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ИЗДАТЕЛСТВО

Съдържание | Content

Convergence Analysis of the Boundary Integral Equation Method for Cracked Functionally Graded Magnetoelectroelastic Media

Yonko D. Stoynov[#1], Tsviatko Rangelov[*2], Petia Dineva[†3]

[#]*Faculty of Applied Mathematics and Informatics, Technical University-Sofia, St. Kliment Ohridski Blvd. № 8, 1000 Sofia, Bulgaria*

[*]*Institute of Mathematics and Informatics, Bulgarian Academy of Sciences, Acad. G. Bonchev St., Block 8, 1113 Sofia, Bulgaria*

[†]*Institute of Mechanics, Bulgarian Academy of Sciences Acad. G. Bontchev St., Block 4, 1113 Sofia, Bulgaria*

[1]ids@tu-sofia.bg, [2]rangelov@math.bas.bg, [3]petia@imbm.bas.bg

Abstract. In this paper a cracked functionally graded magnetoelectroelastic plane, subjected to dynamic time-harmonic anti-plane mechanical and in-plane electric and magnetic load is considered. Boundary Integral Equation Method (BIEM) is developed, validated and applied to evaluate the dynamic stress concentration near the crack tips. A program code in FORTRAN is created. Validation study based on solutions for different number and size of the boundary elements analyzes the convergence of the numerical scheme. A comparison between obtained solutions and results for anisotropic and piezoelectric materials is presented. The BIEM developed tool has application in fracture mechanics and nondestructive evaluation of new multifunctional composite materials used in smart structures technology.

Keywords: magnetoelectroelastic medium, anti-plane crack, boundary integral equation method, SIF.

I. Introduction

Magnetoelectroelastic (MEE) composite materials are increasingly used in modern smart structures as electromagnetic transdusers, ultrasonic/acoustic devices, sensors, hydrophones, etc. Their magnetoelectric property is very large due to the coupling effect of the piezoelectric and piezomagnetic phase. It can be even a hundred times larger than in a single-phase magnetoelectric material.

The composites are highly sensitive to existence of defects like cracks, voids, etc. During service the cracks can reach critical size and thus compromise the structure integrity and/or functional properties of these media. To enhance the promising applications it is very important to study fracture problems in MEE solids. In the recent years dynamic behaviour of cracked MEE has been intensively studied [1-7].

The aim of this work is to investigate the accuracy and convergence of the non-hypersingular traction BIEM for solution of dynamic anti-plane cracked problems of MEE media, basing on comparison with results obtained by other computational techniques for elastic anisotropic, linear piezoelectric and MEE materials.

II. Statement of the Problem

Let's consider an infinite transversely isotropic functionally graded MEE medium with an axis of symmetry along Ox_3 direction of a rectangular coordinate system $Ox_1x_2x_3$. The medium is subjected to an external antiplane mechanical, and inplane electrical and magnetic time-harmonic load. We assume that electric and magnetic fields are potential and the problem is two dimensional – the material properties are the same in all planes perpendicular to the axis of symmetry.

We introduce a generalized tensor of elasticity $C_{iJKl}(x), i,l=1,2;\ J,K=3,4,5$ in the following way:

$$C_{i33l}(x)=\begin{cases}c_{44}(x), i=l\\ 0, i\neq l\end{cases},$$

$$C_{i34l}(x)=C_{i43l}(x)=\begin{cases}e_{15}(x), i=l\\ 0, i\neq l\end{cases},$$

$$C_{i35l}(x)=C_{i53l}(x)=\begin{cases}q_{15}(x), i=l\\ 0, i\neq l\end{cases},$$

$$C_{i44l}(x)=\begin{cases}-\varepsilon_{11}(x), i=l\\ 0, i\neq l\end{cases},$$

$$C_{i45l}(x)=C_{i54l}(x)=\begin{cases}-d_{11}(x), i=l\\ 0, i\neq l\end{cases},$$

$$C_{i55l}(x)=\begin{cases}-\mu_{11}(x), i=l\\ 0, i\neq l\end{cases},$$

where c_{44} is the elastic module, e_{15} is the piezoelectric coefficient, q_{15} is the piezomagnetic coefficient, ε_{11} is the dielectric permittivity, μ_{11} is the magnetic permeability, d_{11} is the magnetoelectric coefficient. Using generalized displacement vector $u_J=(u_3,\varphi,\psi)$ and generalized stress tensor $\sigma_{iJ}=(\sigma_{i3},D_i,B_i)$, where u_3 is the component of the displacement vector along Ox_3, φ and ψ are electric and magnetic potential respectively, σ_{i3} is the mechanical stress, D_i and B_i are the components of the electric displacement and magnetic induction respectively, the constitutive equations for this type of medium can be written in the following way:

$$\sigma_{iJ}=C_{iJKl}u_{K,l} \qquad (1)$$

Here comma means differentiation and summation under repeated indexes is assumed.

The governing equations have the form

$$\sigma_{iJ,i}+\rho_{JK}\omega^2 u_K=0 \qquad (2)$$

In (2) ω is the frequency of the applied time-harmonic load and $\rho_{JK}(x)=\begin{cases}\rho(x), J=K=3\\ 0, J,K=4\ or\ 5\end{cases}$, where ρ is the density. The common multiplier $e^{i\omega t}$ is suppressed, but understood. We suppose that the material properties depend in one and the same manner on $x=(x_1,x_2)$: $C_{iJKl}(x)=C_{iJKl}h(x)$ and $\rho(x)=\rho h(x)$, where the inhomogeneity function $h(x)$ is exponential: $h(x)=e^{2<a,x>}$, $a=(a_1,a_2)$, $<.,.>$ is the scalar product. The boundary conditions for (2) are

$$t_J\Big|_{\Gamma}=0 \qquad (3)$$

where t_J is the total generalized traction defined as $t_J=\sigma_{iJ}n_i$, $n=(n_1,n_2)$ is the normal vector to the crack. $\Gamma=\Gamma^+\cup\Gamma^-$ is the section of the crack and the plane Ox_1x_2, Γ^+ and Γ^- are the upper and lower bound of the crack. The total displacement and traction in any point of the plane can be found using the superposition principle: $u_J=u_J^{in}+u_J^{sc}$ and $t_J=t_J^{in}+t_J^{sc}$. Here u_J^{in} and t_J^{in} are the displacement and traction of the incident wave field and u_J^{sc} and t_J^{sc} are the displacement and traction of the scattered by the crack wave field.

We will solve the boundary value problem (2) and (3) transforming it into an equivalent integro – differential system of equations on the crack Γ and then solve this system numerically.

III. BIEM

Following [8] for the piezoelectric case and [9] for the magnetoelectroelastic homogeneous case we obtain the following boundary integral equation describes the problem:

$$t_J^{in} = -C_{iJKl}(x)n_i(x)\int_{\Gamma^+}[(\sigma^*_{\eta PK}(x,y,\omega)\Delta u_{P,\eta}(y,\omega) - \rho_{QP}\omega^2 u^*_{QK}(x,y,\omega)\Delta u_P(y,\omega))\delta_{\lambda l} - \sigma^*_{\lambda PK}(x,y,\omega)\Delta u_{P,l}(y,\omega)]n_\lambda(y)d\Gamma(y) \qquad (4)$$

u^*_{QK} and σ^*_{iPK} are the fundamental solutions and the respective stress, $t_J^{in}(x,\omega)$ is the incident wave field and $\Delta u_J = u_J\Big|_{\Gamma^+} - u_J\Big|_{\Gamma^-}$ are the unknown crack opening displacements (COD). The fundamental solutions and the incident SH-wave can be found in [10]. We reduce (4) to a system of linear equations and solve it numerically. The traction field in every point $x \in R^2 \setminus \Gamma$ can be found by the corresponding representation formula see [9]. The stress concentration near crack tips is computed using the expression: $K_{III} = \lim_{x_1 \to \pm c} t_3\sqrt{2\pi(x_1 \mp c)}$, where c is the half-length of the crack.

IV. Numerical Studies

The aim of the numerical studies is to evaluate the key parameters controlling the accuracy and convergence of the BIEM for solution of the formulated here mechanical problem.

As far as discretization procedure is used at solution of the integro-differential equation in respect to the generalized crack opening displacement, the main factors for accurate solution is the type and density of the used BEM mesh. It is well known fact that the high computational accuracy of numerical discretization methods demands the satisfaction of the following condition: $\lambda / l > 10$, where λ is the wavelength and l is the BE length. This is the reason we present in Figs. 1-6 BIEM solutions for meshes with parabolic approximation of the field quantities where the number of the BE is 7, 11 and 15 respectively.

Fig. 1 shows normalized SIF versus normalized frequency $\Omega = c\sqrt{\frac{\rho^0\omega^2}{a_0} - |a|^2}$ for functionally graded elastic anisotropic material with the following characteristics: c_{44}=27.1 GPa and ρ=7.55×10^3 kg/m^3, inhomogeneity magnitude β=0.4 and $\alpha = \frac{\pi}{2}$. We compared our results with those of [11], who used BIEM with 5BE. The comparison demonstrates little difference between the authors' results and the results of [11] (not more than 5%) and convergence of the obtained solutions for 7, 11 and 15 BE.

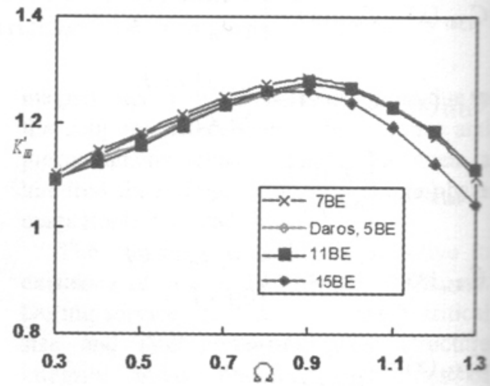

Fig.1 Comparison of the normalized SIF versus normalized frequency for functionally graded anisotropic elastic material.

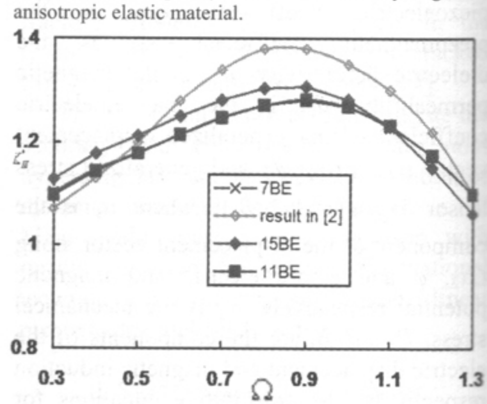

Fig. 2. Comparison of the normalized SIF versus normalized frequency for functionally graded piezoelectric material.

Comparison with the results of [12] for functionally graded piezoelectric material (PEM) PZT6B with inhomogeneity magnitude and direction like the previous ones is made in Fig. 2.

The relative difference is not more than 7% and the close results for 7, 11 and 15BE show the stability of the computational scheme.

In Fig. 3 we present a comparison with the results of [13], who used semi-analytical dual integral equation method. The material used is the homogeneous PEM $BaTiO_3$. We see that increasing of the number of the BE leads to closer results for higher frequencies.

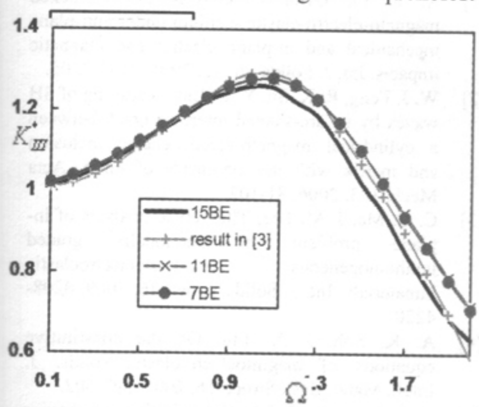

Fig. 3. Comparison of the normalized SIF versus normalized frequency for homogeneous piezoelectric material $BaTiO_3$ obtained in [3].

Numerical results for homogeneous MEE composite $BaTiO_3/CoFe_2O_4$, obtained by discretization mesh with different density is given in Fig. 4. The results converge and the numerical scheme is stable.

As far as we investigate the accuracy of the near-field quantity, i.e. SIFs, the next important for the accuracy factor is the length of the special crack-tip BE used to model the asymptotic behaviour of displacement as $\sqrt{r}$ as r tends to 0. Results obtained by numerical schemes with different size of the crack-tip element is shown in Fig. 5. The maximal percentage difference between the results is about 4%.

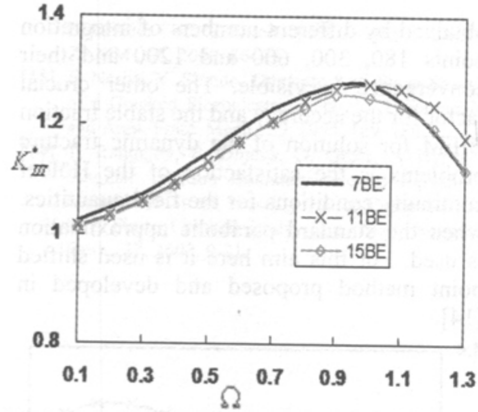

Fig. 4. Normalized SIF versus normalized frequency for homogeneous MEE composite $BaTiO_3/CoFe_2O_4$.

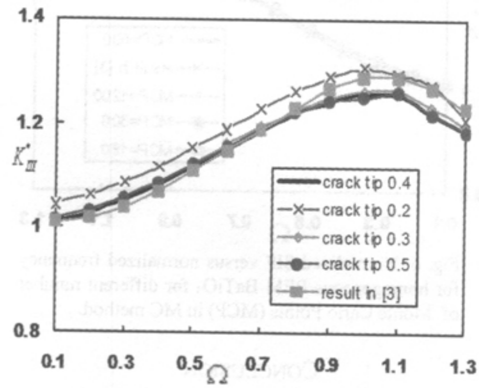

Fig. 5. Comparison of the normalized SIF versus normalized frequency for homogeneous PEM $BaTiO_3$, obtained with 15BE and different length of the crack-tip BE in mm.

The computational accuracy of the solutions for SIF depends strongly also on the accurate solution of all integrals in the BIE for GCOD. There are two types of integrals: regular and singular. Singular integrals are solved analytically based on the asymptotic expansions of the fundamental solutions as *Ln(r)* and its stress as (1/*r*) in the small neighbourhood around the singular point. The regular double integrals are computed by quasi Monte Carlo method (QMCM). The influence of the number of integration points used in the QMCM for solutions of the obtained regular integrals is illustrated in Fig.6. It is drawn results obtained by different numbers of integration points 180, 300, 600 and 1200 and their convergence is visible. The other crucial factor for the accuracy and the stable traction BIEM for solution of the dynamic fracture problems is the satisfaction of the Hölder continuity conditions for the field quantities, when the standard parabolic approximation is used. For this aim here it is used shifted point method proposed and developed in [14].

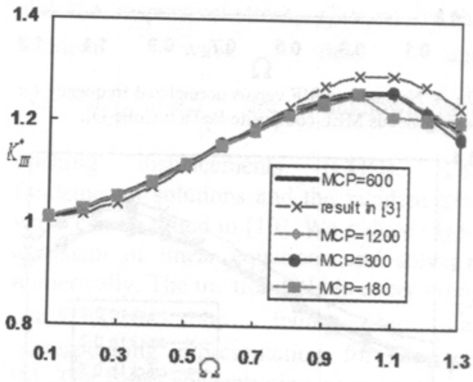

Fig. 6. Normalized SIF versus normalized frequency for homogeneous PEM $BaTiO_3$ for different number of Monte Carlo Points (MCP) in MC method.

Conclusion

The proposed non-hypersingular traction based BIEM for solution of plane dynamic problems of cracked MEE media is validated by comparison with results obtained by other computational tools. It is investigated the sensitivity of the BIEM solution to such factors as the density of the discretization mesh, the size of the special crack-tip BE, the accurate solution of the integrals.

The validated numerical scheme and the accompanied software based on it can be used in the research and applied engineering fields as: fracture mechanics of multifunctional materials, dynamics of structures made by this materials and in the structural health monitoring of the modern intelligent systems and smart structures.

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