Report on the dissertation of Danila Cherkashin "Extremal problems in the Euclidean combinatorial geometry" for the scientific degree "Doctor of Sciences" in Area of higher education: 4. "Natural Sciences, mathematics and informatics" Professional area: 4.5 "Mathematics"

Eugene Stepanov

25 April 2024

1 Scientific data of the reviewer

Eugene Stepanov is a professor of Mathematics at Università di Pisa, Pisa, Italy.

He obtained PhD degree at Scuola Normale Superiore di Pisa, Pisa, Italy in 1999 and D. Sc. degree in Mathematics at the Institute for Information Transmission Problems of Russian Academy of Sciences, Moscow, Russia in 2006.

Eugene Stepanov has written 2 research monographs and more than 50 research papers, most of them published in top journals in mathematics, namely geometry, functional analysis and calculus of variations, namely, Journal of Functional Analysis, Journal of Differential Geometry, Calculus of Variations and Partial Differential Equations, The Journal of Geometric Analysis and others.

2 Scientific biography of the applicant

Danila Cherkashin holds an associate professor position at Institute of Mathematics and Informatics, Bulgarian Academy of Sciences, Sofia, Bulgaria. Before he worked in Saint-Petersburg University, Moscow Institute of Physics and Technology and St. Petersburg Branch of the Higher School of Economics. In particular he was a postdoc under "Gazprom Neft" prize program at Chebyshev Laboratory, Saint-Petersburg University, 2021–2022.

The applicant got his PhD at St. Petersburg Department of Steklov Mathematical Institute of the Russian Academy of Sciences under the supervision of F. V. Petrov and A. M. Raigorodskii in 2018.

Danila Cherkashin is the winner of the annual "Young Mathematician" prize of the St. Petersburg Mathematical Society, 2022.

3 Analysis of the applicants publications

The thesis is based on 8 publications and 2 preprints. Main results are published in respectable international journals like International Mathematics Research Notices, Discrete & Computational Geometry, ESAIM: Control, Optimisation and Calculus of Variations.

The materials presented by the candidate do not intersect those from previous procedures. No plagiarism was found in the thesis and in the related publications.

In total, the applicant has 23 publications according to Scopus database.

4 Scientific contribution

The dissertation deals with several shape optimization problems closely related to the famous Steiner problem of finding a connection (a connected embedded graph in the Euclidean plane) of minimal lengths for a given set of terminals, as well as to some graph theoretic problems of estimating chromatic numbers of some graphs. Both are intimately related to combinatorial geometry in the Euclidean space. These problems are subject of active studies nowadays, some already having a rather long

history (like the Steiner problem itself dating back to 1811), are very difficult and usually require inventing completely new techniques for their solution rather than developing the existing ones. This is probably the reason why some of these problems although seemingly having a taste of just beautiful studies end in itself, in fact, quite surprisingly influence a lot of important and quite distant areas of mathematics and computer science.

What has been said above can be applied to the dissertation under review. It solves some of the difficult known open problems, but, what is more important, does so by introducing quite new ideas and techniques with the potential to influence not only the subject but also quite distant studies. All the results presented in the dissertation seem very interesting and beautiful to me, and therefore I would limit myself to listing the most impressive ones (in my opinion). Those are:

- Theorem 6.1.2 solving Simmons' conjecture on existence of monochromatic points at distance 1 apart in the coloring of the 2-dimensional sphere of diameter at least 1 in 3 colors. This seems to be surprisingly similar to the recent generalizations of the Borsuk–Ulam theorem, but strikingly is not at all of topological nature.

- Theorem 4.3.2 solving (formally only partially, but the solution is still quite important) the horseshoe conjecture for the maximum distance minimization problem. This is one of the few nontrivial examples of an explicit solution for a shape optimization problem. Though being quite natural, its is quite hard to prove (which is more or less a rule in shape optimization, starting from the minimal surface problem), and in this case the proof introduced a quite new technique in the subject.

- Theorem 2.2.1 providing the first known explicit example of a "fractal Steiner tree", a solution to a Steiner problem for an uncountable and even fractal set of terminals. Again, the construction itself is quite natural, but proving it is in fact a solution to the Steiner problem, was open for more than 10 years and several strong attempts have been made to tackle it, each introducing quite new techniques of undoubtful interest in itself.

I would say that even the above mentioned results (to say nothing about the rest of the dissertation, which of course also contains nice and beautiful results) would be enough for a strong D. Sc. thesis in any University in the world.

5 Conclusion

Based on the above discussion, I recommend the scientific Jury to award Danila Cherkashin the scientific degree "Doctor of Science" in Area of higher education: 4. "Natural Sciences, mathematics and informatics" Professional area: 4.5 "Mathematics"

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