{ for } n \longrightarrow \infty.$$

The results are compared with these examples where the asymptotic behaviour of the poles of $r^*_{n,m(n)}$ is known.

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Abstract: We investigate Bernstein type inequalities in L^p norms on special class of sets. This class consists of the finite unions of intervals and has nice potential theoretical properties. The L^1 inequality is as follows. Let $K \subset \mathbf{R}$ be a finite union of intervals and $\omega_K(t)$ is the density function of the equilibrium measure ν_K of K, $\omega_K(t) = \frac{d\nu_K(t)}{dt}$. Then we have

$$\int_K \Big| \frac{P_n'(t)}{n \cdot \pi \cdot \omega(t)} \Big|^\alpha d\nu_K(t) \leq (1 + o(1)) \cdot \int_K |P_n(t)|^\alpha d\nu_K(t) \qquad (1)$$

where P_n is any polynomial of degree n, $o(1)$ is an error term which tends to 0 as $n \to \infty$ and is independent of P_n. We achieve the best possible (linear) growth and constants and discuss the sharpness.