Abstracts of the talks

A generalization of the Leibniz rule

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We present a generalization of the Leibniz Rule. It is useful in the derivation of asymptotic expansions for sequences of approximation operators. As applications we obtain remarkable polynomial identities. The most prominent example is the Abel identity

$$\sum_{k=0}^{n} \binom{n}{k} a\,(a-kc)^{k-1}\,(b+kc)^{n-k} = (a+b)^n\,,$$

published in 1826, as well as its multivariate extension

$$\sum_{\substack{k_1,\cdots,k_r\in\mathbb{Z}^r_{\geq 0}\\ k_1+\cdots+k_r=n}} \binom{n}{k_1,\cdots,k_r} \left[\prod_{i=1}^{r-1}\left(a_i\,(a_i-k_i c)^{k_i-1}\right)\right](a_r+(n-k_r)\,c)^{k_r} = (a_1+\cdots+a_r)^n\,,$$

for $n=0,1,2,\ldots$ and all $\mathbf{a}=(a_1,\ldots,a_r)\in\mathbb{C}^r$, $c\in\mathbb{C}$. This deep generalization of the binomial formula follows as a direct consequence of our intriguing result.

Interval structures in real analysis. Contribution of Blagovest Sendov to the theory of interval functions

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The contribution of great scientists is measured by the impact of their work on future developments: research directions, new theories, new fields. The work of Blagovest Sendov on Interval Analysis can definitely be measured in such terms. Interval structures were first introduced by Sunaga. They got popularity after intervals were introduced in practical applications leading to the design of the so

called validated numerical method. The beginning of this development is associated with Ramon Moore who is also often credited as founder of Interval Analysis. While the main preoccupation of the researches was the numerical computations - algorithms, reliability, speed - Blagovest Sendov saw a different aspect in this new development, namely, the spaces of interval valued functions. This new direction gave a new understanding of Interval Analysis (probably closer to its name) as Analysis of Real Interval Valued Functions with associated concepts of order, limit, algebraic operations, calculus, etc. Naturally, as in all areas of Analysis, the spaces of functions with particular topological, order or algebraic structure are essential role players. The topic of Interval Computations and Interval Analysis was introduced in Bulgaria via a series of meetings and publications initiated by Blagovest Sendov. During this early time of development of the interval ideas in Bulgaria he also defined two of the most important functional spaces, namely the space of S-continuous interval functions and the space of H-continuous interval functions. These two spaces are essential tools in the Sendov's Theory of Hausdorff Approximations [4]. However, future developments showed their importance for other areas of Mathematics like Real Analysis [1] and the General Theory of PDEs [3] not to mention the Interval Analysis itself [2].

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Szegő–Calderon–Klein inequalities for fractional derivatives of trigonometric polynomials

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We will discuss sharp estimates of integral functionals for linear operators on the set $\mathbf{T}_n$ of real trigonometric polynomials f_n of order $n \geq 1$ by the uniform norm $\|f_n\|_{C_{2\pi}}$ of the polynomials and similar questions for algebraic polynomials on the unit circle of the complex plane. S. N. Bernstein, G. Szegö, P. Erdös, A. P. Calderon,

G. Klein, L. V. Taikov, A. I. Kozko, Q. I. Rahman, and many others investigated such inequalities (see the bibliography in [1]). In this talk, we, in particular, will show that, for $0 \leq q < 1$, the sharp inequality $\|D^\alpha f_n\|_{L_q} \leq n^\alpha \|\cos t\|_{L_q} \|f_n\|_\infty$ holds on the set $\mathbf{T}_n$ for the Weyl fractional derivatives $D^\alpha f_n$ of order $\alpha \geq 1$. For $q = \infty$ ($\alpha \geq 1$), this fact was proved by P. I. Lizorkin (1965). For $1 \leq q < \infty$ and positive integer α, the inequality was proved by L. V. Taikov (1965); however, in this case, the inequality follows from results of an earlier paper by A. P. Calderon and G. Klein (1951).

The results were obtained by the author jointly with P. Yu. Glazyrina [1].

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Best uniform approximation to a continuous function by piecewise-linear continuous functions with restrictions on the angles between adjacent segments

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We study the problem of the best uniform approximation to a continuous function on a closed interval by the class of piecewise-linear continuous functions with fixed knots and restrictions on angles between adjacent segments. An alternance criterion for best approximation is obtained.

This kind of approximation could be applied to the best navigation trajectory construction (see, e.g., [1]).

The similar problem for continuous function on an open interval with absolutely continuous derivative of the $(n-1)$-th order and bounded derivative of the order n was studied by A. V. Mironenko (see, e.g., [2]).

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On adaptivity in point-cloud approximation

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We consider non-parametric methods for high-dimensional approximation that consist of adaptive partitioning of the input space followed by fitting of the point cloud locally. The fitting procedure uses simple local models like piecewise constant or linear approximations. The idea of being content with piecewise constants is supported by classical concentration of measure results according to which a well-behaved function (e.g., Lipschitz-continuous) in very high dimensions deviates much from its mean or median only on sets of small measure. A typical example for such a recovery strategy is to determine for any given query point its k nearest neighbors in the given data and to use their average as the approximate function value. At first glance this kind of memory-based learning does not seem to require any training process except of reading and storing the incoming data. Unfortunately, the exact solution of the problem of finding the nearest neighbors requires either a preprocessing time which is exponential in the dimension d or a single query time which is linear in the number of points m. Actually, for function recovery purposes one would also be satisfied with an approximate solution that could be achieved much more efficiently. Therefore, in situations where a fast answer to a query matters alternative strategies may be preferable.

In view of these considerations we develop and investigate methods, which are primarily designed to be fast, to deal efficiently with large data sets, and to provide fast algorithms for evaluation. The motivation behind our approach is to explore the potential of multiresolution ideas for high spatial dimension ($d > 20$). For this purpose we propose new methods based on what we call *sparse occupancy trees* and piecewise linear schemes on adaptive subdivision over hyper-rectangular or simplex partitions. In order to ensure reliable approximation even in the presence of local singularities, we pay special attention to the question *how to adapt the number of degrees of freedom* used in the approximation *to the local behavior of the point cloud.*

Maximal properties of kernels for Vilenkin-like systems[1]

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Keywords and phrases: Walsh series, Vilenkin system, generalized Fourier series, Dirichlet and Fejér kernel functions.

The investigation of kernel functions is an important part of the Fourier analysis. The maximal values of the n-th Dirichlet and Fejér kernels for Walsh-Paley, Vilenkin and some other systems are n and $\frac{n-1}{2}$, respectively. In this talk we will deal with more general systems; in these cases the situation can be different. From these results we work out methods to assign the properties of concrete systems.

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The five point problem on the sphere as a weighted external field problem in the plane

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The arrangement of even a small number of points interacting according to a reciprocal s-power law (Riesz potential) so that they assume minimal Riesz energy is challenging from both the numerical and the theoretical point of view. For 2, 3, 4, 6 and 12 points rigorously proven minimizing configurations are known. These point

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systems are in fact all universally optimal with respect to the class of completely monotonic energy functionals that include the Riesz s-energy with $s > -2$ whereas the minimizing 5 point set is not (see Cohn and Kumar, J. Amer. Math. Soc. 20 (2007)). Only partial results are known for 5 points.

We recast the constraint optimization problem of arranging 5 points on the sphere that have minimal Riesz s-energy as an unconstrained rotationally symmetric external field optimization problem for 4 points in the (complex) plane. This provides a uniform approach to the potential-theoretical regime regarding the s parameter. Explicit expressions of involved potential functions in terms of special functions are obtained.

This research is based in part on joint work with R. Nerattini (Università di Firenze) and M. K.-H. Kiessling (Rutgers) and in part on joint work with P. Dragnev (IPFW) and Ed Saff (Vanderbilt).

Smooth convex resolution of unity on triangulated polygonal domains with Hermite interpolation at the triangulation vertices using GERBS

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In [1] a method was proposed for construction of C^m-smooth resolution of unity, $m = 2, 3, \dots$, including the C^∞-case, on a very general class of partitions of n-dimensional domains, which provided also the possibility to Hermite-interpolate at a scattered-point set consistent with the considered domain partition. This construction was based on radial generalized expo-rational B-splines (GERBS), the supports of which contained the respective elements of the domain partition. In [2], for the special case of triangulated polygonal two-dimensional domains, the method of [1] was enhanced in such a way that the C^m-smooth or C^∞-smooth resolution from [1] was upgraded to a resolution with the same smoothness, but minimally supported, i.e., the supports of the B-splines in the new resolution of unity were exactly the $\star_1$-neighbourhoods of the triangulation vertices. This enhanced construction allows Hermite interpolation in the vertices of the triangulation of up to transfinite order. The present work enhances further the construction of [2] in two important new aspects, as follows.

(1) Here we find all possible upgrades of the construction in [1]. Denoting by N the number of vertices in the triangulation, we show that there are up to $N! + 2$ possible (minimally supported in the above-said sense) upgrades of the construction in [1]. $N!$ of these upgrades depend on the ordering of the triangulation vertices, and the other two upgrades are order-independent.

The first one of the order-independent upgrades is the construction from [2]; the second one of these is obtained by arithmetic-means averaging over all order-dependent upgrades. While for general triangulations the second order-independent variant is clearly computationally inefficient (unlike the variant from [2] which is computationally cheap for any triangulation), in the presence of local symmetries in the triangulation the number of different order-dependent variants drastically decreases, and computing the second order-independent variant may become of interest. For example, in the case of uniform triangulation of the whole plane (in which case $N = +\infty$), the second order-independent variant is obtained by averaging over only 6 essentially different order-dependent cases, and is of considerable interest, e.g., in relevance to dyadic refinement schemes.

(2) In the present study we also show that if, in the C^m-smooth case with finite $m = 1, 2, 3, \dots$, the radial-based construction in [1] be replaced by its tensor-product variant considered in [3], and the underlying univariate GERBS in the constructions in [3, 2] is chosen to be a C^m-smooth GERBS of the type of Beta-function B-splines (BFBS), then the C^m-smooth convex resolution of unity constructed in [2] and in item 1 of the present list is *piecewise rational* everywhere on the triangulated domain.

In all considered cases, all variants of the construction obtained in [2] and in items 1 and 2 of the above list are rational forms. Due to this, introducing weights and a further NURBS-type rational upgrade is possible and meaningful in all considered cases. This new NURBS-type version has three essential advantages compared to classical NURBS on triangulations: (a) it can be infinitely smooth, (b) it can achieve up to transfinite order of Hermite interpolation at the triangulation vertices and, most important, (c) it is minimally supported on the respective $\star_1$-neighbourhoods of these vertices. In view of this, it has the potential of becoming a new, highly versatile, computational and modelling tool of Computer-Aided Geometric Design, as well as in Finite Element and Iso-geometric Analysis.

The construction can be efficiently extended to dimensions higher than 2, and in the concluding part of the presentation one of the currently known two possible ways to do this will be briefly discussed.

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On Hardy spaces of distributions in the framework of Dirichlet spaces

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We discuss Hardy spaces defined on Dirichlet spaces that have a Markovian heat kernel with small time Gaussian bounds and Hölder continuity. In particular, this covers the cases of Lie groups or homogeneous spaces with polynomial volume growth, complete Riemannian manifolds with Ricci curvature bounded from below and satisfying the volume doubling condition, and in various other nonclassical setups. Naturally, it covers the more classical cases on the sphere, interval, ball, and simplex with weights.

The talk is based on a joint work with Gerard Keryachairan, George Kyriazis and Pencho Petrushev.

Nikol'skii inequality for algebraic polynomials on a multidimensional Euclidean sphere and on a closed interval

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We study the sharp Nikol'skii inequality between the uniform norm and L_q norm of algebraic polynomials of a given (total) degree $n \geq 1$ on the unit sphere $\mathbb{S}^{m-1}$ of the Euclidean space $\mathbb{R}^m$ for $1 \leq q < \infty$. We prove that the polynomial ϱ_n in one variable with unit leading coefficient, that deviates least from zero in the space $L_q^{\psi}(-1,1)$ of functions f such that $|f|^q$ is summable on $(-1,1)$ with the Jacobi weight $\psi(t) = (1-t)^{\alpha}(1+t)^{\beta}$, $\alpha = (m-1)/2$, $\beta = (m-3)/2$, as a zonal polynomial in one variable $t = \xi_m$, $x = (\xi_1, \xi_2, \ldots, \xi_m) \in \mathbb{S}^{m-1}$, is (in a certain sense, unique) extremal in the Nikol'skii inequality on the sphere $\mathbb{S}^{m-1}$. The corresponding one-dimensional inequalities for algebraic polynomials on a closed interval are discussed. For $q = 1$, the results were proved in [1], except for the uniqueness of the polynomial.

The results were obtained by the author jointly with V. V. Arestov [2].

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Weighted uniform approximation by the Szász-Mirakjan operator

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We establish a sharp characterization of the error of the Szász-Mirakjan operator in weighted uniform norm in terms of a K-functional. We also define an unweighted fixed-step modulus of smoothness, which is equivalent to this K-functional. The weights we consider are of the form $w(x) = x^{\gamma_0}(x+1)^{\gamma_\infty - \gamma_0}$, where γ_0 satisfies the natural restrictions $-1 \le \gamma_0 \le 0$, and γ_∞ is an arbitrary real number.

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Keywords and phrases: Szász-Mirakjan operator, modulus of smoothness, K-functional, rate of convergence, weighted uniform approximation.

Mathematics Subject Classification 2010: Primary 41A25; Secondary 41A15, 41A27.

Optimal s-energy configurations under external field

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A configuration of points on the sphere that minimizes discrete Riesz s-energy in the presence of an external field Q is called Q-optimal s-configuration. We show that for $d-2 \leq s < d$ and under some general assumptions on Q such configurations are well-separated.

Joint work with J. S. Brauchart - UNSW and E. B. Saff - Vanderbilt.

Smooth orthogonal projections on sphere

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Keywords and phrases: smooth orthogonal projection, sphere, Parseval frame.

Mathematics Subject Classifiction 2010: 42C40.

We construct a decomposition of the identity operator on the sphere $\mathbb{S}^d$ as a sum of smooth orthogonal projections subordinate to an open cover of $\mathbb{S}^d$.

Convergence of dyadic wavelet expansions in $L^p(\mathbb{R}_+)$

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As usual, let $\mathbb{R}_+ = [0,\infty)$ be the positive half-line, $\{w_k \mid k \in \mathbb{Z}_+\}$ be the Walsh system on $\mathbb{R}_+$, $\widehat{f}$ be the Walsh-Fourier transform of $f \in L^2(\mathbb{R}_+)$, $\|\cdot\|_p$ be the norm in $L^p(\mathbb{R}_+)$, and let $\oplus$ denote the dyadic addition on $\mathbb{R}_+$ (see [1]). Suppose that φ is the scaling function for a multiresolution analysis $\{V_j\}$ in $L^2(\mathbb{R}_+)$ such that $\widehat{\varphi}(0) = 1$ and φ satisfies the refinement equation

$$\varphi(x) = \sum_{k=0}^{2^n-1} c_k \varphi(2x \oplus k), \quad x \in \mathbb{R}_+.$$

Then, for each $j \in \mathbb{Z}$, the functions

$$\varphi_{jk}(x) = 2^{j/2}\, \varphi(2^j x \oplus k), \quad k \in \mathbb{Z}_+,$$

form an orthonormal basis of V_j, the union $\bigcup V_j$ is dense in $L^2(\mathbb{R}_+)$ and $V_j \subset V_{j+1}$ for all $j \in \mathbb{Z}$ (cf. [2],[3]). Let us denote by $P_j f$ the projection of a function f on the subspace V_j. The L^p dyadic modulus of continuity of f is defined by

$$\omega_p(f;\delta) := \sup_{0<h<\delta} \|f(\cdot \oplus h) - f(\cdot)\|_p.$$

We prove that if $1 \le p < \infty$, then there exists a constant C_p such that for all $f \in L^p(\mathbb{R}_+)$,

$$\|f - P_j f\|_p \le C_p\, \omega_p(f; 2^{-j}) \tag{1}$$

(in particular, $\|f - P_j f\|_p \to 0$ when $j \to \infty$).

For any $\alpha > 0$, let

$$\mathrm{Lip}^{(p)}(\alpha) := \{f \in L^p(\mathbb{R}_+) \mid \omega_p(f;\delta) = O(\delta^\alpha) \quad \text{as} \quad \delta \to 0\}.$$

From (1) we see that if $f \in \mathrm{Lip}^{(p)}(\alpha)$, then $\|f - P_j f\|_p = O(2^{-j\alpha})$ when $j \to \infty$. As a converse result, we prove the following

Theorem (cf. [4, p.351]). *Let $f \in L^p(\mathbb{R}_+)$, $1 \le p < \infty$ and $0 < \alpha < 1$. Suppose that the scaling function φ satisfies the Lipschitz condition*

$$|\,\varphi(x \oplus h) - \varphi(x)| \le Ch, \quad x, h \in \mathbb{R}_+,$$

and, for all $j \in \mathbb{Z}_+$, we have $\|f - P_j f\|_p \le C 2^{-j\alpha}$. Then $f \in \mathrm{Lip}^{(p)}(\alpha)$.

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Subsequences of partial sums of one and two-dimensional Walsh-Fourier series

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Mathematics Subject Classifiction 2010: 42C10.

Let x be an element of the unit interval $I := [0, 1)$. The $\mathbb{N} \ni n$th Walsh function is

$$\omega_n(x) := (-1)^{\sum_{k=0}^{\infty} n_k x_k} \quad (n = \sum_{k=0}^{\infty} k_i 2^i, \ x = \sum_{k=0}^{\infty} \tfrac{x_i}{2^{i+1}}).$$

The Walsh-Fourier coefficients, the n-th partial sum of the Fourier series, the n-th $(C, 1)$ mean of $f \in L^1(I)$:

$$\hat{f}(n) := \int_I f(x)\omega_n(x)dx, \quad S_n f := \sum_{k=0}^{n-1} \hat{f}(k)\omega_k, \quad \sigma_n f := \frac{1}{n}\sum_{k=0}^{n-1} S_k f.$$

It is of prior interest that how to reconstruct a function from the partial sums of its Walsh-Fourier series. Fine proved [1] that for each integrable function in L^1 the almost everywhere convergence of Fejér means $\sigma_n f \to f$. An old question of Zalcwasser [4] that what happens if we have only a subsequence of the partial sums. That is, how "rare" can $a(n)$ be such that

$$\frac{1}{N}\sum_{n=1}^{N} S_{a(n)} f \to f.$$

In the talk we give a brief résumé of the recent results (see e.g. [3]) not only for one dimensional functions, but also for two variable ones. We also talk about the convergence properties of one and two-dimensional (C, α), the logarithmic, Marczinkiewicz

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means and their generalizations (see e.g. [2]) with respect to this Zalcwasser's issue above, with respect to Walsh-Fourier series.

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Bernstein–Szegő inequality for trigonometric polynomials in the spaces L_p, $0 \le p < 1$

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Let T_n be the set of triginometric polynomials

$$f_n(t) = a_0 + \sum_{k=1}^{n} (a_k \cos kt + b_k \sin kt), \quad a_k, b_k \in \mathbb{R},$$

and $D^\alpha_\beta f_n$ be the Weyl fractional derivative of order $\alpha \in \mathbb{R}$ with shift β of a polynomial f_n,

$$D^\alpha_\beta f_n(t) = \sum_{k=1}^{n} k^\alpha \big(a_k \cos(kt + \alpha\pi/2 + \beta) + b_k \sin(kt + \alpha\pi/2 + \beta) \big).$$

Denote by $C^\alpha_\beta(n,p)$ the best (i.e., the least possible) constant in the inequality

$$\left\| D^\alpha_\beta f_n \right\|_p \le C^\alpha_\beta(n,p) \|f_n\|_p, \qquad f_n \in \mathrm{T}_n, \tag{1}$$

where

$$\|f_n\|_q = \left(\frac{1}{2\pi} \int_0^{2\pi} |f_n(t)|^q dt \right)^{1/q}, \quad 0 < q < \infty,$$

$$\|f_n\|_\infty = \|f_n\|_{C_{2\pi}} = \lim_{q \to +\infty} \|f_n\|_q = \max\{|f_n(t)| : \ t \in \mathbb{R}\},$$

$$\|f_n\|_0 = \lim_{q \to 0+} \|f_n\|_q = \exp\left(\frac{1}{2\pi} \int_0^{2\pi} \ln |f_n(t)| dt \right).$$

It is known that $C^\alpha_\beta(n,p) = n^\alpha$ (i.e., $C^\alpha_\beta(n,p)$ is independent of β and p) in the following cases:

1) $1 \le p \le \infty$, $\alpha \in \mathbb{R}$, $\alpha \ge 1$, and $\beta \in \mathbb{R}$;

2) $0 \le p < 1$, $\alpha \in \mathbb{N}$, and $\beta = 0$;

3) $0 \le p < 1$, $\alpha \in \mathbb{N}$, $\alpha \ge \ln 2n/\ln(n/(n-1))$, and $\beta = \pm\pi/2$.
In these cases, inequality (1) turns into an equality only for the polynomials $A\cos(nt+a)$, $A, a \in \mathbb{R}$.

Inequality (1) was obtained by Bernstein [3] for $p = \infty$, $\alpha \in \mathbb{N}$, and $\beta = 0$; by Szegő [6] for $p = \infty$, $\alpha \in \mathbb{N}$, and $\beta \in \mathbb{R}$, by Zygmund [7, 8] for $1 \le p < \infty$, $\alpha \in \mathbb{N}$, and $\beta \in \mathbb{R}$; by Lizorkin [5] for $1 \le p \le \infty$, $\alpha \in \mathbb{R}$, and $\beta = 0$; by Kozko [4] for $\beta \in \mathbb{R}$; by Arestov [1, 2] for $0 \le p < 1$.

In the case $0 \le p < 1$, we prove that $C^\alpha_\beta(n,p) = n^\alpha$ for $\beta \in \mathbb{R}$ and all real $\alpha \ge \ln 2n/\ln(n/(n-1))$. This result was obtained by the author jointly with V. V. Arestov.

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Strong summability of quadratic partial sums of double Fourier series

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In my talk I will discuss some strong convergence and divergence properties of the logarithmic means of quadratic partial sums of double Fourier series of function in the measure, almost everywhere and in the Lebesgue norm.

On "two-sided Hölder" Green functions

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A. Volberg posed a problem about the existence of a compact set on the line with the corresponding "two-sided Hölder" Green function. This means that, given a point near the compact set, the Green function at this point has lower and upper estimates of Hölder type with the same exponent for all points. A solution to this problem is suggested by means of Cantor-type sets that are intersections of the level domains for a simple sequence of polynomials.

It is possible also to present Green functions that are "two-sided" with respect to other moduli of continuity.

Infinite products of positive linear operators

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Infinite products of positive linear operators reproducing linear functions are considered from a quantitative point of view. Refining and generalizing convergence theorems of Gwozdz-Lukawska, Jachymski, Gavrea, Ivan, among others, it is shown that infinite products of such operators, all taken from a finite set of mappings reproducing linear functions, converge to the first Bernstein operator (i.e., linear interpolation at 0 and 1). A discussion of products of Meyer- Konig and Zeller operators is included. The talk is based upon joint work with Ioan Rasa (UT Cluj-Napoca).

Keywords: infinite operator products, positive linear operators, degree of approximation, second order modulus of smoothness, Bernstein operators, Meyer-Konig and Zeller operators.

Mathematics Subject Classification (2010): 41A17, 41A25, 41A36.

n-term approximation and Besov regularity for elliptic PDEs on polyhedral domains

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We investigate the Besov regularity for solutions of elliptic PDEs on polyhedral domains. This is based on regularity results in weighted Sobolev spaces (Babuska-Kondratiev spaces) $\mathcal{K}_a^m(D)$. Following the argument of Dahlke and DeVore, we first prove an embedding of these spaces into the scale $B_{\tau,\tau}^r(D)$ of Besov spaces with $\frac{1}{\tau} = \frac{r}{d} + \frac{1}{2}$. The latter scale is known to be closely related to n-term approximation w.r.t. wavelet systems, and recently this has been extended to adaptive Finite Element approximation. This ultimately yields the rate $n^{-r/d}$ for $u \in \mathcal{K}_a^m(D) \cap H^s(D)$ for $r < r^* \leq m$.

In order to obtain the full rate $n^{-m/d}$ we subsequently leave the scale $B_{\tau,\tau}^r(D)$ and instead consider the spaces $B_{p,\infty}^m(D)$. We show that under appropriate conditions we now have an embedding $\mathcal{K}_a^m(D) \cap H^s(D) \hookrightarrow B_{p,\infty}^m(D)$ for some $0 < p \leq p_0 \leq 2$, $\frac{1}{p_0} = \frac{m}{d} + \frac{1}{2}$, which in turn yields the desired approximation rate.

Generalized normal multi-scale transforms for curves

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Normal multi-scale transforms (MTs) [1] allow for efficient computational processing of densily sampled (or analytically given) geometric objects of co-dimension one. In the planar setting, given a curve $\mathcal{C}$ in $\mathbb{R}^2$, an initial sequence of vertices $\mathbf{v}^0 \subset \mathcal{C}$ (creating a polygonal line interpolating $\mathcal{C}$), and a linear subdivision operator S, a normal MT produces denser vertex sets $\mathbf{v}^j \subset \mathcal{C}$ and polygonal lines associated with them according to

$$\mathbf{v}^j = S\mathbf{v}^{j-1} + d^j \mathbf{n}^j, \qquad j \in \mathbb{N}.$$

Here $\mathbf{n}^j$ is a set of unit "normals" that can be computed solely from $\mathbf{v}^{j-1}$, and d^j is the scalar "detail" sequence of the signed distances between the prediction points $S\mathbf{v}^{j-1}$ and the corresponding imputation points $\mathbf{v}^j$. Normal MTs based on interpolating and approximating operators S have been analyzed in [2] and [3], respectively.

For smooth $\mathcal{C}$, the general theory provides two different restrictions on the detail decay rate of the associated normal MT: the regularity of $\mathbf{v}^j$, and the polynomial exactness order of the prediction operator. Both properties depend on S, and for fixed size of the mask there is a tradeoff between them. However, the first restriction deals with *parameter information* and is affected by tangential displacements of the data, while the second one deals with *geometry information* and is affected by normal displacements of the data. In this talk we introduce *combined* prediction rules (S, T), with different subdivision operators S and T used for tangential and normal directions, respectively. In particular, for $\mathcal{C} \in C^4$ we construct a new 4-point scheme that leads to local well-posedness, $C^{2,1}$ normal re-parameterization of $\mathcal{C}$, and detail decay order 4 of its associated normal MT. We also consider a "geometrical" extension of this 4-point scheme, based on circle interpolation, that again leads to detail decay order 4 of the associated normal MT.

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Some results for Szász-Mirakjan type operators

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We consider different modifications of the classical Szász-Mirakjan operators

$$S_n(f,x) \quad = \quad \sum_{k=0}^{\infty} s_{n,k}(x) f\left(\frac{k}{n}\right), \quad s_{n,k}(x) = \frac{(nx)^k}{k!} e^{-nx},\, x \in [0,\infty),$$

such as the genuine Szász-Mirakjan-Durrmeyer operators

$$\widetilde{S}_n(f,x) = e^{-nx} f(0) + n \sum_{k=1}^{\infty} s_{n,k}(x) \int_0^{\infty} s_{n,k-1}(t) f(t) dt,$$

and the Durrmeyer type modification

$$\overline{S}_n(f,x) = n \sum_{k=0}^{\infty} s_{n,k}(x) \int_0^{\infty} s_{n,k-1}(t) f(t) dt.$$

We discuss different aspects of their approximation behavior also in regard to linear combinations and quasiinterpolants.

Cubature rules for harmonic functions based on Radon projections

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We construct a class of cubature formulae for harmonic functions on the unit disk based on line integrals over $2n+1$ distinct chords. These chords are assumed to have constant distance t to the center of the disk, and their angles to be equispaced over the interval $[0, 2\pi]$. If t is chosen properly, these formulae integrate exactly all harmonic polynomials of degree up to $4n+1$, which is the highest achievable degree of precision for this class of cubature formulae. For more generally distributed chords, we introduce a class of interpolatory cubature formulae which we show to coincide with the previous formulae for the equispaced case. We give an error estimate for a particular cubature rule from this class.

Recent advances in spherical needlet approximations

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Effective methods for solving two related problems on the unit 2-d sphere are presented. The first problem is for evaluation of high degree spherical polynomials (band-limited functions) at many scattered points on the sphere, and the second is for reconstruction of spherical polynomials on the sphere from their values at irregular sampling points.

Our methods rely on the sub-exponential localization of the spherical father needlets and their compatibility with spherical harmonics. They are fast, local, memory efficient, numerically stable and with guaranteed (prescribed) accuracy of up to 10^{-11} for polynomials of degree 5000.

Software realization of the algorithms and numerical experiments are also presented. Targeted applications of these algorithms are to Geopotential Modeling and, in particular, to fast and memory efficient evaluation of the Geoid Undulation.

Analysis on convex subset of a Riemannian manifold and classical polynomial approximation

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It is a classical topic to look for orthonormal basis of polynomials on a compact set X of R^d, with respect to some Radon measure μ. For example: the one dimensional interval (Jacobi), the unit sphere (Spherical harmonics), the ball and the simplex (work of Petrushev, Xu, ...) In this framework, one can be interested in the best approximation of functions by polynomials of fixed degree, in $L_p(\mu)$, and to built a

suitable frame for characterization of function spaces related to this approximation. These constructions have been carried using special functions estimates.

We will be interested in spaces where the polynomials give the spectral spaces of some positive selfadjoint operator. Under suitable conditions, a "natural" metric ρ could be defined on X such that (X, ρ, μ) is a homogeneous space, and if the associated semi-group has a good "Gaussian" behavior, then we could apply the procedure developed in recent work by P. Petrushev, T. Coulhon and George Kyriazis, to built such frames, and such function spaces.

Actually we will show that analysis on "convex" open set in a Riemannian manifold can help to understand these classical topics.

Hardy-Petrovitch-Hutchinson's problem and partial theta function

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A real polynomial $p(x) = 1 + a_1 x + \cdots + a_n x^n$ is *hyperbolic* if it has all roots real. Consider polynomials p with positive coefficients. Such a polynomial is *section-hyperbolic* if all its sections (i.e. truncations) $1 + a_1 x + \cdots + a_i x^i$, $i = 1, \ldots, n$, are hyperbolic. Denote by Δ_n the set of all such degree n section-hyperbolic polynomials and set $\Delta := \cup_{n=1}^{\infty} \Delta_n$. Set $m_i := \inf_{p \in \Delta} a_i^2 / a_{i-1} a_{i+1}$. The series $\Theta(q, x) := \sum_{j=0}^{\infty} q^{j(j+1)/2} x^j$ defines a *partial theta function.* For each $q \in [0, 1)$ fixed this is an entire function in x having infinitely real negative zeros.

Theorem 1 *(1) The sequence $\{m_i\}$ is strictly decreasing and tending to $1/\tilde{q} = 3.2336\ldots$, where $\Theta(q, .)$ belongs to the Laguerre-Pólya class $\mathcal{L} - \mathcal{PI}$ exactly if $q \in (0, \tilde{q})$.*

(2) The number $\tilde{q} = 0.3092493386\ldots$ is the solution belonging to $(0, 1)$ of the equation $1/x - 2/(1 - x) = \sum_{j=1}^{\infty} x^j / (1 - x^{j+1})$.

(3) There exists a sequence $\tilde{q} = \tilde{q}_1 < \tilde{q}_2 < \cdots < 1$ tending to 1 such that $\Theta(\tilde{q}_j, .)$ has a single real double zero y_j which is the rightmost of its real zeros (all the rest of them being simple). For $q \in (\tilde{q}_j, \tilde{q}_{j+1})$ the function $\Theta(q, .)$ has exactly j conjugate pairs of zeros counted with multiplicity. For $q \neq \tilde{q}_i$, $i = 1, 2, \ldots$, all real zeros of $\Theta(q, .)$ are simple.

(4) One has $\tilde{q}_j = 1 - (\pi/2j) + o(1/j)$ and $\lim_{j \to \infty} y_j = -e^{\pi} = -23.1407\ldots$.

Padé approximants of functions of Markov's type

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Given be a positive Borel measure σ with an infinite support $S \subset \Re$, set

$$\widetilde{\sigma}(z) := \int \frac{d\sigma(x)}{z-x},\ z \in \overline{\mathbb{C}} \setminus S.$$

Let $\pi_n = [n/n]_\sigma, n = 1, 2, \cdots$ be the diagonal Padé approximants of f at infinity. As known, the denominators of $[n/n]_\sigma$, say Q_n, $n = 0, 1, 2, \cdots$ are orthogonal with respect to the measure σ.

By the classical theorem of A. A. Markov,

$$[n/n]_\sigma \rightrightarrows \widetilde{\sigma}$$

uniformly on compact subsets of $\overline{\mathbb{C}} \setminus [\alpha, \beta]$, $[\alpha, \beta]$ - the convex hull of S.

The theorem of Markov is the background of the present talk.

Suppose that the support S of σ consists of a finite number of (disjoint) intervals E_j, $j = 1, \cdots, g+1$, $g \geq 0$, $E_j := [e_{2j-1}, e_{2j}]$ and set $h(x) = \prod(x - e_k)$. Suppose further that σ is absolutely continuous with respect to Lebesque measure on S and

$$\frac{\sigma(x)}{dx} = \frac{1}{\pi} \frac{\rho(x)}{\sqrt{-h(x+i0)}} > 0 \text{ on S},$$

$\rho-$ a trigonometric weight function, $\sqrt{h(z)}/z^{g+1} \to 1$ as $z \to \infty$.

In the present talk, results about the strong asymptotic of the free poles of the diagonal Padé approximants of the function

$$f(z) = \tilde{\sigma}(z) + r(z)$$

will be provided, where r is a rational function with no zeros on the convex hull of S, $r(\infty) = 0$ and $r \subset \Re(z)$.

The results are due to A. A. Gonchar, Rakhmanov, Suetin and the author.

MRA-based wavelets with symmetry[3]

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Mixed Extension Principle is a well known general scheme for the construction of dual wavelet frames. However, generally speaking, this scheme leads to MRA-based dual wavelet systems $\{\psi_{ik}^{(\nu)}\}$, $\{\widetilde{\psi}_{ik}^{(\nu)}\}$ which are not necessary frames in $L_2(R^d)$ and may even consist of tempered distributions. Some properties of these systems such as frame-type expansion (with convergence in different senses) and their approximation order were investigated in [1]. Such wavelet systems were called frame-like.

A symmetry is one the most desirable properties for wavelet systems in applications. For an arbitrary symmetry group H, an appropriate matrix dilation and any integer n we give explicit formulas for refinable masks that are H-symmetric and have sum rule of order n. Under some additional conditions on the symmetry group H, an explicit construction of wavelet masks that have the H-symmetry property and generate a frame-like wavelet system providing approximation order n is presented for each H-symmetric refinable mask.

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On the construction of bases and frames for general distribution spaces

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Methods are developed for construction of bases and frames for general distribution spaces. These methods allow the freedom to prescribe the nature and properties of the basis or frame elements. Two particular applications will be considered with the first being the construction of a basis consisting of rational functions of uniformly bounded degrees for Besov and Triebel-Lizorkin spaces of holomorphic functions in the unit disc. As a second application we discuss the construction of compactly

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supported frames for Besov and Triebel-Lizorkin spaces in the general framework of Dirichlet spaces.

The talk is based on a joint work with S. Dekel, G. Kerkyacharian and P. Petrushev.

Localization of dyadic functions[4]

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We introduce a notion of localization for dyadic functions $f \in L_2([0, \infty))$. It is characterized by a functional UC similar to the uncertainty constant used for real-line functions $f \in L_2(\mathbb{R})$.

Let $x = \sum_{n\in\mathbb{Z}} x_j 2^{-j-1}$ be a dyadic expansion of $x \in [0, \infty)$, where $x_j \in \{0, 1\}$. For $x = p2^n$, $p \in \mathbb{N}$, $n \in \mathbb{Z}$ we chose $x_j \to 0$ as $j \to \infty$. The dyadic sum of x and y is defined by $x \oplus y := \sum_{n\in\mathbb{Z}} |x_j - y_j| 2^{-j-1}$. Then $[0, \infty)$ is metrizable with the distance between x, y defined to be $x \oplus y$.

Definition. Let $\tilde{f}$ be the Walsh-Fourier transform. The functional

$$UC(f) := \frac{\Delta_f \Delta_{\tilde{f}}}{p_{f,\tilde{f}}}$$

is called *the dyadic uncertainty constant*, where

$$x_0 := \frac{1}{\|f\|^2} \int_0^\infty x|f(x)|^2\,dx, \quad \Delta_f^2 = \frac{1}{\|f\|^2} \int_0^\infty (x \oplus x_0)^2 |f(x)|^2\,dx,$$

$$\omega_0 := \frac{1}{\|\tilde{f}\|^2} \int_0^\infty \omega|\tilde{f}(\omega)|^2\,d\omega, \quad \Delta_{\tilde{f}}^2 = \frac{1}{\|\tilde{f}\|^2} \int_0^\infty (\omega \oplus \omega_0)^2 |\tilde{f}(\omega)|^2\,d\omega,$$

$$p_{f,\tilde{f}} := \frac{1}{\|\tilde{f}\|\|f\|} \left| \int_0^\infty (x \oplus x_0)(x \oplus \omega_0) f(x) \overline{\tilde{f}(x)}\,dx \right|. \square$$

The dyadic uncertainty principle takes the form $UC(f) \geq 1$ for $f \in L_2([0, \infty))$. Some examples of UC's for dyadic scaling functions and wavelets will be given.

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Functions of bounded Λ-variation and integral smoothness

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We obtain the necessary and sufficient condition for embeddings of integral Lipschitz classes $\mathrm{Lip}(\alpha; p)$ into spaces ΛBV of functions of bounded Λ-variation. This answers a question of Wang [2].

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Semidefiite extreme points in a polynomial space

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Let Δ be the triangle in $\mathbb{R}^2$ bounded by the lines $x = 0$, $y = 0$, $x + y = 1$ (the standard simplex in $\mathbb{R}^2$). Denote by π_2 the set of all real bivariate algebraic polynomials of total degree at most two. Let B_Δ be the unit ball of the space π_2 endowed with the supremum norm on Δ.

We describe the semidefinite extreme points of B_Δ. This completes the description of the set E_Δ of all extreme points of B_Δ started by L. Milev and N. Naidenov in two papers, concerning strictly definite and indefinite elements of E_Δ.

Sharp Bernstein inequality in integral norms

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The classical Bernstein inequality estimates the derivative of a polynomial at a fixed point with the supremum norm and a factor depending on the point only. Recently, it was generalized to large class of compact sets on the real line. That generalization uses potential theorety. It also gives a hint how a sharp L^α Bernstein inequality should look like.

In this talk we present this inequality, briefly sketch its proof and we also discuss its sharpness from various points of view.

This is a joint work with Ferenc Toókos.

Restricted summability of the two-dimensional Walsh- and Walsh-Kaczmarz-Fejér means

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We discuss the almost everywhere convergence of the two-dimensional Fejér means of Walsh- and Walsh-Kaczmarz-Fourier series, where the set of the indices is inside a cone-like set [2, 3]. Moreover, we investigate the properties of the maximal operator of two-dimensional Walsh-Fejér means [7] and Walsh-Kaczmarz-Fejér means [3], with cone-like restriction. We treat that the maximal operator σ_L^* is bounded from H_p to L_p for $1/2 < p$ and is of weak type $(1, 1)$. As special cases we have the theorem of Weisz [6], Gát [1] and Simon [5] on the a.e. convergence of cone-restricted two-dimensional Fejér means of integrable functions. The endpoint case $p = 1/2$ is investigated, too [3, 4].

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On the path-connectedness of the set of extreme points in a polynomial space

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Let Δ be the triangle in $\mathbb{R}^2$ bounded by the lines $x = 0$, $y = 0$, $x + y = 1$ (the standard simplex in $\mathbb{R}^2$). Denote by π_2 the set of all real bivariate algebraic polynomials of total degree at most two. Let B_Δ be the unit ball of the space π_2 endowed with the supremum norm on Δ.

Recently L. Milev and N. Naidenov completed the description of the set E_Δ of all extreme points of B_Δ. We present here a result concerning the path-connectedness of E_Δ.

Some new results on approximation with redundant dictionaries

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Data approximation using sparse linear expansions from overcomplete dictionaries has become a central theme in signal and image processing with applications ranging from data acquisition (compressed sensing) to denoising and compression.

For a given dictionary, we can also study best m-term approximation rates for any specific function. Interestingly, the notions of sparse expansions and certain asymptotic approximation rates are closely linked in the case of nice non-redundant dictionaries (e.g., an orthonormal basis in a Hilbert space.)

In this talk, I will explore the link between sparse expansions from an overcomplete dictionary and asymptotic approximation rates. Redundancy complicates the analysis, and we show that the close link between the two notions fails in general. However, using a probabilistic approach, we show that the close link is retained for 'many' redundant dictionaries.

An extension of Turán's inequality for ultraspherical polynomials

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The m-th ultraspherical polynomial $P_m^{(\lambda)}$, $m \in \mathbb{N}$, is orthogonal in $[-1,1]$ with respect to the weight function $w_\lambda(x) = (1-x^2)^{\lambda-1/2}$, $\lambda > -1/2$, and is normalized by $P_m^{(\lambda)}(1) = \binom{m+2\lambda-1}{m}$. Set

$$p_m(x) = p_m^{(\lambda)}(x) := P_m^{(\lambda)}(x)/P_m^{(\lambda)}(1), \quad m = 0, 1, \dots, \tag{1}$$

where, for the sake of brevity, the superscript $^{(\lambda)}$ will be oppressed hereafter. We prove the following extension of Turán's inequality:

Theorem. *Let p_n be defined by (1), and $\lambda \in (-1/2, 1/2]$. Then, for every $n \in \mathbb{N}$,*

$$|x| p_n^2(x) - p_{n-1}(x) p_{n+1}(x) \ge 0 \quad \textit{for every} \;\; x \in [-1,1]. \tag{2}$$

The equality in (2) holds only for $x = \pm 1$ and, if n is even, for $x = 0$. Moreover, (2) fails for every $\lambda > 1/2$ and $n \in \mathbb{N}$.

This variation of Turán's inequality was introduced by Gerhold and Kauers [1] and proven in the limit case $\lambda = 1/2$, i.e., for the Legendre polynomials. We present both analytical and computer proof of this result. An extension to Jacobi polynomials is discussed.

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Properties of some functionals associated with h-convex and quasilinear functions

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We consider quasilinearity of the functional $(h \circ v) \cdot \left(\Phi \circ \frac{g}{v}\right)$ where Φ is a monotone h-convex (h-concave) function, v and g are functionals with certain super(sub)additivity properties. These general results are applied on some special functionals such as Jensen's, Jensen's-Mercer and Chebyshev's functionals.

Stable space splittings and their applications

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Decomposition of linear spaces into simple building blocks is at the heart of classical Fourier Analysis and Approximation Theory, and has numerous theoretical and applied aspects. In the Hilbert space case, there is a simple yet powerful theory of splitting a large space into many small spaces, possibly in a redundant manner but controlled by a stability condition that generalizes the notion of a frame. In this talk, I review some old and new aspects of stable space splittings, mostly related to solving variational problems. The starting point is a much cited paper from CTF Varna 1991, the newer developments concern the theory of quarkonial frames and randomized solvers for linear systems.

I will conclude with formulating an unsolved problem on trigonometric approximation of functions with values in matrix Lie-groups such as $SU(N)$ or $SO(N)$.

Weighted approximation by a class of Baskakov-type operators

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The class of Baskakov-type operators discussed here are given for natural n by

$$\tilde{B}_n(f,x) = \sum_{k=0}^{n} b_{n,k}(f) P_{n,k}(x),$$

$P_{n,k}(x) = \binom{n+k-1}{k} x^k (1+x)^{-n-k}$, $k \in \mathbb{N}_0 = \mathbb{N} \cup \{0\}$, denote the Baskakov basic functions and the functionals $b_{n,k}(f)$ satisfy the following conditions

$$b_{n,0}(f) = f(0);$$
$$b_{n,k}(f) \text{ are linear and positive;}$$
$$\tilde{B}_n(E_i,x) = E_i(x) \text{ for i=0 and i=1;}$$
$$\tilde{B}_n(E_2,x) = E_2(x) + \alpha(n)x.$$

Here E_i (for $i = 0, 1, 2$) are the functions $E_i(x) = \frac{x^i}{1+x}$ and $\lim_{n\to\infty} \alpha(n) = 0$.

The functional $b_{n,k}(f)$ for $1 \le k \le \infty$ in the operators $\tilde{B}_n$ takes place of $f\left(\frac{k}{n}\right)$ in the classical Baskakov operators.

Direct theorem in terms of the weighted K-functional for the uniform weighted approximation errors of these operators are obtained for functions from $C(w)[0,\infty]$ with weight of the form $\left(\frac{x}{1+x}\right)^{\beta_0} \left(\frac{1}{1+x}\right)^{\beta_\infty}$ for $\beta_0, \beta_\infty \in [-1, 0]$.

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Finding the minimum of a function

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We discuss strategies for finding an approximation to the minimum value of a continuous function f defined on $[0,1]^d$. We assume that we are able to query f by asking for its value at any point. The algorithms we study give an adaptive procedure to determine the points to query f. We estimate the rate of convergence of these algorithms under various model assumptions on the function f.

Sub-exponentially localized frames for spaces of distributions in the general setting of Dirichlet spaces

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The general setting of a Dirichlet space with a doubling measure and local scale-invariant Poincare inequality will be introduced and the construction of frames with band limited elements of sub-exponential space localization will be presented. These frames will be used for characterization of Besov and Triebel-Lizorkin spaces with complete range of indices in the framework of Dirichlet spaces. Nonlinear n-term approximation from the frames will be considered. This theory, in particular, allows developing sub-exponentially localized frames in the context of Lie groups, Riemannian manifolds, and other nonclassical settings.

The talk is based on a joint work with Gerard Kerkyacharian.

Optimal polynomial admissible meshes for the closure of bounded $\mathscr{C}^{1,1}$ domains of $\mathbb{R}^d$

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Admissible Meshes (**AM**) [1] are sequences $\{A_n\}_{n\in\mathbb{N}}$ of finite subsets of a given compact $K \subset \mathbb{R}^d$ (or $\mathbb{C}^d$) such that the following inequality holds for any polynomial $p \in \mathscr{P}^n(\mathbb{R}^d)$ of degree at most n

$$\|p\|_{\mathscr{C}(K)} \leq C \max_{A_n} |p| \tag{1}$$

and moreover $\mathrm{Card}(A_n)$ grows polynomially w.r.t. n.

AM was introduced in [1] as suitable sets where perform the sampling for the *discrete least square* polynomial approximation. In [2] A. Kroó defines **Optimal AMs** as AMs which cardinality grows at optimal rate $\mathcal{O}(n^d)$. He also provides an existence result for star-like smooth compact sets in $\mathbb{R}^d$.

In this talk we present our recent result [5][Th. 3.7]. *If* $\Omega \subset \mathbb{R}^d$ *is a bounded* $\mathscr{C}^{1,1}$ *domain, then* $\overline{\Omega}$ *has an Optimal Admissible Mesh.* The proof is fully constructive and relays on the regularity property of the *distance function* w.r.t. $\complement\Omega$ instead of the star shape of the set.

The main features of AMs are also discussed, both as theoretical motivations and applications. Namely, starting from an AM, one can extract by standard Linear Algebra quasi-optimal interpolation arrays, say *Approximate Fekete* and *Leja points* [3]. Moreover the sequence of discrete probability measures $\{\mu_n\}_{\mathbb{N}}$ canonically associated to an AM can be used to compute the *Pluripotential Equilibrium Measure* of the given compact and the *Siciak Extremal Function*. Finally given an holomorphic function f the sequence of $L^2_{\mu_n}$-least squares polynomials p_n is *maximally convergent* to f on K (see [4]).

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On some estimates of singular number of integral operator of Hilbert-Schmidt

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Let be A an integral operator of Hilbert-Schmidt with kernel $a(t,s)$ acting in the space $L_2[0;1]$ and let $s_k(A)$ its singular values. We establish that for all $r = 2, 3, \ldots$ the estimate

$$\sum_{k=r}^{\infty} s_k^2(A) \leq 2\omega^2 \left(\frac{1}{r-1}, a\right)_2$$

holds. Here $\omega(\delta, a)_2$ is the modulus of continuity of the kernel $a(t,s)$ given by the formulas

$$\omega(\delta, a)_2 := \sup_{0 \leq \ h \leq \delta} \left(\int_0^1 \int_0^{1-h} |a(t+h, s) - a(t,s)|^2 \, dt \, ds \right)^{\frac{1}{2}}, \quad 0 < \delta \leq 1$$

Also we have established similar estimates in terms of the modulus of continuity more then one order in the case, when domain of image of the operator A belongs to the space of continuity functions.

For all $m = 2, 3, \ldots$ the estimate

$$\text{(1)} \qquad \sum_{k=r}^{\infty} s_k^2(A) \leq 25\omega_m^2 \left(\frac{2}{r}, a\right), \quad r = m+1, m+2, \ldots$$

holds. Here $\omega_m(\delta, a)$ is the modulus of continuity of the kernel $a(t,s)$ given by the formulas

$$\omega_m(\delta, a) := \sup_{f \in L_2, \|f\|_2 = 1} \ \sup_{0 \leq \ h \leq \delta} \ \sup_{0 \leq \ t \leq 1-hm} |\sum_{q=0}^{m} (-1)^q C_m^q (Af)(t+hq)|.$$

The formula (1) yields the following proposition. Let the domain of values of the operator A belong to a space of continuous functions, and $m = 2, 3, \ldots$

$$s_r(A) \leq \left(\frac{10m}{r}\right)^{\frac{1}{2}} \omega_m \left(\frac{4}{r}, a\right), \quad r = m+1, m+2, \ldots$$

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The Carathédoroy-Fejér extremal problem on LCA groups

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Given a locally compact Abelian group G, a point $z \in G$, a zero-symmetric open set $\Omega \ni z$, we investigate the following extremal problem.

$$\mathcal{C}_G(\Omega, z) := \sup\{|f(z)| \ : \ f(0) = 1, \ \operatorname{supp} f \Subset \Omega, \ f \text{ positive definite}\}.$$

This extremal problem was investigated in $\mathbb{R}$ and $\mathbb{R}^d$ and for Ω a 0-symmetric convex body in a paper of Boas and Kac in 1944. Arestov and Berdysheva extended the investigation to $\mathbb{T}^d$, where $\mathbb{T} := \mathbb{R}/\mathbb{Z}$. Kolountzakis and Révész gave a more general setting, considering arbitrary open sets, in all the classical groups above. Also they observed, that such extremal problems occurred in certain special cases and in a different, but equivalent formulation already a century ago in the work of Carathéodory and Fejér.

Moreover, following observations of Boas and Kac, Kolountzakis and Révész showed how the general problem can be reduced to equivalent discrete problems of “Carathéodory-Fejér type” on $\mathbb{Z}$ or $\mathbb{Z}_m := \mathbb{Z}/m\mathbb{Z}$.

We extend the results of Kolountzakis and Révész to locally compact Abelian groups.

On new families of radial basis functions[7]

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In this talk, we present two new families of conditionally positive definite radial basis functions. Denote as $\mathcal{R}_m^d$ the set of all conditionally positive definite radial basis functions of order m on $\mathbb{R}^d$ and specify $\mathcal{R}_m^\infty = \bigcap_{d=1}^\infty \mathcal{R}_m^d$. Functions from $\mathcal{R}_m^\infty$ are strongly connected with completely monotonic functions ($f \in C^\infty(0, \infty)$ is called the completely monotonic function if $(-1)^k f^{(k)} \geq 0$ for every $k \in \mathbb{Z}_+$).

We denote as $\mathcal{M}_m$ the set of functions $f \in C^\infty(0, \infty)$ with completely monotonic mth derivative $(-1)^m f^{(m)}$. It is well known that if a function $f \in \mathcal{M}_m$ is bounded at zero and $f^{(m)}$ is not a constant, the function $f(r^2)$ belongs to $\mathcal{R}_m^\infty$. Conversely,

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$\phi \in \mathcal{R}_m^\infty \Rightarrow \phi(\sqrt{\cdot}\,) \in \mathcal{M}_m$. It is also clear that if $f \in \mathcal{M}_m$ is unbounded at zero and $f^{(m)}$ is not a constant, $f(r^2+c^2)$, $c \neq 0$, belongs to $\mathcal{R}_m^\infty$.

We call a function $f \in \mathcal{M}_m$ to be *a generic function* for $\phi \in \mathcal{R}_m^\infty$ if $\phi(r) = f(a^2(r^2+c^2)) + p_{m-1}(r^2)$ for given $a \in \mathbb{R} \setminus \{0\}$, $c \in \mathbb{R}$, and p_{m-1} be a polynomial of a degree less than m. The following family of functions is generic for many well-known radial basis functions:

$$f_\nu(t) = \begin{cases} \Gamma(-\nu)t^\nu, & \nu \in \mathbb{R} \setminus \mathbb{Z}_+, \\ (-1)^{\nu+1}t^\nu \ln t, & \nu \in \mathbb{Z}_+. \end{cases}$$

For example, f_ν, $\nu \in \mathbb{R}\setminus\mathbb{Z}_+$, is the generic function for multiquadric $(-1)^{\lfloor\nu\rfloor+1}(r^2+c^2)^\nu$, $\nu > 0$, and unverse multiquadric $(r^2+c^2)^\nu$, $\nu < 0$, and f_1 is the generic function for radial basis function $r^2 \ln r$ of the thin-plate spline. It is clear that $f_\nu \in \mathcal{M}_{(\lfloor\nu\rfloor+1)_+}$, where $(k)_+ = \max\{k, 0\}$.

The first new family of generic functions generalizes the constructions of tension spline and regularized spline from L. Mitáš and H. Mitášová, *General variational approach to the interpolation problem*, in Comput. Math. Appl., 16/12 (1988), 983–992. Denote $h_\nu(t) = t^{\nu/2}K_\nu(\sqrt{t})$,

$$\tilde{h}_\nu(t) = \begin{cases} \Gamma(-\nu)t^\nu/2^{\nu+1}, & \nu \in \mathbb{R} \setminus \mathbb{Z}_+, \\ (-1)^{\nu+1}t^n[\ln(t/4) - \psi(1) - \psi(n+1)]/(n!\,2^{n+1}), & \nu \in \mathbb{Z}_+. \end{cases}$$

Here K_ν is the modified Bessel function of the second kind of order $\nu \in \mathbb{R}$, $\psi(n) = -\gamma + \sum_{k=1}^{n-1} k^{-1}$, γ is the Euler constant, and $\Gamma(x)$ is the Gamma-function. We prove that $h_\nu \in \mathcal{M}_0$ for every $\nu \in \mathbb{R}$. We introduce a new two-parametric family of functions $h_{\nu,n}$, $\nu \in \mathbb{R}$, $n \in \mathbb{Z}_+$:

$$h_{\nu,0}(t) = \tilde{h}_\nu(t) - h_\nu(t); \qquad h_{\nu,n}(t) = \frac{\tilde{h}_{\nu+n}(t)}{n!\,2^n} - h_{\nu,n-1}(t), \quad n = 1, 2, \ldots$$

and prove that $h_{\nu,n} \in \mathcal{M}_{(\lfloor\nu\rfloor+n+1)_+}$ and $h_{\nu,n}$ is bounded at zero for $\nu + n + 1 > 0$.

The second new family of generic functions generalizes the construction of completely regularized spline from H. Mitášová and L. Mitáš, *Interpolation by regularized splines with tension: I. Theory and implementation*, in Mathematical Geology, 25/6 (1993), 641–655. Denote

$$g_\nu(t) = t^\nu\Gamma(-\nu, t), \qquad \tilde{g}_\nu(t) = \begin{cases} \Gamma(-\nu)t^\nu, & \nu \in \mathbb{R} \setminus \mathbb{Z}_+, \\ (-1)^{\nu+1}t^\nu[\ln t - \psi(\nu+1)]/\nu!, & \nu \in \mathbb{Z}_+. \end{cases}$$

Here $\Gamma(a,t) = \int_t^\infty e^{-x}x^{a-1}\,dx$ is the incomplete Gamma-function. It is well-known that $g_\nu \in \mathcal{M}_0$ for every $\nu \in \mathbb{R}$. We introduce a new two-parametric family of functions $g_{\nu,n}$, $\nu \in \mathbb{R}$, $n \in \mathbb{Z}_+$:

$$g_{\nu,0}(t) = \tilde{g}_\nu(t) - g_\nu(t); \qquad g_{\nu,n}(t) = g_{\nu+1,n-1}(t) - g_{\nu,n-1}(t), \quad n = 1, 2, \ldots$$

and prove that $g_{\nu,n} \in \mathcal{M}_{(\lfloor\nu\rfloor+n+1)_+}$ and $g_{\nu,n}$ is bounded at zero for all $\nu \in \mathbb{R}$, $n \in \mathbb{Z}_+$.

Chebyshev-Grüss-type inequalities -a new approach

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The classical form of Grüss' inequality gives an estimate of the difference between the integral of the product and the product of the integrals of two functions in $C[a, b]$.

The aim of this talk is to introduce Chebyshev-Grüss-type inequalities via discrete oscillations. We apply these inequalities to well-known positive linear operators and compare the new results with some old ones. Some examples are also presented, in order to underline the advantages of the new approach.

Minimal energy and maximal polarization

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While this could be a lecture about the U.S. Congress, it instead deals with problems that are asymptotically related to best-packing and best-covering. In particular, we discuss how to efficiently generate N points on a d-dimensional manifold that have the desirable qualities of well-separation and optimal order covering radius, while asymptotically having a prescribed distribution. Even for certain small numbers of points like N=5, optimal arrangements with regard to energy and polarization can be a challenging problem.

A fast algorithm for computing a truncated orthonormal basis for RBF native spaces

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In the setting of radial basis functions approximation it is well known that several problems arise when dealing with large amount of (scattered) data or when the shape parameter is not properly chosen. We propose a new way to partially overcome these difficulties by treating the kernel matrix with an approach based on the *GMRES algorithm*. Altough this method is yet widely studied for similar problems (see [1]), we consider some new results from *Krylov-based algorithms* (see [3]) which allows to approximate in a fast way the relevant segment of the spectrum of the kernel matrix. On the *native space* side this allows to compute a short sequence of orthonormal functions which is a proper truncation of an orthonormal basis (see [2]). In this way we can solve effectively the approximation problem by means of a least-squares approximants.

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A new representation theorem for Bernstein functions with applications to convexity and characteristic functions

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Bernstein functions have been the subject of investigation for decades. A function $f : [0, \infty) \to [0, \infty)$ is called Bernstein if it is infinitely times differentiable and $(-1)^{k-1} f^{(k)}(x) \geq 0$ for all $k = 1, 2, \ldots$ These functions find numerous application in areas such as Lévy processes, potential theory, and monotone operators, among

others. In this talk we show that for every $\alpha \in (0, 2/3)$ there is a unique measure μ_α on $(0, \infty)$ and constants $a, b \geq 0$ such that

$$f(x) = a + bx^{1/\alpha} + \int_{(0,\infty)} (1 - e^{-tx^{1/\alpha}})\mu_\alpha(dt).$$

Moreover, the measure μ_α has *harmonically concave tail*, that is, the function $t \mapsto t\mu_\alpha(t, \infty)$ is concave on $(0, \infty)$.

Applications range from convexity results to results about characteristic functions of random variables.

This is a joint work with Shen Shan.

Approximation numbers of Sobolev embeddings

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First we investigate optimal linear approximations (approximation numbers) in the context of isotropic periodic Sobolev spaces $H^s(\mathbb{T}^d)$ of fractional smoothness $s > 0$ for various equivalent norms including the natural one. The error is always measured in $L_2(\mathbb{T}^d)$. Particular emphasize is given to the dependence of all constants on the dimension d. We capture the exact decay rate in n and the exact decay order of the constants with respect to d, which is in fact polynomial.
Secondly, we consider the approximation numbers with respect to the pairs $H^s_{mix}(\mathbb{T}^d)$ and $L_2(\mathbb{T}^d)$. Here $H^s_{mix}(\mathbb{T}^d)$ denotes the Sobolev space of dominating mixed smoothness of order s. Again we are interested in the dependence of these numbers on n and d.
This is joined work with Thomas Kühn (Leipzig) and Tino Ullrich (Bonn).

Convergence questions of sequences of two-dimensional (C,α) means of 2-adic Fourier series of integrable functions

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The talk investigates convergence questions on a special locally compact totally disconnected non-Archimedean normed field: on the 2-adic field. The basics are carried out in terms of UDMD product systems. The concepts, notations and propositions of Schipp and Wade[3] and Scipp-Wade-Simon [2], Hewitt-Ross [1] and Zygmund [4] are considered.

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A sharp Markov–Nikolski type inequality in the spaces L_∞ and L_1 with Chebyshev weight

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The inequality between the uniform norm of the derivative of an algebraic polynomial of degree $n \geq 2$ and the L_1-norm with Chebyshev weight $(1-x^2)^{-1/2}$ of the polynomial itself on the closed interval $[-1, 1]$ is studied. For all derivatives, the exact constant and the extremal polynomial are written out.

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Approximation by band-limited scaling and wavelet expansions

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The well-known sampling theorem (which is often called Kotel'nikov or Shannon theorem) states that

$$f(x) = \sum_{n\in\mathbb{Z}} f(2^{-j}n)\,\frac{\sin\pi(2^j x - n)}{\pi(2^j x - n)} \tag{1}$$

for any function $f \in L_2(\mathbb{R})$ whose Fourier transform is supported on $[-2^{j-1}, 2^{j-1}]$. This formula is very useful for engineers. It was just Kotel'nikov and Shannon who started to apply the formula for signal processing, respectively in 1933 and 1949. Up to now, an overwhelming diversity of digital signal processing applications and devices are based on it and more than successfully use it. Without sampling theorem it would be impossible to make use of internet, make photos and videos.

From the point of view of wavelet theory, (1) is not a theorem, it is just an illustration for the Shannon MRA. Indeed, the function $\phi(x) = \frac{\sin\pi x}{\pi x}$ is a scaling function for this MRA, and a function f belongs to the sample space V_j if and only if its Fourier transform is supported on $[-2^{j-1}, 2^{j-1}]$. So, such a function f can be expanded as $f = \sum_{n\in\mathbb{Z}}\langle f, \phi_{jn}\rangle\phi_{jn}$, where $\phi_{jn}(x) = 2^{j/2}\phi(2^j x + n)$, which coincides with (1). Also, since $\{V_j\}_{j\in\mathbb{Z}}$ is an MRA, any $f \in L_2(\mathbb{R})$ can be represented as

$$f = \lim_{j\to+\infty}\sum_{n\in\mathbb{Z}}\langle f, \phi_{jn}\rangle\phi_{jn}. \tag{2}$$

Moreover, (2) has an arbitrary large approximation order. This happens because the function $\phi(x) = \frac{\sin\pi x}{\pi x}$ is band-limited, a similar property cannot be valid for other natural classes of ϕ, in particular, for compactly supported ϕ.

We study operators $Q_j f = \sum_{n\in\mathbb{Z}}\langle f, \widetilde{\phi}_{jn}\rangle\phi_{jn}$ for a class of band-limited functions ϕ and a wide class of tempered distributions $\widetilde{\phi}$. Convergence of $Q_j f$ to f as $j\to+\infty$ in L_2-norm is proved under a very mild assumption on ϕ, $\widetilde{\phi}$, and the rate of convergence is equal to the order of Strang-Fix condition for ϕ. To study convergence in L_p, $p > 1$, we assume that there exists $\delta \in (0, 1/2)$ such that $\widehat{\widetilde{\phi}}\widehat{\phi} = 1$ a. e. on $[-\delta, \delta]$, $\widehat{\phi} = 0$ a.e. on $[l-\delta, l+\delta]$ for all $l \in \mathbb{Z}\setminus\{0\}$. For appropriate band-limited or compactly supported functions $\widetilde{\phi}$, the estimate $\|f - Q_j f\|_p \le C\omega_r(f, 2^{-j})_{L_p}$, where ω_r denotes the r-th modulus of continuity, is obtained for arbitrary $r \in \mathbb{N}$. For tempered distributions $\widetilde{\phi}$, we prove that $Q_j f$ tends to f, $f \in S$, in L_p-norm, $p \ge 2$, with an arbitrary large approximation order. In particular, for some class of differential operators L, we consider $\widetilde{\phi}$ such that $Q_j f = \sum_{n\in\mathbb{Z}} Lf(2^{-j}\cdot)(n)\phi_{jn}$. The corresponding wavelet frame-type expansions are found, for example, the following

$$f(x) = \sum_{k\in \mathbb{Z}} f(-k)\phi(x+k) + \sum_{j=0}^{\infty}\sum_{k\in \mathbb{Z}} f(-2^{-j-1}(2k+1))\psi^{(1)}(2^j x+k) + f(-2^{-j-1}(2k))\psi^{(2)}(2^j x+k),$$

where $\psi^{(1)}(x) = \sqrt{2}\phi(2x+1)$, $\psi^{(2)}(x) = \sqrt{2}(\phi(2x) - \phi(x))$.

Replacing $\langle f, \widetilde{\phi}_{jk}\rangle$ by $\langle \widehat{f}, \widehat{\widetilde{\phi}_{jk}}\rangle$ for some $\widetilde{\phi}$, we extend these results for an essentially lager class of functions f.

On the class of operators U_n^{ϱ} linking the Bernstein and the genuine Bernstein-Durrmeyer operators

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We consider the class of operators U_n^{ϱ} introduced and investigated by Păltănea and Gonska and study further properties, such as variation diminuition and global smoothness preservation. We also consider the difference of such operators and the commutators. The results we provide here come as a natural extension of the known results for both the Bernstein and the genuine Bernstein-Durrmeyer operators.

Bernstein polynomials for arbitrary points of interpolation

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Following a 1939 article of Favard we consider the composition of classical Bernstein operators and piecewise linear interpolation at mutually distinct knots in $[0,1]$, not necessarily equidistant. We prove direct theorems in terms of the classical and the Ditzian-Totik modulus of second order. This is a joint work of H. Gonska and G. Tachev.

Mathematics Subject Classification: 41A10, 41A15, 41A17, 41A25, 41A36.

Convergence in L^p-norm of Fourier series on the complete product of quaternion groups with bounded orders

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In Fourier analysis several properties and results differ considerably if they are defined on non-abelian groups. For instance, Walsh and Vilenkin-systems have only functions with module 1, but a representative product system on the complete product of finite non-abelian groups is not necessarily uniformly bounded and it takes the value 0. These systems were introduced in [1].

A well known result for Vilenkin systems is the fact that for all $1 < p < \infty$ the n-th partial sums of Fourier series of all functions in the space L^p converge to the function in L^p-norm. This statement is known as Paley's theorem and it is not true either for non-abelian groups in general. Indeed, in [2] the author proved that there are representative product systems such that Paley's theorem is only valid for $p = 2$. Until this moment we did not know products of non-abelian groups where Paley's theorem is true.

Now we have obtained the first positive examples on the complete product of quaternion groups. In my talk I deal with properties and arrangements concerning generalized quaternion groups in order to obtain Paley's theorem for all $1 < p < \infty$.

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Interpolation and fitting by harmonic polynomials based on Radon projections

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Given the line integrals of a harmonic function on a finite set of chords of the unit circle, we consider the problem of fitting these Radon projections type of data by a harmonic polynomial in the unit disk. In particular, we focus on the overdetermined case where the amount of given data is greater than the dimension of the polynomial space. We prove sufficient conditions for existence and uniqueness of a harmonic polynomial fitting the data by using least squares method. Combining with recent results on interpolation with harmonic polynomials, we obtain an algorithm of practical application. We extend our results to fitting of more general mixed data consisting of both Radon projections and function values. Numerical results are presented and discussed.

Keywords and Phrases: multivariate interpolation, Radon transform, harmonic polynomials, least-squares fitting.

Least-squares fitting with Birkhoff interpolation at the vertices of triangulations using smooth GERBS

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In the present study we address one of the posible applications of a recently introduced in [1, 2] construction of smooth convex resolutions of unity on triangulated polygonal domains. Important features of the B-spline-type basis functions generated via this construction are that there is a 1-1 correspondence between them and the vertices in the triangulation, each one of these basis functions is supported at the $\star_1$-neighbourhood of its respective vertex where it peaks with value 1, these basis functions are sufficiently smooth on the whole triangulated domain (C^∞-smooth or C^m-smooth, $m = 0, 1, 2, \ldots$, depending on the context); moreover, all of their existing partial derivatives vanish at all knots, hence, multiplying these basis functions with local Taylor polynomials of total degree $k \leq m$ 'centered' at their respective vertex has the effect of Hermite interpolation of total order k at this vertex (which, in the C^∞-smooth case, can be transfinite, i.e., the Taylor polynomial can be replaced by a Taylor series). Using these properties, we show that it is possible to incorporate the problem about Birkhoff interpolation at the vertices of the triangulation within the more general problem setting of finding the optimal solution of a linear-quadratic optimization problem for least-squares fitting with linear constraints of equality type. One advantage of this setting is that the matrix of the resulting linear system of equations (part of the unknowns of which are partial derivatives at the vertices which are not included in the Birkhoff interpolation conditions, and the remaining unknowns are Lagrange multipliers) has the structure of a bordered Hessian, for which there is a necessary and sufficient condition for its non-singularity. This criterion, which is a generalization of Sylvester's necessary and sufficient criterion for existence and uniqueness of the optimal solution of an unconstrained least-squares problem, is particularly efficient to verify when the number of unknown partial derivatives is small compared to the number of partial derivatives determined by the Birkhoff interpolation constraints, i.e., when these constraints are tight. We shall illustrate how this approach works on some examples, one of which will be Hermite interpolation. Although this approach was specifically designed to handle fitting and interpolation on scattered-point sets in the multivariate case, it is of interest in the univariate case, too.

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On approximation by algebraic version of the trigonometric Jackson integrals $G_{S,N}$ in weighted integral metric

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We characterize the errors of the algebraic version of trigonometric Jackson integrals $G_{s,n}$ in weighted integral metric. We prove direct and strong converse theorem in terms of the weighted K-functional.

Mathematics Subject Classification (2010): 41A36, 41A25, 41A17.

Key words and phrases: Linear operator, Direct theorem, Strong converse theorem, K-functional.

Entire functions in the Laguerre–Pólya class

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We shall discuss some topics on the Laguerre-Pólya class of entire functions, denoted by $\mathcal{LP}$. It consists of entire functions which are limits, in the sense of local uniform convergence, of polynomials with only real zeros. An old conjecture of Pólya states that the real line attracts the zeros of the entire functions of order less than two under differentiation. Though the conjecture was settled by Craven, Csordas and Smith, further results indicate that once such a function is differentiated, the zeros not only approach the real line but also become equally spaced.

Interesting and important functions from $\mathcal{LP}$ are those represented as Fourier transforms of certain positive and symmetric measures. The problem to characterize the latter subclass was posed by George Pólya in 1926 who was motivated by his efforts to settle the Riemann hypothesis. Essentially the same question arose in Statistical Mechanics and it is related to the celebrated Lee–Yang circle theorem, for which Lee and Yang were awarded the 1957 Nobel Prize in physics. We report a result which provides a characterization of the so-called Lee–Yang measures and a solution of Pólya's problem too, in terms of the polynomials, orthogonal with respect to the measure.