Short report

for the award of the title Doctor of Sciences at the Institute of Mathematics and Informatics, Bulgarian Academy of Sciences in the area of science 4. Natural sciences, mathematics and informatics, professional field 4.5 Mathematics, prepared by Prof. Dr. Sc. Mladen Svetoslavov Savov, "Probability, Operations Research and Statistics", Faculty of Mathematics and Informatics, Sofia University "St. Kliment Ohridski" and "Operations research, Probability and Statistics", Institute of Mathematics and Informatics, Bulgarian Academy of Sciences, appointed member of the scientific panel by order No. 68/25.03.2024 issued by the director of the Institute of Mathematics and Informatics, Bulgarian Academy of Sciences upon a decision of the Scientific council of Institute of Mathematics and Informatics, Bulgarian Academy of Sciences contained in its protocol No. 3/22.03.2024, and writing a short report according to the decision taken by the scientific panel during its first meeting on 27.03.2024.

This short report was prepared in compliance with order No. 68/25.03.2024 by the director of the Institute of Mathematics and Informatics, Bulgarian Academy of Sciences issued on the basis of the decision by the Scientific council of the Institute of Mathematics and Informatics, Bulgarian Academy of Sciences contained in Protocol 3/22.03.2024 and in accordance with the requirements of the Institute of Mathematics and Informatics, Bulgarian Academy of Sciences for the award of the title Doctor of Sciences in mathematics which exceed in quantitative value the minimal national requirements.

As a member of the scientific jury I have received all the documents submitted by the candidate Danila Dmitrievich Cherkashin (Institute of Mathematics and Informatics, Bulgarian Academy of Sciences).

1. Biographical data about the candidate

Danila Cherkashin is born in 1992. In 2018 he defended his PhD thesis "Extremal problems in hypergraph colorings" under the supervision of A. Raigorodskii and F. Petrov. Danila Cherkashin has held the following research positions: from 2016 to 2021 at Moscow Institute of Physics and Technology (MIPT), from 2021 to 2023 at St. Petersburg Department of Steklov Mathematical Institute of the Russian Academy of Sciences and from 2015 to 2023 at the Saint-Petersburg University (SPbU). He joined the Institute of Mathematics and Informatics, Bulgarian Academy of Sciences in 2022 where he was elected as an associate professor in 2023. Danila Cherkashin is a very productive mathematician as evidenced by his list of publications.

2. Fulfillment of the minimal requirements

Assoc. Prof. Cherkashin has furnished 10 papers for the competition out of which: 6 are published in WoS and Scopus, 1 is indexed in zbMATH and 3 are submitted or accepted manuscripts. For this procedure the candidate has selected 24 citations in WoS and Scopus and they accumulate more than 100 points as needed. All minimal requirements in all categories are exceeded and all additional institutional requirements are met as well.

3. Scientific contributions of the thesis

The thesis is devoted to extremal problems at the intersection of combinatorics and Euclidean geometry. It consists of 7 chapters with original results of Danila Cherkashin presented in chapters 2-7 and chapter 1 is introductory to the topic. The results are based on 10 publications, 1 out of which is single authored. I would assume that the contribution to each paper by each author is equal. The authors have offered a number of directions for future work and particular open problems.

For the purpose of this short report I would go briefly through some of the main results in each chapter.

The main result of Chapter 2 is Theorem 2.1.1 which shows for the planar case of Steiner trees that the $n \geq 4$ point configurations in $\mathbb{R}^2$ with more than one Steiner tree have a Hausdorff dimension of $2n-1$ in $(\mathbb{R}^2)^n \setminus diag$. This result is state of the art in the literature and proves a conjecture by Edelsbrunner and Strelkova. The proof is technical and makes clever use of notions such as semianalytic and subanalytic sets and analytic manifolds which arise from the function evaluating the length of the embeddings of a topology of tree, and results available in the literature. I would like to point out the lack of more detailed proofs which would have made the reading easier for non-specialists.

The main result of Chapter 3 is Theorem 3.3.1 which for *admissible* cost functions proves that flows in $\mathbb{R}^2$ that minimize the Gilbert functional do not have branching points of degree higher than 3. The proof is very interesting and involves an embedding in a suitable Hilbert space whose norm depends on the cost function, say C. Then by considering sums of angles of configurations pertaining to one branching point one gets a contradiction, if one assumes more than 3 edges, via estimates and inequalities for the involved angles. As a probabilist I find it very intriguing that in fact the measure in 3.1 is precisely a Lévy measure and the cost functions are related to the exponent of the associated Lévy process which is a negative definite function itself.

Chapter 4 is the lengthiest of all and it contains solid results. Its main topic is about maximal distance minimizers and it offers a very detailed introduction and literature review. Parallels are made with average distance minimizers. Motivated by various applications the

research on the topic of Chapter 4 seems to be very active. The author himself has papers not included in this thesis and his co-authors also have various additional contributions. Sections 4.6 and 4.7 contain the bulk of the proofs and I shall briefly comment on their content. The first one considers minimizers for planar convex closed smooth curves with minimal radius of curvature exceeding the target distance r. A number of structural and technical lemmas describes the properties of the minimizers. The main theorem is Theorem 4.6.1 and it reveals the structure of the closure of the connected components of the minimizer which turns out to have at most three entering points. Its proof is rather technical and its results are a small portion of all obtained properties. Parts of this Section are not optimally written and it makes the reading for non-specialists challenging. Section 4.7 deals with the setting of Section 4.6 when $R > 5r$. Then the main theorem is Theorem 4.3.2 which describes completely in a neat way the shape of the minimizer which turns out to be the horseshoe. The proof is lengthy and relies on a sequence of previous results and geometrical arguments which require some time to follow. I think the whole of Chapter 4 is illustrative of the mathematical prowess of D. Cherkashin.

Chapter 5 deals with the independence number of Johnson-type graphs in some special cases. Theorems 5.1.7-8 provide precise formulae for the size of the maximal independent set of the aforementioned graphs when they are constructed by taking into account negative integer scalar products. Theorem 5.1.9 considers the case of scalar product of value 0. The remaining results deal with estimates of the chromatic number of those graphs. The proofs of these results are in the domain of graph theory with Lemma 5.2.1 being extensively used. One has to note the skillful way some of the computations have been done.

Chapter 6 contains the proof of a hypothesis formulated by Simmons which states that the chromatic number of a two-dimensional sphere with radius larger than 1/2 is at least 4. The proof consists of two steps. The first one establishes that if the chromatic number was 3 then all three colours are dense in the respective set. The second is to provide an embedding of a graph which if coloured by three colours would yield a contradiction. Chapter 7 studies the chromatic number of particular slices. The proofs are again technical and are an attestation of the strong mathematical abilities of the candidate.

4. Recommendation and overall evaluation

Overall, my opinion for the dissertation is positive. It contains a number of recent results some of which are published in respectable journals and are already appreciated by the mathematical community. Besides the chapters of the thesis are coherent in terms of topic and correspond to the title of the dissertation, and as such present a consistent body of work that a Dr. Sciences dissertation must be. Also, the candidate has a number of papers on the

topic that are not included in this procedure. This is a result of the activity of D. Cherkashin and the rich opportunities the area presents.

The thesis is written with quite some care regarding the illustrations and some of the explanations but the overall style is for specialists working predominantly in this area. The latter made it hard for me to follow at some places. In particular, Chapter 5 is the direct incorporation of published papers which brings in many typos and this style is not friendly to mathematicians that are not very close to the topic. The same is valid for other results of the thesis. Thus could be justified provided the abstract was more detailed but it is not and at places the dissertation is a collection of papers which is not the general purpose of the procedure. I assume that this is a result of the fact that D. Cherkashin became a mathematician in a different academic system and our procedure is not well-streamlined.

5. Conclusion

According to the applied documents the candidate Assoc. Prof. Danila Cherkashin satisfies all the minimal requirements set by the Act for the Development of the Academic Staff in the Republic of Bulgaria, the Regulations for its implementation and the Regulations of the Institute of Mathematics and Informatics, Bulgarian Academy of Sciences, stipulating the specific additional requirements for the award of scientific titles and academic positions. My professional opinion concerning his work is fully confirmed by the applied documents which clearly demonstrate that Assoc. Prof. Danila Cherkashin is a very good specialist in the research area of his dissertation.

Therefore, I give an overall positive evaluation of the dissertation and the work of Assoc. Prof. Danila Cherkashin and I **recommend** that the scientific panel awards Assoc. Prof. Danila Cherkashin the academic title Doctor of Sciences in the area of science 4. Natural sciences, mathematics and informatics, professional field 4.5 Mathematics.

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Prof. Dr.Sc. Mladen Savov

Sofia

26.04.2024